X – MATHS

Name :

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$EXERCISE\ 1.1$

Example 1.1 $\boldsymbol{If\ A = \{1,3,5\}\ and\ B = \{2,3\}\ then\ (i)\ A \times B\ and\ B \times A.}$
$\boldsymbol{(ii)\ Is\ A \times B = B \times A\ ?\ If\ not\ why?}$
$\boldsymbol{(iii)\ Show\ that\ n(A \times B) = n(B \times A) = n(A) \times n(B)}$

$Given\ that\ A = \{1,3,5\}\ and\ B = \{2,3\}$

$(i)\ A \times B = \{1,3,5\} \times \{2,3\}$

$A \times B = \{(1,2),(1,3),(3,2),(3,3),(5,2),(5,3)\}$

$B \times A = \{2,3\} \times \{1,3,5\}$

$B \times A = \{(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)\}$

$(ii)\ From\ (1) and\ (2) we\ conclude\ that\ A \times B \neq B \times A\ as\ (1,2) \neq (2,1)\ and\ (1,3) \neq (3,1)\ etc.$

$(iii)\ n(A) = 3;\ n(B) = 2.$

$n(A) \times n(B) = 3 \times 2 = 6$

$n(A \times B) = n(B \times A) = 6$

$n(A \times B) = n(B \times A) = n(A) \times n(B)$

Example 1.2 $\boldsymbol{If\ A \times B = \{(3,2),(3,4),(5,2),(5,4)\}\ then\ find\ A\ and\ B}$

$A \times B = \{(3,2),(3,4),(5,2),(5,4)\}$

$A = \{set\ of\ all\ first\ coordinates\ of\ elements\ of\ A \times B\}$

$\therefore A = \{3,5\}$

$B = \{set\ of\ all\ second\ coordinates\ of\ elements\ of\ A \times B\}$

$\therefore B = \{2,4\}$

$A = \{3,5\}\ and\ B = \{2,4\}.$

Example 1.3 $\boldsymbol{Let\ A = \{x \in \mathbb{N} \setminus 1 < x < 4\}, B = \{x \in \mathbb{W} \setminus 0 \leq x < 2\}\ and\ C = \{x \in \mathbb{N} \setminus x < 3\}\ Then\ verify\ that\ (i)\ A \times (B \cup C) = (A \times B) \cup (A \times C)}$
$\boldsymbol{(ii)\ A \times (B \cap C) = (A \times B) \cap (A \times C)}$

$A = \{2,3\},\ B = \{0,1\}\ and\ C = \{1,2\}$

$(i)\ A \times (B \cup C) = (A \times B) \cup (A \times C)$

$L.H.S = A \times (B \cup C)$

$B \cup C = \{0,1\} \cup \{1,2\}$

$B \cup C = \{0,1,2\}$

$A \times (B \cup C) = \{2,3\} \times \{0,1,2\}$

$A \times (B \cup C) = \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\} \ldots (1)$

$R.H.S = (A \times B) \cup (A \times C)$

$A \times B = \{2,3\} \times \{0,1\}$

$A \times B = \{(2,0),(2,1),(3,0),(3,1)\}$

$A \times C \;= \{2,3\} \times \{1,2\}$

$\boxed{A \times C = \{(2,1),(2,2),(3,1),(3,2)\}}$

$(A \times B) \cup (A \times C) = \{(2,0),(2,1),(3,0),(3,1)\} \cup \{(2,1),(2,2),(3,1),(3,2)\}$

$\boxed{(A \times B) \cup (A \times C) = \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}} \ldots(1)$

$From\ (1)\ and\ (2)\ A \times (B \cup C) = (A \times B) \cup (A \times C)$

$(ii)\ A \times (B \cap C) = (A \times B) \cap (A \times C)$

$A = \{2,3\},\ B = \{0,1\}\ and\ C = \{1,2\}$

$L.H.S = A \times (B \cap C)$

$B \cap C = \{0,1\} \cap \{1,2\} \implies B \cap C = \{1\}$

$A \times (B \cap C) = \{2,3\} \times \{1\}$

$\boxed{A \times (B \cap C) = \{(2,1),(3,1)\}}\ldots(1)$

$R.H.S = (A \times B) \cap (A \times C)$

$A \times B \;= \{2,3\} \times \{0,1\}$

$\boxed{A \times B = \{(2,0),(2,1),(3,0),(3,1)\}}$

$A \times C = \{2,3\} \times \{1,2\}$

$\boxed{A \times C = \{(2,1),(2,2),(3,1),(3,2)\}}$

$(A \times B) \cap (A \times C) = \{(2,0),(2,1),(3,0),(3,1)\} \cap \{(2,1),(2,2),(3,1),(3,2)\}$

$\boxed{A \times (B \cup C) = \{(2,1),(3,1)\}}\ldots(2)$

$From\ (1)\ and\ (2)\ A \times (B \cap C) = (A \times B) \cap (A \times C)$

1. Find $A \times B, A \times A$ and $B \times A$ (i) $A = \{2,-2,3\}$ and $B = \{1,-4\}$ (ii) $A = B = \{p,q\}$ (iii) $A = \{m,n\}$, $B = \emptyset$

(i) $A = \{2,-2,3\}$ and $B = \{1,-4\}$

$A \times B \;= \{2,-2,3\} \times \{1,-4\}$

$A \times B = \{(2,1),(2,-4),(-2,1),(-2,-4),(3,1),(3,-4)\}$

$A \times A \;= \{2,-2,3\} \times \{2,-2,3\}$

$A \times A = \{(2,2),(-2,-2),(2,3),(-2,2),(-2,-2),(-2,3),(3,2),(3,-2),(3,3)\}$

$B \times A = \{1,-4\} \times \{2,-2,3\}$

$B \times A = \{(1,2),(1,-2),(1,3),(-4,2),(-4,-2),(-4,3)\}$

(ii) $A = B = \{p,q\}$

$A \times B \;= \{p,q\} \times \{p,q\}$

$A \times B = \{(p,p),(p,q),(q,p),(q,q)\}$

$A \times A = \{p,q\} \times \{p,q\}$

$A \times A = \{(p,p),(p,q),(q,p),(q,q)\}$

$B \times A = \{p,q\} \times \{p,q\}$

$B \times A = \{(p,p),(p,q),(q,p),(q,q)\}$

$\boldsymbol{(iii)\ A = \{m,n\}\ ,B = \emptyset}$

$A \times B\ = \{m,n\} \times \{\quad\}$

$A \times B = \{\quad\}$

$A \times A\ = \{m,n\} \times \{m,n\}$

$A \times A = \{(m,m),(m,n),\ (n,m),(n,n)\}$

$B \times A = \{\ \} \times \{m,n\}$

$B \times A = \{\ \}$

$\boldsymbol{6. Let\ A = \{x \in \mathbb{W} \setminus x < 2\}, B = \{x \in \mathbb{N} \setminus 1 < x \leq 4\}\ and\ C = \{3,5\}.}$
$\boldsymbol{verify\ that\ (i)\ A \times (B \cup C) = (A \times B) \cup (A \times C)}$
$\boldsymbol{(ii)\ A \times (B \cap C) = (A \times B) \cap (A \times C)\ (iii)\ (A \cup B) \times C = (A \times C) \cup (B \times C)}$

$A = \{0,1\}, B\ = \{2,3,4\}\ and\ C = \{3,5\}$

$(i)\ A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{2,3,4\} \cup \{3,5\}$

$B \cup C\ = \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$A \times (B \cup C)\ = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$ ... (1)

$R.H.S = (A \times B) \cup (A \times C)$

$A \times B = \{0,1\} \times \{2,3,4\}$

$A \times B = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$\boxed{A \times C\ = \{(0,3),(0,5),(1,3),(1,5)\}}$

$(A \times B) \cup (A \times C)\ = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$
$\cup$
$\{(0,3),(0,5),(1,3),(1,5)\}$

$\boxed{(A \times B) \cup (A \times C) = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\ \}}$ ... (2)

$From\ (1)\ and\ (2)\ A \times (B \cup C) = (A \times B) \cup (A \times C)$

$\boldsymbol{(i)}$ **A = {1, 2, 3, 4, 5, 6, 7}, B = {2, 3, 5, 7} and C = {2}**

$L.H.S = (A \cap B) \times C$

$\mathrm{A \cap B} = \{1,2,3,4,5,6,7\} \cap \{2,3,5,7\}$

$\mathrm{A \cap B} = \{2,3,5,7\}$

$(\mathrm{A \cap B}) \times \mathrm{C} =\ \{2,3,5,7\}\ \times \{2\}$

$= \{(2,2),(3,2)\ ,(5,2)\ ,(7,2)\ \}$ ... (1)

$\mathrm{R.H.S} = (\mathrm{A} \times \mathrm{C}) \cap (\mathrm{B} \times \mathrm{C})$

$A \times C\ = \{1,2,3,4,5,6,7\} \times \{2\}$

$A \times C\ = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{2,3,5,7\} \times \{2\}$

$B \times C = \{ (2,2) , (3,2) , (5,2) , (7,2) \}$

$\text{R.H.S} = (\text{A} \times \text{C}) \cap (\text{B} \times \text{C})$

$= \{ (2,2) , (3,2) , (5,2) , (7,2) \} \ldots (2)$

From (1)and (2) $\therefore$ L. H. S = R. H. S

$(\boldsymbol{ii})\ \mathbf{A} \times (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) - (\boldsymbol{A} \times \boldsymbol{C})$

$\text{L.H.S} = \text{A} \times (\text{B} - \text{C})$

$\text{B} - \text{C} = \{2,3,5,7\} - \{2\}$

$\text{B} - \text{C} = \{3,5,7\}$

$\text{A} \times (\text{B} - \text{C}) = \{1,2,3,4,5,6,7\} \times \{2\}$

$= \{(1,3) , (2,3) , (3,3) , (4,3) , (5,3) , (6,3) , (7,3),$

$(1,5) , (2,5) , (3,5) , (4,5) , (5,5) , (6,5) , (7,5),$

$(1,7) , (2,7) , (3,7) , (4,7) , (5,7) , (6,7) , (7,7)\} \ldots (1)$

$\text{R.H.S} = (A \times B) - (A \times C)$

$\text{A} \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

$\text{A} \times B = \{(1,2) , (2,2) , (3,2) , (4,2) , (5,2) , (6,2) , (7,2),$

$(1,3) , (2,3) , (3,3) , (4,3) , (5,3) , (6,3) , (7,3),$

$(1,5) , (2,5) , (3,5) , (4,5) , (5,5) , (6,5) , (7,5),$

$(1,7) , (2,7) , (3,7) , (4,7) , (5,7) , (6,7) , (7,7)\}$

$\text{A} \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$\text{A} \times C = \{(1,2) , (2,2) , (3,2) , (4,2) , (5,2) , (6,2) , (7,2) \}$

$(A \times B) - (A \times C) = \{(1,3) , (2,3) , (3,3) , (4,3) , (5,3) , (6,3) , (7,3),$

$(1,5) , (2,5) , (3,5) , (4,5) , (5,5) , (6,5) , (7,5),$

$(1,7) , (2,7) , (3,7) , (4,7) , (5,7) , (6,7) , (7,7)\} \ldots (2)$

From (1)and (2) $\therefore$ L. H. S = R. H. S

7. *Let A = The set of all natural numbers less than* 8,
***B = The set of all prime numbers less than* 8,**
C = The set of even prime number. Verify that

$(\boldsymbol{i})\ (\boldsymbol{A} \cap \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \times \boldsymbol{C}) \cap (\boldsymbol{B} \times \boldsymbol{C})$

$(\boldsymbol{ii})\ \boldsymbol{A} \times (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) - (\boldsymbol{A} \times \boldsymbol{C})$

$(\boldsymbol{i})\ \mathbf{A} = \{\mathbf{1, 2, 3, 4, 5, 6, 7}\}, \mathbf{B} = \{\mathbf{2, 3, 5, 7}\}$ **and** $\mathbf{C} = \{\mathbf{2}\}$

$L.H.S = (A \cap B) \times C$

$\text{A} \cap \text{B} = \{1,2,3,4,5,6,7\} \cap \{2,3,5,7\}$

$\text{A} \cap \text{B} = \{2,3,5,7\}$

$(\mathrm{A} \cap \mathrm{B}) \times \mathrm{C} = \{2,3,5,7\} \times \{2\}$

$= \{(2,2), (3,2), (5,2), (7,2)\} \ldots (1)$

$\mathrm{R.H.S} = (\mathrm{A} \times \mathrm{C}) \cap (\mathrm{B} \times \mathrm{C})$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$A \times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{2,3,5,7\} \times \{2\}$

$B \times C = \{(2,2), (3,2), (5,2), (7,2)\}$

$\mathrm{R.H.S} = (\mathrm{A} \times \mathrm{C}) \cap (\mathrm{B} \times \mathrm{C})$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\} \cap \{(2,2), (3,2), (5,2), (7,2)\}$

$= \{(2,2), (3,2), (5,2), (7,2)\} \ldots (2)$

$From\ (1) and\ (2) \therefore L.H.S = R.H.S$

Example 1.4 Let $A = \{3,4,7,8\}$ and $B = \{1,7,10\}$. Which of the following sets are relations from A to B?

(i) $R_1 = \{(3,7),(4,7),(7,10),(8,1)\}$ (ii) $R_2 = \{(3,1),(4,12)\}$

(iii) $R_3 = \{(3,7),(4,10),(7,7),(7,8),(8,11),(8,7),(8,10)\}$

$A = \{3,4,7,8\}$ and $B = \{1,7,10\}$

$A \times B = \{3,4,7,8\} \times \{1,7,10\}$

$A \times B = \{(3,1),(3,7),(3,10),(4,1),(4,7),(4,10),(7,1),(7,7),(7,10),(8,1),(8,7),(8,10)\}$

(i) $R_1 = \{(3,7),(4,7),(7,10),(8,1)\}$

$\therefore R_1$ is a relation from A to B

$R_1 \subseteq A \times B$

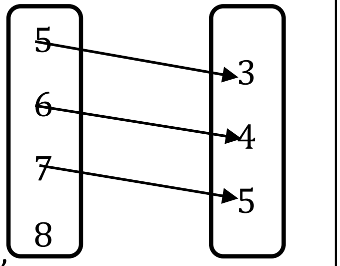
(ii) $R_2 = \{(3,1),(4,12)\}$

Here, $(4,12) \in R_2$ but $(4,12) \notin A \times B$. So, R_2 is not a relation from A to B.

(iii) $R_3 = \{(3,7),(4,10),(7,7),(7,8),(8,11),(8,7),(8,10)\}$

Here, $(7,8) \in R_3$ but $(7,8) \notin A \times B$. So, R_3 is not a relation from A to B.

Example 1.5 The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



(i) Set builder form of

$R = \{(x,y) \setminus y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5,3),(6,4),(7,5)\}$

(iii) Domain of $R = \{5,6,7\}$ and range of $R = \{3,4,5\}$

A relation which contains no element is called a "Null relation".

If $n(A) = p, n(B) = q$, then the total number of relations that exist between A and B is 2^{pq}.

$EXERCISE\ 1.2$

1 . $Let\ A = \{1, 2, 3, 7\}\ and\ B = \{3, 0, -1, 7\}. Which\ of\ the\ following\ sets\ are\ relations\ from\ A\ to\ B?$

$(i)\ \boldsymbol{R_1 = \{(2,1),(7,1)\}}\ \ (ii)\ \boldsymbol{R_2 = \{(-1,1)\}}$

$(iii)\ \boldsymbol{R_3 = \{(2,-1),(7,7),(1,3)\}}\ \ \ (iv)\ \boldsymbol{R_4 = \{(7,-1),(0,3),(3,3),(0,7)\}}$

$A = \{1, 2, 3, 7\} and\ B = \{3, 0, -1, 7\}.$

$A \times B = \{1, 2, 3, 7\} \times \{3, 0, -1, 7\}$

$A \times B = \{\ (1,3),(1,0),(1,-1),(1,7),(2,3),(2,0),(2,-1),(2,7),(3,3),(3,0),$
$(3,-1),(3,7),(7,3)\ ,(7,0),(7,-1),(7,7)\ \}$

$(i)\ \boldsymbol{R_1 = \{(2,1), (7,1)\}}$

$1 \notin B$ Since $(2,1)$ and $(7,1) \notin A \times B$ $. Hence\ it\ is\ not\ a\ relation$

$(ii)\ \boldsymbol{R_2} = \{(\boldsymbol{-1, 1})\}$

$Here, (-1, 1) \in R_2\ but\ (-1, 1) \notin A \times B.$
$So, R_2\ is\ not\ a\ relation\ from\ A\ to\ B.$

$(iii)\ \boldsymbol{R_3 = \{(2,-1), (7,7), (1,3)\}}$

R_3 is a relation

$(iv)\ \boldsymbol{R_4} = \{(\boldsymbol{7, -1}), (\boldsymbol{0, 3}), (\boldsymbol{3, 3}), (\boldsymbol{0, 7})\}$

$Here, (0, 3) \in R_3\ but\ (0, 3) \notin A \times B.\ So, R_3\ is\ not\ a\ relation\ from\ A\ to\ B.$

2 . $Let\ A = \{1, 2, 3, 4, \ldots, 45\} and\ R\ be\ the\ relation\ defined\ as$ "$is\ square\ of$" $on\ A. Write\ R\ as\ a\ subset\ of\ A \times A. Also, find\ the\ domain\ and\ range\ of\ R.$

$A = \{1, 2, 3, 4, \ldots, 45\}$

$Relation\ defined\ as$ "$is\ square\ of$" $from\ A\ to\ A$

$R\ is\ subset\ of\ A \times A$

$R = \{\ (1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6)\}$

$Domain\ of\ R\ = \{1, 4, 9, 16, 25, 36\}$

$Range\ of\ R = \{1, 2, 3, 4, 5, 6\}$

3 . $A\ Relation\ \ R\ is\ given\ by\ the\ set \{(x, y)\ / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}. Determine\ its\ domain\ and\ range.$

$R = \{(x, y)/y = x + 3\}\ and\ x \in \{0, 1, 2, 3, 4, 5\}$

$y = x + 3$

$x = 0,\ \ y = 0 + 3 \Longrightarrow \boxed{y = 3}$

$x = 1,\ \ y = 1 + 3\ \Longrightarrow \boxed{y = 4}$

$x = 2,\ \ y = 2 + 3\ \Longrightarrow \boxed{y = 5}$

$x = 3,\ \ y = 3 + 3 \Longrightarrow \boxed{y = 6}$

$x = 4,\ y = 4 + 3 \implies \boxed{y = 7}$

$x = 5,\ y = 5 + 3 \implies \boxed{y = 8}$

$R = \{(0,3),(1,4),(2,5),(3,6),(4,7),(5,8)\}$

$Domain\ of\ R = \{0,1,2,3,4,5\}$

$Range\ of\ R = \{3,4,5,6,7,8\}$

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x,y) | x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$

(ii) $\{(x,y) | y = x + 3, x, y \text{ are natural numbers} < 10\}$

(i) $\{(x,y) | x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$

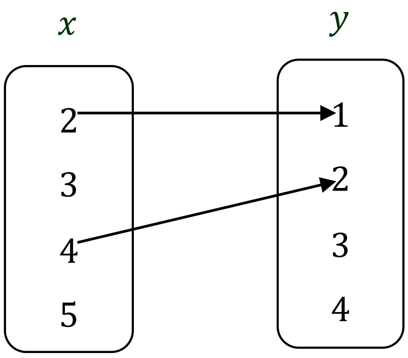
$2y = x \implies y = \frac{x}{2}$

$x = 2;\ y = \frac{2}{2} = 1 \implies \boxed{y = 1}$

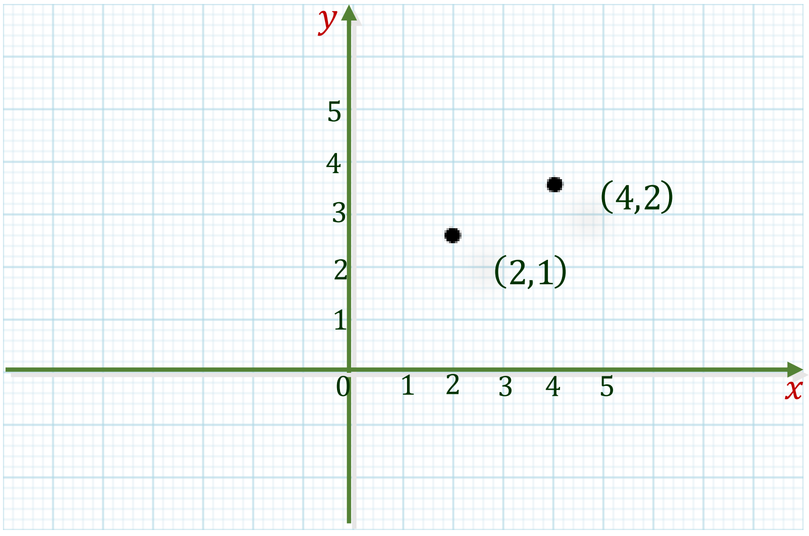
$x = 4;\ y = \frac{4}{2} \implies \boxed{y = 2}$

$R = \{(2,1),(4,2)\}$

a) Arrow diagram



b) graph



a) Roster form

$R = \{(2,1),(4,2)\}$

(ii) $\{(x,y) | y = x + 3, x, y \text{ are natural numbers} < 10\}$

$x = \{1,2,3,4,5,6,7,8,9\}$

$y = \{1,2,3,4,5,6,7,8,9\}$

$y = x + 3$

$x = 1,\ y = 1 + 3 \implies \boxed{y = 4}$

$x = 2,\ y = 2 + 3 \implies \boxed{y = 5}$

$x = 3,\ y = 3 + 3 \implies \boxed{y = 6}$

$x = 4,\ y = 4 + 3 \implies \boxed{y = 7}$

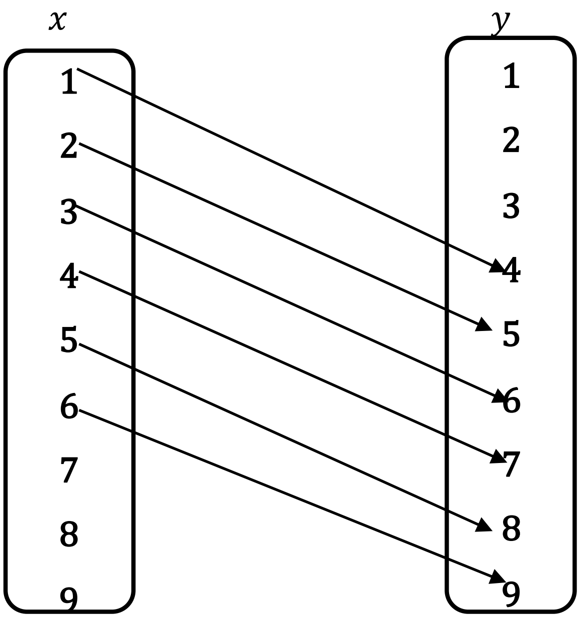
$x = 5,\ y = 5 + 3 \implies \boxed{y = 8}$

$x = 6,\ y = 6 + 3 \implies \boxed{y = 9}$

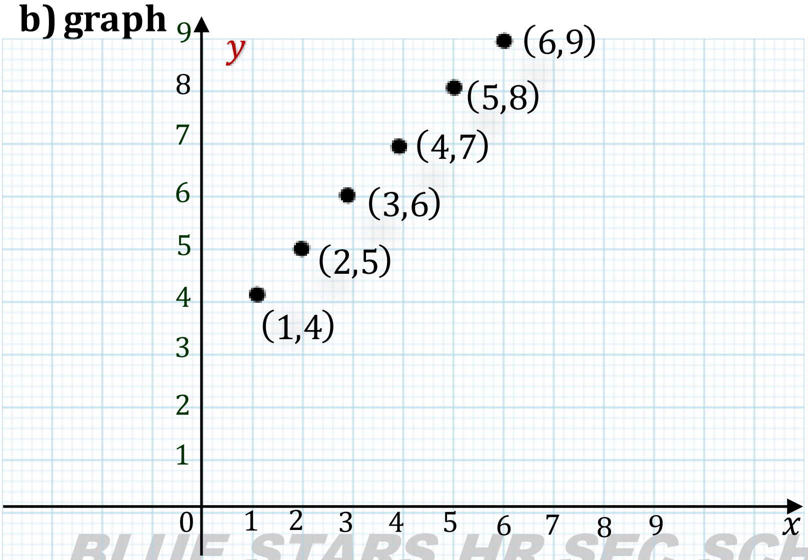
$x = 7,\ y = 7 + 3 \implies \boxed{y = 10} \notin y$

$R = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

$\boldsymbol{a})$ ***Arrow diagram***



b) graph



$a)$ ***Roster form***

$R = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide Rs .10,000, Rs. 25,000, Rs .50,000 and Rs.1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by $x\mathrm{R}y$, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

$\text{Assistants (A)} = \{A_1, A_2, A_3, A_4, A_5\}$

$\text{Clerks (C)} = \{C_1, C_2, C_3, C_4\}$

$\text{Manager (M)} = \{M_1, M_2, M_3\}$

$\text{Executive Officer (E)} = \{E_1, E_2\}$

$Salary \longrightarrow \text{Categories}$

$10{,}000 \longrightarrow \text{Assistants } \{A_1, A_2, A_3, A_4, A_5\}$

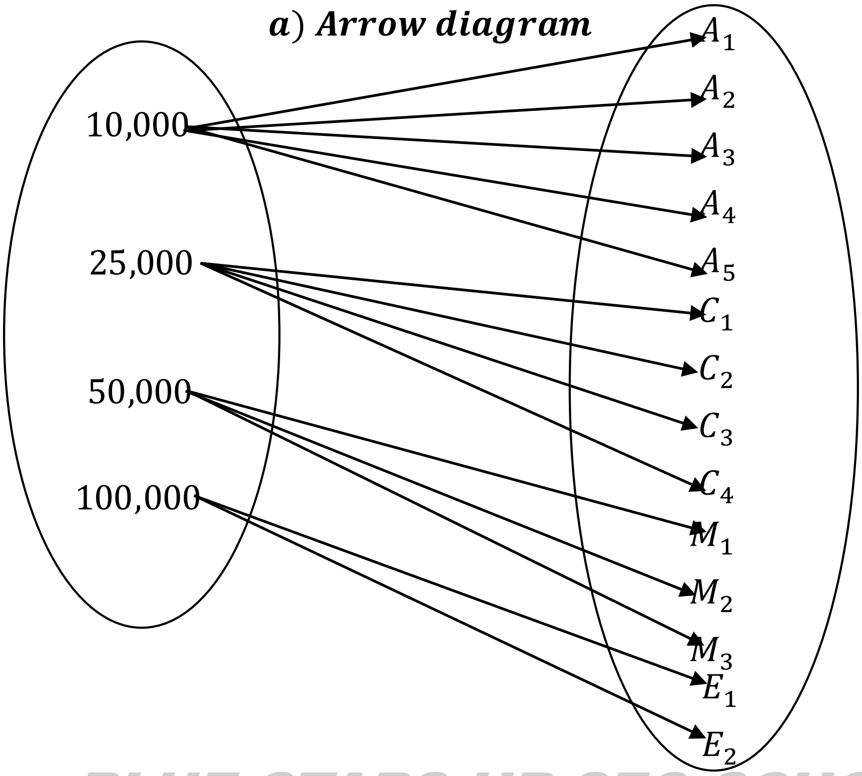
$25{,}000 \longrightarrow \text{Clerks } \{C_1, C_2, C_3, C_4\}$

$50{,}000 \longrightarrow \text{Manager } \{M_1, M_2, M_3\}$

$100{,}000 \longrightarrow \text{Executive Officer } \{E_1, E_2\}$

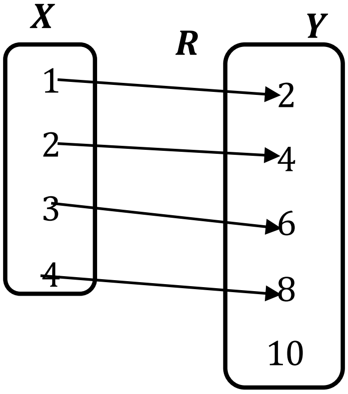
$$\begin{aligned}R = \{&(10{,}000, A_1), (10{,}000, A_2), (10{,}000, A_3), (10{,}000, A_4), (10{,}000, A_5),\\ &(25{,}000, C_1), (25{,}000, C_2), (25{,}000, C_3), (25{,}000, C_4), (50{,}000, M_1),\\ &(50{,}000, M_2), (50{,}000, M_3), (100{,}000, E_1), (100{,}000, E_2)\}\end{aligned}$$

$a)$ ***Arrow diagram***



$Exercise\ 1.3$

Example: 1.6 $\boldsymbol{Let\ x = \{1,2,3,4\}\ and\ y = \{2,4,6,8,10\}\ and\ R = \{(1,2),(2,4),(3,6),(4,8)\}.}$ **Show that R is a function and find its domain, co − domain and range?**



$Given: x = \{1,2,3,4\},\ y = \{2,4,6,8,10\}$

$R = \{(1,2),(2,4),(3,6),(4,8)\}.$

All the elements in X have Only one image in Y

$\therefore R$ *is a function*

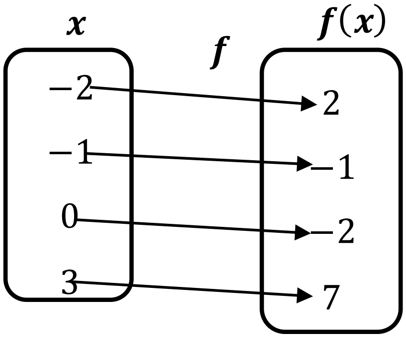
$Domain: X = \{1,2,3,4\}$

$Co - Domain: Y = \{2,4,6,8,10\}$

$Range\ of\ f = \{2,4,6,8\}$

Example: 1.7 **A relation ‘f’ is defined by** $\boldsymbol{f(x) = x^2 - 2}$ **where** $\boldsymbol{x \in \{-2,-1,0,3\}}$ **(i) List the elements of f (ii) Is f a function?**

$Given: f(x) = x^2 - 2\ where\ x \in \{-2,-1,0,3\}$



$$f(x) = x^2 - 2$$

$$x = -2 \Rightarrow f(-2) = (-2)^2 - 2$$
$$= 4 - 2 = 2$$

$$x = -1 \Rightarrow f(-1) = (-1)^2 - 2$$
$$= 1 - 2 = -1$$

$$x = 0 \Rightarrow f(0) = (0)^2 - 2$$
$$= 0 - 2 = -2$$

$$x = 3 \Rightarrow f(3) = (3)^2 - 2$$
$$= 9 - 2 = 7$$

$\therefore f = \{(-2,2),(-1,-1),(0,-2),(3,7)\}$

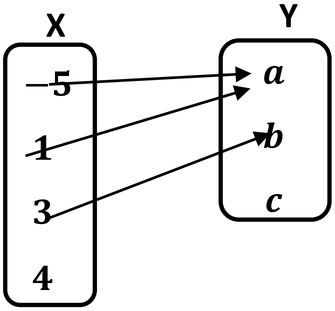
$\therefore$ *Each element in the domain of f has unique image*

$\therefore f$ *is a function*

Example: 1.7

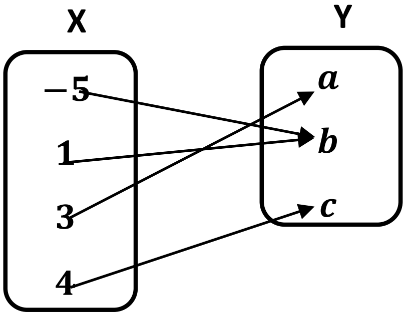
If $X = \{-5,1,3,4\}$ and $Y = \{a,b,c\}$, then which of the following relations are functions from X to Y ?

(i) $\boldsymbol{R_1 = \{(-5,a),(1,a),(3,b)\}}$



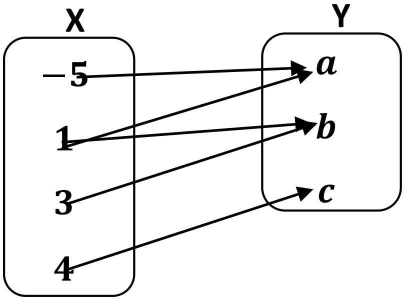
R_1 *is not a function as* $4 \in X$ *does not have an image in Y*

$(ii)\ \boldsymbol{R_2 = \{(-5,b),(1,b),(3,a),(4,c)\}}$



R_2 *is a function as each element in* x *has an unique image in* y.

$\boldsymbol{(iii)\ R_3 = \{(-5,a),(1,a),(3,b),(4,c),(1,b)\}}$



R_3 *is not a function because* $1 \in X$ *has two image* $a \in Y$ *and* $b \in Y$

$\boldsymbol{Example: 1.9\ Given : f(x) = 2x - x^2\ find\ (i)\ f(1)(ii)f(x+1)}$
$\boldsymbol{(iii)\ f(x) + f(1)}$

$Given: f(x) = 2x - x^2$

$(i)\ f(1) = 2(1) - (1)^2 = 2 - 1$

$$\boxed{f(1) = 1}$$

$(ii)\ f(x+1) = 2(x+1) - (x+1)^2$

$= 2x + 2 - [x^2 + 1^2 + 2x(1)]$

$= 2x + 2 - [x^2 + 1 + 2x]$

$= \cancel{2x} + 2 - x^2 - 1 - \cancel{2x}$

$= 1 - x^2$

$\boldsymbol{(iii)\ f(x) + f(1)}$

$f(x) + f(1) = 2x - x^2 + 1$

1. $\boldsymbol{Let\ f = \{(x,y) \setminus x,y \in \mathbb{N}\ and\ y = 2x\}\ be\ a\ relation\ on\ N. Find\ the\ domain, co-domain\ and\ range. Is\ this\ relation\ a\ function?}$

$Given: f = \{(x,y) | x, y \in \mathbb{N}\ and\ y = 2x\}$

$y = f(x) = 2x$

$x = 1;\ f(1) = 2(1) = 2$

$x = 2;\ f(2) = 2(2) = 4$

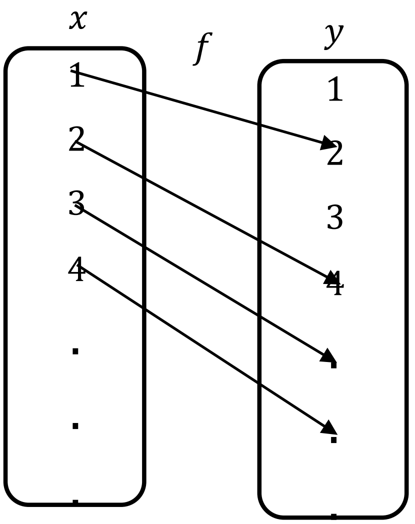
$x = 3;\ f(3) = 2(3) = 6$

$\vdots \qquad\qquad \vdots$

$Domain = \{1,2,3, \dots\}$

$Co - Domain = \{1,2,3, \dots\}$

$Range = \{2,4,6,8, \dots \dots \dots\}$



$\boldsymbol{Each\ element\ in\ x\ has\ only\ one\ \ image\ in\ y. Thus, f\ is\ a\ function.}$

2. $\boldsymbol{Let\ X = \{3,4,6,8\}. Determine\ whether\ the\ relation}$
$\boldsymbol{R = \{(x,f(x)) |\ x \in X, f(x) = x^2 + 1\}\ is\ a\ function\ from\ X\ to}\ \mathbb{N}$?

$Given : x = \{3,4,6,8\}$

$f(x) = x^2 + 1$

$x = 3 \Longrightarrow\ f(3) = 3^2 + 1 = 9 + 1$

$\boxed{f(3) = 10}$

$x = 4 \Longrightarrow\ f(4) = 4^2 + 1 = 16 + 1$

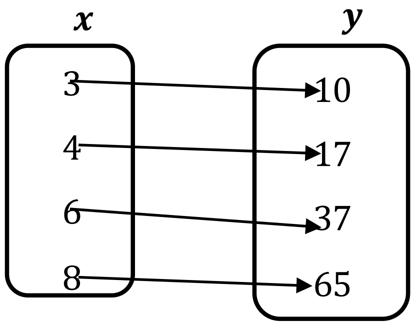
$\boxed{f(4) = 17}$

$x = 6\ \Longrightarrow f(6) = 6^2 + 1 = 36 + 1$

$\boxed{f(6) = 37}$

$x = 8 \Longrightarrow\ f(8) = 8^2 + 1 = 64 + 1$

$\boxed{f(6) = 65}$



$\therefore R: X \rightarrow N\ is\ a\ function$

$R = \{(3,10), (4,17), (6,37), (8,65)\}$

3. $\boldsymbol{Given\ the\ function\ f : x \rightarrow x^2 - 5x + 6, evaluate}$
$\boldsymbol{(i) f(-1) \qquad (ii) f(2a) \qquad (iii) f(2) \qquad (iv) f(x - 1)}$

$Given\ f : x \rightarrow x^2 - 5x + 6$

$f(x) = x^2 - 5x + 6$

$(i)\ \ f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

$(ii)\ f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

$\boxed{(a - b)^2 = a^2 - 2ab + b^2}$

$(iii)\ \ f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

$\boxed{(x - 1)^2 = x^2 - 2 \times x \times 1 + 1^2}$

$\boxed{= x^2 - 2x + 1}$

$(iv)\ f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6 = x^2 - 7x + 12$

4. A graph representing the functions $f(x)$ is given in figure it is clear that $f(9) = 2$.

(i) Find the following values of the function

(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$

(ii) For what value of x is $f(x) = 1$?

(iii) Describe the following (a) Domain (b) Range.

(iv) What is the image of 6 under f?

(i) Find the following values of the function

(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$

Given: $y = f(x)$

$x = 0$

From the Graph, $\boldsymbol{y = f(0) = 9}$

(b) $f(7)$

Given: $x = 7$

From the Graph, $y = f(7) = 6$

(c) $f(2)$

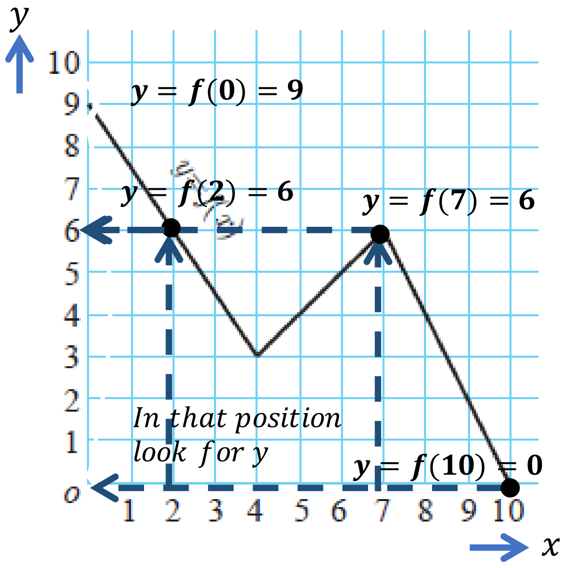
$x = 2$

From the Graph, $\boldsymbol{y = f(2) = 6}$

(d) $f(10)$

$x = 10$

From the Graph, $y = f(10) = 0$



(ii) For what value of x is $f(x) = 1$?

Given: $y = f(x) = 1$

From the Graph for the value $y = 1$

$\therefore$ The value x at $f(x) = 1$ is 9.5

(iii) Describe the following

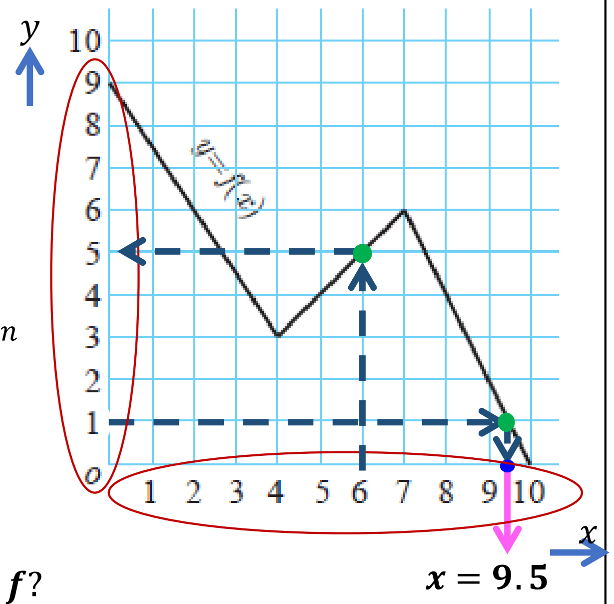
(a) Domain (b) Range

In the Graph, all values of x is called Domain

Domain: $\{x; 0 \le x \le 10, x \in R\}$

In the Graph, maximum range of value of y is called Range

Range: : $\{x; 0 \le x \le 9, x \in R\}$



(iv) What is the image of 6 under f?

x

Image of $6 = f(6) = 5$

5. Let $f(x) = 2x + 5$. If $x \ne 0$ then find $\dfrac{f(x+2) - f(2)}{x}$

Find $f(x+2)$:

$$f(x+2) = 2(x+2) + 5 = 2x + 4 + 5$$

$$\boxed{f(x+2) = 2x + 9}$$

Find $f(2)$:

$$f(2) = 2(2) + 5 = 4 + 5$$

$$\boxed{f(2) = 9}$$

$$\frac{f(x+2) - f(2)}{x} = \frac{2x + \cancel{9} - \cancel{9}}{x} = \frac{\cancel{2x}}{\cancel{x}} = 2$$

6. A function f is defined by $f(x) = 2x - 3$

(i) Find $\frac{f(0)+f(1)}{2}$ (ii) Find x such that $f(x) = 0$

(iii) Find x such that $f(x) = x$ (iv) Find x such that $f(x) = f(1-x)$

(i) Find $\frac{f(0)+f(1)}{2}$

Given: $f(x) = 2x - 3$

Put $x = 0$ *in* $f(x) = 2x - 3$

$$f(0) = 2(0) - 3 = 0 - 3$$

$$f(0) = -3$$

Put $x = 1$ *in* $f(x) = 2x - 3$

$$f(1) = 2(1) - 3 = 2 - 3$$

$$f(1) = -1$$

$$\frac{f(0)+f(1)}{2} = \frac{(-3)+(-1)}{2} = \frac{-\cancel{4}}{\cancel{2}} = -2$$

(ii) Find x such that $f(x) = 0$

$$2x - 3 = 0 \implies 2x = 3 \implies x = \frac{3}{2}$$

(iii) Find x such that $f(x) = x$

$f(x) = x$, *Here* $f(x) = 2x - 3$

$$2x - 3 = x \implies 2x - x = 3$$

$$\boxed{x = 3}$$

(iv) Find x such that $f(x) = 1 - x$

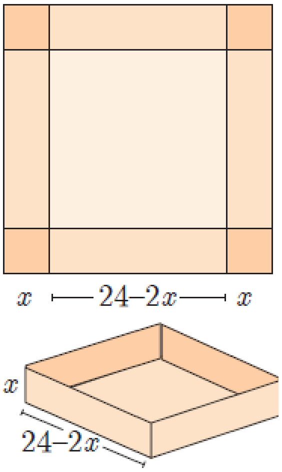
Given: $f(x) = 1 - x$, *Here* $f(x) = 2x - 3$

$$2x - 3 = 1 - x \implies 2x - 3 - 1 + x = 0$$

$$3x - 4 = 0 \implies 3x = 4$$

$$x = \frac{4}{3}$$

7. An open box is to be made from a square piece of materia, 24cm on a side, by cutting equal squares from the corners and turning up the sides as shownin the figure. Express the volume V of the box as a function of x.



In a Square, All sides are equal

$$Length(l) = Breadth(b)$$

$$Given : b = (24-2x)cm \text{ and } Height(h) = x\ cm$$

$$Length(l) = Breadth(b) = (24-2x)cm$$

$$\therefore Volume\ of\ the\ box, V = \ell \times b \times h$$

$$= (24-2x) \times (24-2x) \times x$$

$$= (24-2x)^2 \times x$$

$$= [(24)^2 - 2 \times 24 \times 2x + (2x)^2] \times x$$

$$= (576 - 96x + 4x^2) \times x$$

$$\boxed{V = 4x^3 - 96x^2 + 576x}$$

8. A function f is defined by $f(x) = 3-2x$. Find x such that $f(x^2) = (f(x))^2$

$$Given:\ f(x) = 3-2x$$

$$Replacing:\ x \longrightarrow x^2 \Longrightarrow\ f(x^2) = 3-2(x^2)$$

$$f(x^2) = 3-2x^2 \ \ldots(1)$$

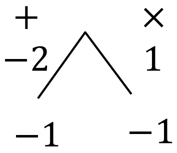
$$f(x) = 3-2x$$

Squaring on both sides

$$(f(x))^2 = (3-2x)^2$$

$$= (3)^2 - 2 \times 3 \times 2x + (2x)^2$$

$$(f(x))^2 = 9 - 12x + 4x^2 \ \ldots(2)$$



$$f(x^2) = (f(x))^2$$

$$3 - 2x^2 = 9 - 12x + 4x^2 \Longrightarrow 0 = 9 - 12x + 4x^2 + 2x^2 - 3$$

$$\underset{\div 6}{6x^2 - 12x + 6 = 0} \ \Longrightarrow x^2 - 2x + 1 = 0 \Longrightarrow (x-1)(x-1) = 0$$

$$x - 1 = 0 \ \Longrightarrow \boxed{x = 1}$$

9. A plane is flying at a speed of 500km per hour. Express the distance 'd' travelled by the plane as function of time 't' in hours.

$$Given: Speed\ of\ the\ plane = 500\ km/hr$$

$$Distance = 'd' km\ , Time = \ 't' \ hrs$$

$$Speed = \frac{Distance}{Time} \Longrightarrow \ Distance = \ Speed \ \times Time$$

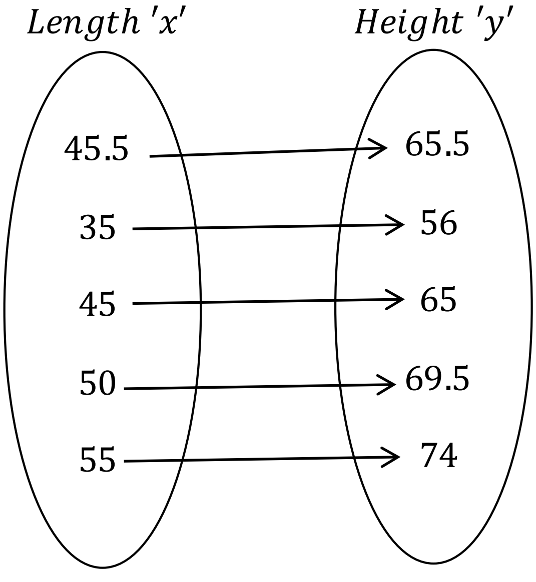
$$\therefore d = 500\ t$$

10. ***The data in the adjacent table depicts the length of a woman's forehand and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants.***

Length 'x' of forhand (in cm)	**Height 'y' (in inches)**
45.5	**65.5**
35	**56**
45	**65**
50	**69.5**
55	**75**

(i) Check if this relation is a function.
(ii) Find a and b
(iii) Find the height of a woman whose forehand length is 40cm
(iv) Find the length of forehand of a woman if her height is 53.3 inches.

(i) Check if this relation is a function



From the arrow diagram, each domain has a unique image

Hence the given relation is a function

(ii) Find 'a' and 'b'

Given: $y = ax + b$ ... (1)

$x = 55, y = 75$

$55a + b = 75$ ... (2)

$x = 45, y = 65$ *Sub in eqn* (1)

$45a + b = 65$ ... (3)

$\boldsymbol{Solve\ (2)\ and\ (3)}$

$$\begin{array}{rcl} 55a + \cancel{b} &=& 75 \\ (-)\quad (\cancel{-}) && (-) \\ 45a + \cancel{b} &=& 65 \\ \hline 10a &=& 10 \end{array} \Rightarrow a = \frac{10}{10} \Rightarrow \boxed{a = 1}$$

$Sub\ a = 1\ in\ eqn\ (2)\ \ 55a + b = 75$

$$55(1) + b = 75 \Rightarrow 55 + b = 75$$

$$b = 75 - 55 \Rightarrow \boxed{b = 20}$$

$$\therefore a = 1, b = 20$$

$\boldsymbol{(iii)\ Find\ the\ height\ of\ a\ woman\ whose\ forehand\ length\ is\ 40cm}$

$When\ x = 40;\ y = ?$

$sub\ a = 1, b = 20\ in\ y = ax + b$

$$y = x + 20$$

$When\ x = 40\ in\ y = x + 20$

$$y = 40 + 20 \Rightarrow \boxed{y = 60}$$

$\boldsymbol{(iv)\ Find\ the\ length\ of\ forehand\ of\ a\ woman\ if\ her\ height\ is\ 53.3\ inches}$

$When\ y = 53.3\ , x = ?$

$Sub\ a = 1, b = 20\ in\ y = ax + b$

$$y = x + 20$$

$When\ y = 53.3\ in\ y = x + 20$

$$53.3 = x + 20 \Rightarrow 53.3 - 20 = x$$

$$\boxed{x = 33.3}$$

Example 1.11 Let $A = \{1, 2, 3, 4\}$; $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f: A \rightarrow B$ be a function given by $f(x) = 3x - 1$. Represent this function

(i) as an arrow diagram **(ii) in a table form**

(iii) as a set of ordered pairs **(iv) in a graphical form**

Given : $A = \{1,2,3,4\}, B = \{2,5,8,11,14\}$

$$f(x) = 3x - 1$$

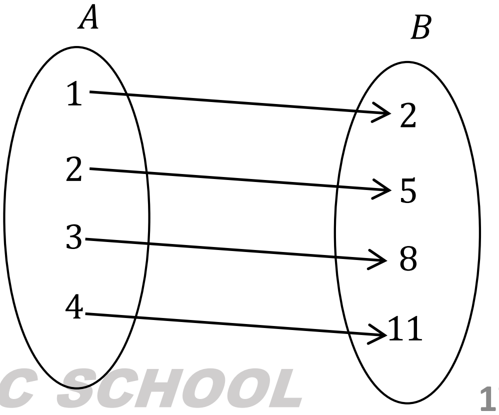
$$x = 1;\ f(1) = 3(1) - 1 = 3 - 1 = 2$$

$$x = 2;\ f(2) = 3(2) - 1 = 6 - 1 = 5$$

$$x = 3;\ f(3) = 3(3) - 1 = 9 - 1 = 8$$

$$x = 4;\ f(4) = 3(4) - 1 = 12 - 1 = 11$$

$\boldsymbol{(i)\ Arrow\ Diagram}$



(ii) ***Table Form***

x	1	2	3	4
$f(x)$	2	5	8	11

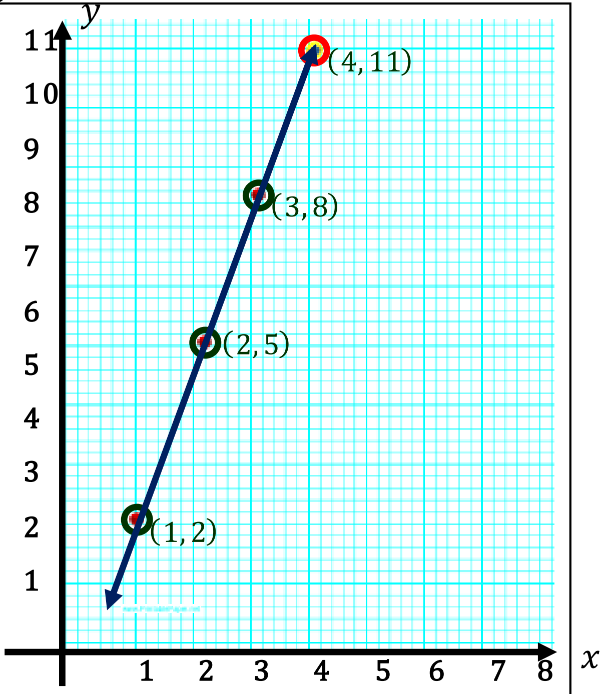
(iii) ***set of ordered pairs***

$f = \{(1,2),(2,5),(3,8),(4,11)\}$

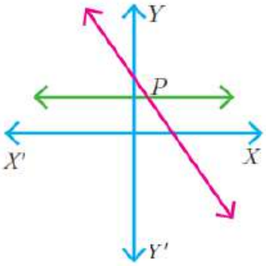
(iv) ***Graphical Form***

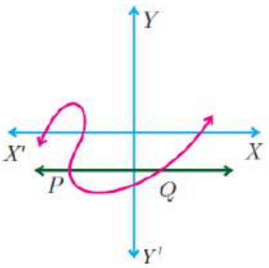
Points to be plotted on the $x - y$ *plane*

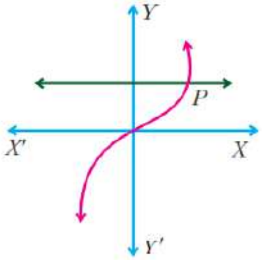
$\mathbf{(1,2),(2,5),(3,8),(4,11)}$



Example 1.12. Using horizontal line test, determine which of the following functions are one-one.

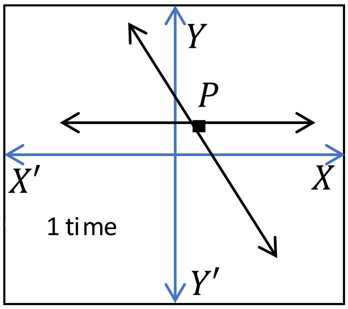






Horizontal Test:

If the horizontal line meets the curves in only one point, then the curve is one-one function



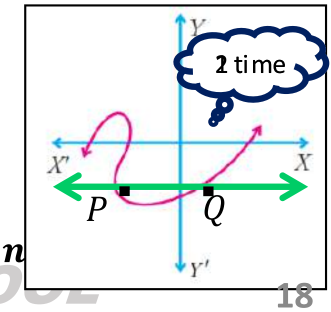
Re − draw the given Graph

Draw the horizontal line (parallel to the X *− axis) on the Graph*

Find the number of times the horizontal line cuts the graph

Here, the horizontal line cuts the graph only one time

Hence the given function is one − one function



Re − draw the given Graph

Draw the horizontal line (parallel to the $X - X'$*) on the Graph*

Find the number of times the horizontal line cuts the graph

Here, the horizontal line cuts the graph in two times

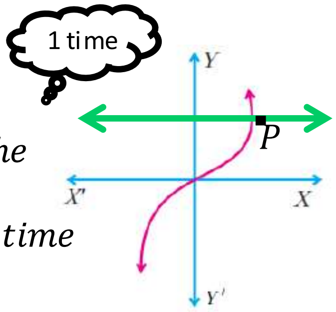
Hence the given function is not a one − one function

$Re - draw\ the\ given\ Graph$

$Draw\ the\ horizontal\ line\ (parallel\ to\ the\ X - X')$ $on\ the\ Graph$

$Find\ the\ number\ of\ times\ the\ horizontal\ line\ cuts\ the\ graph$

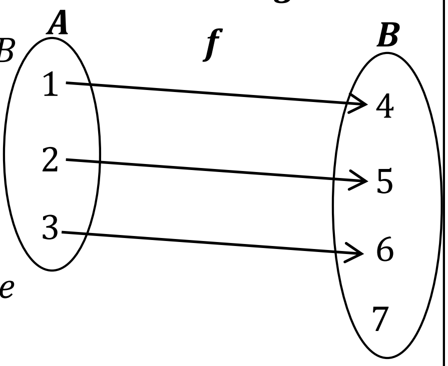
$Here, the\ horizontal\ line\ cuts\ the\ graph\ in\ only\ one\ time$



$\boldsymbol{Hence\ the\ given\ function\ is\ one - one\ function}$

$\boldsymbol{Example\ 1.13\ Let\ A = \{1,2,3\}, B = \{4,5,6,7\} and\ f = \{(1,4),(2,5),(3,6)\}}$ $\boldsymbol{be\ a\ function\ from\ A\ to\ B. Show\ that\ f\ is\ one - one\ but\ not\ onto\ function.}$

$\boldsymbol{Arrow\ Diagram}$



$Given: A = \{1,2,3\}, B = \{4,5,6,7\}$

$f = \{(1,4),(2,5),(3,6)\}$ $is\ a\ function\ from\ A\ to\ B$

$From\ the\ arrow\ diagram,$

$For\ every\ elements\ of\ A\ have\ a\ different\ images\ in\ B$

$Hence\ f\ is\ one - one\ function$

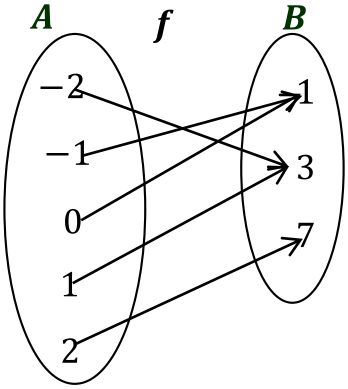
$But, Element\ 7\ in\ the\ co - domain\ does\ not\ have\ any\ pre\ image\ in\ domain$

$Hence\ f\ is\ not\ onto\ function$

$Hence\ f\ is\ one - one\ but\ not\ onto\ function$

$\boldsymbol{Example\ 1.14. If\ A = \{-2,-1,0,1,2\}\ and\ f: A \to B\ is\ an\ onto\ function\ defined\ by\ f(x) = x^2 + x + 1\ then\ find\ B}$

$\boldsymbol{Arrow\ Diagram}$



$Given: A = \{-2,-1,0,1,2\}\ and\ f(x) = x^2 + x + 1$

$x = -2,\ f(-2) = (-2)^2 + (-2) + 1 = 4 - 2 + 1 = 3$

$x = -1,\ f(-1) = (-1)^2 + (-1) + 1 = 1 - 1 + 1 = 1$

$x = 0,\ f(0) = (0)^2 + (0) + 1 = 0 + 1 + 1 = 1$

$x = 1,\ f(1) = (1)^2 + (1) + 1 = 1 + 1 + 1 = 3$

$x = 2,\ f(2) = (2)^2 + (2) + 1 = 4 + 2 + 1 = 7$

Onto function:

$Every\ element\ in\ B\ has\ a\ pre - image\ in\ A$

$\because f\ is\ an\ onto\ function, range\ of\ f = B = co - domian\ of\ f$

$\therefore B = \{1,3,7\}$

$\boldsymbol{Example\ 1.15. Let\ f\ be\ a\ function}$ **f**$: \mathbb{N} \longrightarrow \mathbb{N}$ $\boldsymbol{defined\ by\ f(x) = 3x + 2,}$ $\boldsymbol{x \in \mathbb{N}\ (i) Find\ the\ images\ of\ 1,2,3\quad (ii)\ Find\ the\ pre - images\ 29, 53}$ $\boldsymbol{(iii)\ Identify\ the\ type\ of\ function}$

$Given: f: \mathbb{N} \longrightarrow \mathbb{N}\ defined\ by\ f(x) = 3x + 2$

$\boldsymbol{(i)\ Find\ the\ images\ of\ 1,2,3}$

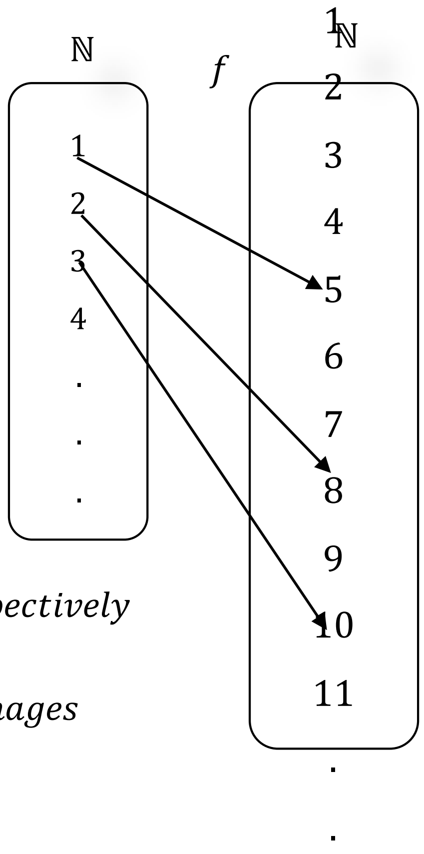
$x = 1,\ f(1) = 3(1) + 2 = 3 + 2 = 5$

$x = 2,\ f(2) = 3(2) + 2 = 6 + 2 = 8$

$x = 3,\ f(3) = 3(3) + 2 = 9 + 2 = 11$

$The\ image\ of\ 1,2,3\ are\ 5,8,11\ respectively$

$\boldsymbol{(ii)\ Find\ the\ pre-\ image\ of\ 29\ ,53}$



$If\ x\ is\ the\ preimage\ of\ 29\ ,then\ f(x) = 29$

$$3x + 2 = 29 \Longrightarrow 3x = 29 - 2$$

$$3x = 27 \Longrightarrow x = \frac{\cancel{27}}{\cancel{3}}\ 9$$

$$\boxed{x = 9}$$

$If\ x\ is\ the\ pre\ image\ of\ 53, then\ f(x) = 53$

$$3x + 2 = 53 \Longrightarrow 3x = 53 - 2$$

$$3x = 51 \Longrightarrow x = \frac{\cancel{51}}{\cancel{3}}\ 17$$

$$\boxed{x = 17}$$

$\therefore The\ pre\ image\ of\ 29\ and\ 53\ are\ 9\ and\ 17\ respectively$

$\boldsymbol{(iii)\ Identify\ the\ type\ of\ function}$

$(iii)\ Since, Different\ elements\ of\ \mathbb{N}\ different\ images$
$in\ the\ co-domain,$

$\boldsymbol{Thus\ f\ is\ one\ -\ one\ function}$

$Co-domain\ of\ f\ is\ \mathbb{N}$

$But\ the\ range\ of\ f\ = \{5,8,11,14,17,\ldots\}\ is\ a\ subset\ of\ \mathbb{N}$

$Therefore, f\ is\ not\ an\ onto\ function.$
$(i.e)\ f\ is\ an\ into\ function.$

$\boldsymbol{Thus\ f\ is\ one\ -one\ and\ into\ function}$

$\boldsymbol{Example\ 1.16. Forensic\ scientists\ can\ determine\ the\ height\ (in\ cms)\ of\ a\ person\ based\ on\ the\ length\ of\ their\ thigh\ bone. They\ usually\ do\ so\ using\ the\ function\ h(b) = 2.47b + 54.10\ where\ b\ is\ the\ length\ of\ thigh\ bone\ .(i)\ Check\ if\ the\ function\ h\ is\ one-one}$

$\boldsymbol{(ii)\ Also\ find\ the\ height\ of\ a\ person\ if\ the\ length\ of\ his\ thigh\ bone\ is\ 50cms.}$

$\boldsymbol{(iii)\ Find\ the\ length\ of\ the\ thigh\ bone\ if\ the\ height\ of\ a\ person\ is\ 147.96\ cms.}$

$\boldsymbol{(i)\ Check\ if\ the\ function\ h\ is\ one-one}$

$Given\ function: h(b) = 2.47b + 54.10$

$We\ assume\ that\ h(b_1) = h(b_2)$

$$2.47b_1 + \cancel{54.10} = 2.47b_2 + \cancel{54.10} \implies \cancel{2.47}b_1 = \cancel{2.47}b_2$$

$$b_1 = b_2 \ \therefore h \text{ is one} - \text{one}$$

(ii) Also find the height of a person if the length of his thigh bone is 50cms.

$Given: b = 50$

$$h(50) = (2.47 \times 50) + 54.10 \implies h(50) = 123.5 + 54.10$$

$$\therefore Height = 177.6 \, cms$$

(iii) Find the length of the thigh bone if the height of a person is 147.96 cms.

$Given: h(b) = 147.96$

$$2.47b + 54.10 = 147.96 \implies 2.47b = 147.96 - 54.10$$

$$2.47b = 147.96 - 54.10 \implies 2.47b = 93.86$$

$$b = \frac{93.86}{2.47} \implies b = \frac{9386}{247} \implies b = 38$$

$$\begin{array}{r} 247\ )\,9386\,(38 \\ 741 \\ \hline 1976 \\ 1976 \\ \hline 0 \end{array}$$

$$\therefore Length = 38 \, cms$$

Example 1.17. Let f be a function from R to R defined by f(x) = 3x − 5. Find the values of ‘a’ and ‘b’ given that (a,4) and (1,b) belong to f.

$Given : f(x) = 3x - 5$

$$(i)\ \underset{(x, f(x))}{(a, 4)} \implies x = a \text{ and } f(x) = 4$$

$$f(x) = 4 \implies 3x - 5 = 4$$

$$\text{where } x = a$$

$$3a - 5 = 4 \implies 3a = 4 + 5$$

$$\cancel{3}a = \underset{3}{\cancel{9}} \implies \boxed{a = 3}$$

$$(ii)\ \underset{(x, f(x))}{(1, b)} \implies x = 1 \text{ and } f(x) = b$$

$$f(x) = b \implies 3x - 5 = b$$

$$\text{where } x = 1$$

$$3(1) - 5 = b \implies b = 3 - 5$$

$$\boxed{b = -2}$$

Example 1.18. The distance S (in kms) travelled by a particle in time ‘t’ hours is given by $S(t) = \frac{t^2 + t}{2}$. Find the distance travelled by the particle after. (i) three and half hours (ii) eight hours and fifteen minutes

Given: The distance travelled by the particle in time t hours is given by

$$S(t) = \frac{t^2 + t}{2}$$

(i) ***Three and Half Hours***

$$\text{Three hours} = 3$$

$$\text{Half hours (30 Minutes)} = \underline{0.5}$$

$$\text{Total Hours} = \underline{3.5}$$

60 Minutes → 1 Hour
30 Minutes → 0.5 Hour

$t = 3.5;$

$$S(t) = \frac{(3.5)^2 + 3.5}{2} = \frac{12.25 + 3.5}{2} = \frac{\overset{7.875}{\cancel{15.75}}}{\cancel{2}} = 7.875$$

Distance travelled in 3.5 *hours is* 7.875*kms*

(ii) ***Eight Hours and Fifteen Minutes***

$$\text{Eight Hours} = 8$$

$$\text{Fifteen Minutes} = \underline{0.25}$$

$$\text{Total Hours} = \underline{8.25}$$

60 Minutes → 1 Hour
30 Minutes → 0.5 Hour
15 Minutes → 0.25 Hour

$$S(t) = \frac{t^2 + t}{2}$$

$t = 8.25;$

$$S(t) = \frac{(8.25)^2 + 8.25}{2} = \frac{68.0625 + 8.25}{2} = \frac{\overset{38.15625}{\cancel{76.3125}}}{\cancel{2}} = 38.15625$$

Distance travelled in 8.25 *hours is* 38.16*kms (approximately)*

***Example* 1.19. *If the function* $f:\mathbb{R} \to \mathbb{R}$ defined by**

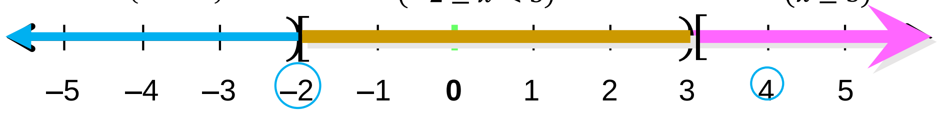
$$f(x) = \begin{cases} 2x + 7, x < -2 \\ x^2 - 2, -2 \le x < 3 \\ 3x - 2, x \ge 3 \end{cases}$$

Then find the values of $(i) f(4)$ $(ii) f(-2)$ $(iii) f(4) + 2f(1)$

(iv) $\dfrac{f(1) - 3f(4)}{f(-3)}$

$f(x) = 2x + 7$ | $f(x) = x^2 - 2$ | $f(x) = 3x - 2$

Interval – I $(x < -2)$ | *Interval – II* $(-2 \le x < 3)$ | *Interval – III* $(x \ge 3)$



(i) $f(4)$

$x = 4;$ *it lies in Interval – III*

$\therefore f(x) = 3x - 2$

$f(4) = 3 \times 4 - 2 = 12 - 2$

$$\boxed{f(4) = 10}$$

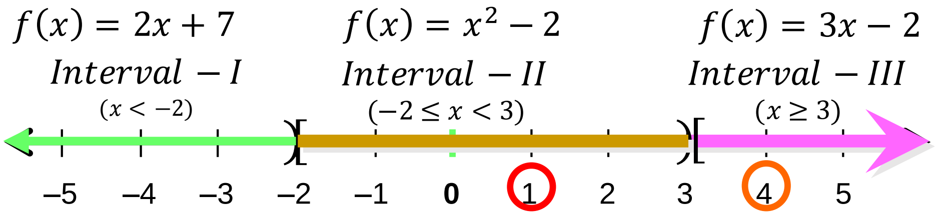
(ii) $f(-2)$

$x = -2;$ *it lies in Interval – II*

$\therefore f(x) = x^2 - 2$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

$$\boxed{f(-2) = 2}$$



$(\boldsymbol{iii})\, \boldsymbol{f(4) + 2f(1)}$

$To\ find : f(4)$

$x = 4;\ it\ lies\ in\ Interval\ \ - III$

$\therefore\ f(x) = 3x - 2$

$f(4)\ = 3 \times 4 - 2 = 12 - 2$

$f(4) = 10$

$To\ find : f(1)$

$x = 1;\ \ it\ lies\ in\ Interval\ \ - II$

$\therefore\ f(x) = x^2 - 2$

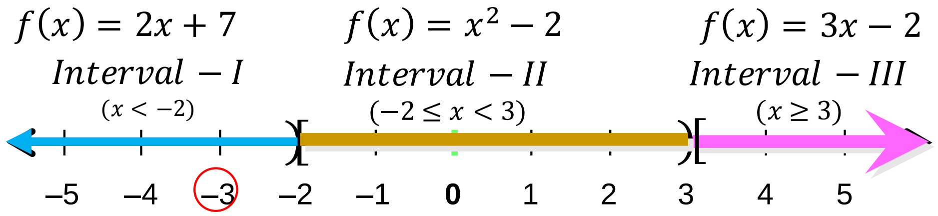
$f(1)\ = 1^2 - 2\ \ = 1 - 2$

$f(1) = -1$

$$f(4) + 2f(1)\ \ = 10 + 2(-1)$$

$$= 10 - 2$$

$$\boxed{f(4) + 2f(1) = 8}$$



$(\boldsymbol{iii})\ \dfrac{\boldsymbol{f(1) - 3f(4)}}{\boldsymbol{f(-3)}}$

$To\ Find\ f(-3)$

$x = -3; it\ lies\ in\ Interval - II$

$\therefore\ f(x) = 2x + 7$

$f(-3)\ = 2(-3) + 7 = -6 + 7$

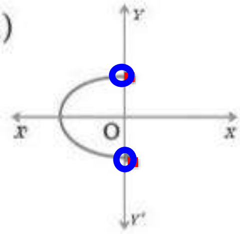
$f(-3) = 1$

$$\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1 - 3(10)}{1}\ \ = \frac{-1 - 30}{1}$$

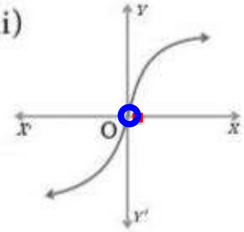
$$\boxed{\frac{f(1) - 3f(4)}{f(-3)} = -31}$$

1. Determine whether the graph given below represent functions. Give reason for your answer concerning each graph.

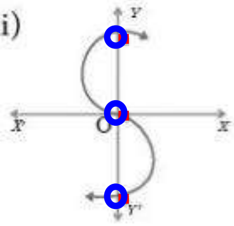
(i)



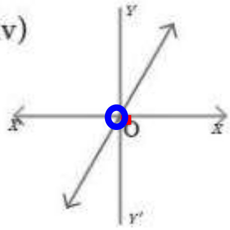
(ii)



(iii)



(iv)



(i) *The curve do not represent a function since it meets y axis at 2 points*

(ii) *The curve represents a function as it, meets x − axis or y − axis at only one point*

(iii) *The curve do not represent a function since it meets y axis at 2 points*

(iv) *The line represents a function as it meets axes at origin*

2. Let f: $A \longrightarrow B$ be a function define by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by (i) set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a Graph

Given : $A = \{2,4,6,10,12\}, B = \{0,1,2,4,5,9\}$

$$f(x) = \frac{x}{2} - 1$$

$$x = 2;\ f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$$

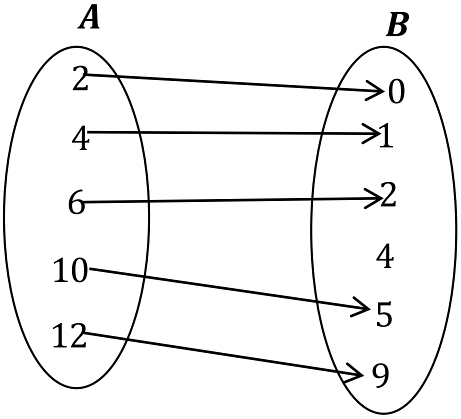
$$x = 4;\ f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$$

$$x = 6;\ f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$$

$$x = 10;\ f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$$

$$x = 12;\ f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$$

(i) Arrow Diagram



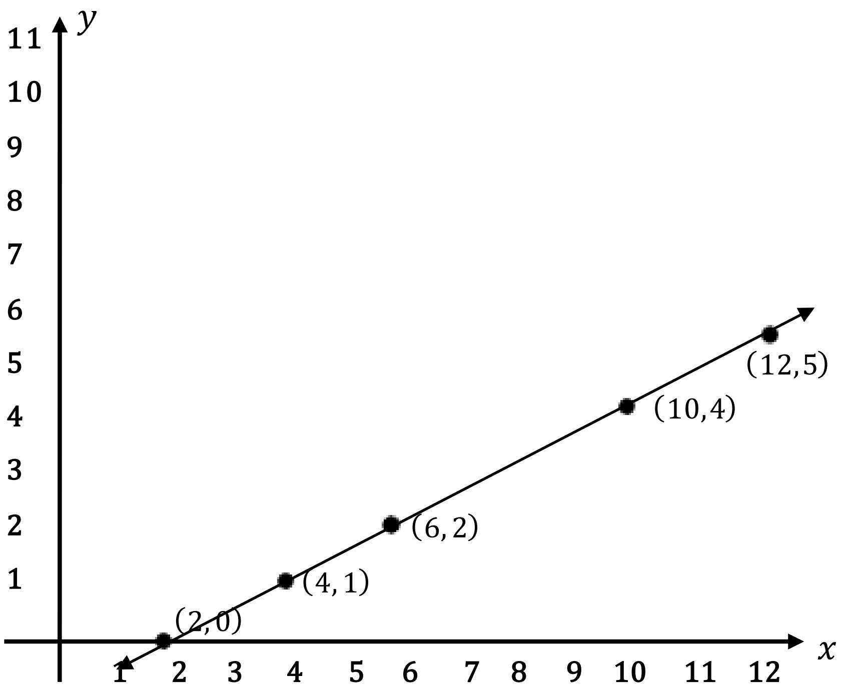
(ii) Table Form

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) set of ordered pairs $f = \{(2,0), (4,1), (6,2), (10,4), (12,5)\}$

(iv) Graphical Form

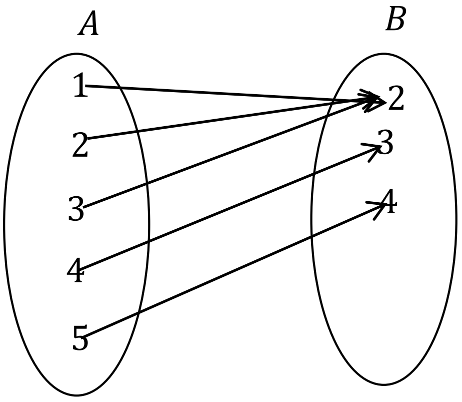
Points to be plotted are $(\mathbf{2}, \mathbf{0}), (\mathbf{4}, \mathbf{1}), (\mathbf{6}, \mathbf{2}), (\mathbf{10}, \mathbf{4}), (\mathbf{12}, \mathbf{5})$



3. ***Represent the function*** $\boldsymbol{f(x) = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}}$ ***through i) an arrow diagram (ii) a table form (iii) a Graph***

Given: $f(x) = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$

(i) Arrow Diagram



(ii) Table Form

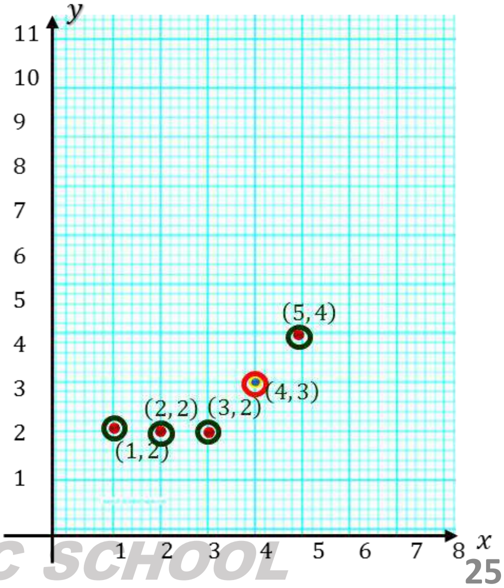
x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) set of ordered pairs

$f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$

(iv) Graphical Form

Points to be plotted on the x − y plane are
$\boldsymbol{(1,2),(2,2),(3,2),(4,3),(5,4)}$



4. $\boldsymbol{Show\ that\ the\ function\ f: N \to N\ defined\ f(x) = 2x - 1\ is\ one - one\ but\ not\ onto.}$

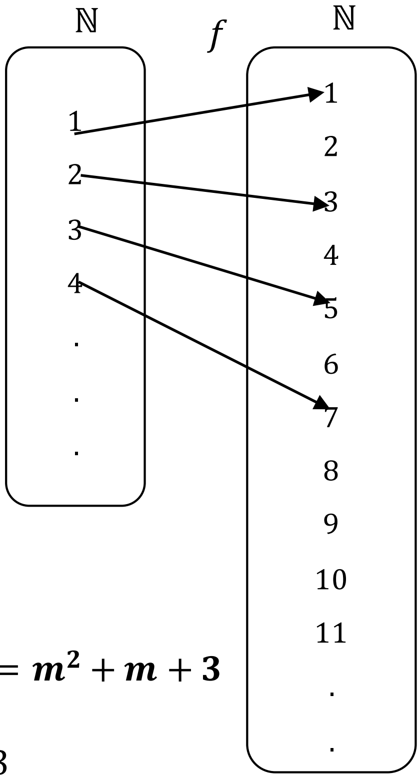
$Given: f: N \to N\ \&\ defined\ by\ f(x) = 2x - 1$

$x = 1 \Longrightarrow f(1) = 2(1) - 1 = 2 - 1 = 1$

$x = 2 \Longrightarrow f(2) = 2(2) - 1 = 4 - 1 = 3$

$x = 3 \Longrightarrow f(3) = 2(3) - 1 = 6 - 1 = 5$

$x = 4 \Longrightarrow f(4) = 2(4) - 1 = 8 - 1 = 7$



$f: N \to N\ and\ for\ different\ elements\ in\ domain$

$having\ different\ images\ in\ co - domain$

$\therefore \boldsymbol{f\ is\ one - one function}$

$co - domain\ in\ N$

$Range = \{1,3,5,7, \dots \dots\}$

$Range \neq Co - domain$

$\therefore \boldsymbol{f\ is\ not\ onto\ function}$

5. Show that the function f:N → N defined by $\boldsymbol{f(m) = m^2 + m + 3}$ is one–one function.

$Given: f: N \to N\ \&\ defined\ by\ f(m) = m^2 + m + 3$

$$f(m) = m^2 + m + 3$$

$m = 1 \Longrightarrow f(1) = 1^2 + 1 + 3 = 1 + 1 + 3 = 5$

$m = 2 \Longrightarrow f(2) = 2^2 + 2 + 3 = 4 + 2 + 3 = 10$

$m = 3 \Longrightarrow f(3) = 3^2 + 3 + 3 = 9 + 3 + 3 = 15$

$For\ different\ elements\ of\ domain\ having\ different\ image\ in\ co\ domain$

$\therefore \boldsymbol{f\ is\ one - one\ function}$

6. $\boldsymbol{Let\ A = \{1, 2, 3, 4\}\ and\ B = N. Let\ f: A \to B\ be\ defined\ by\ f(x) = x^3\ then, (i) find\ the\ range\ of\ f\ (ii)\ identity\ the\ type\ of\ function}$

$\text{A} = \{1, 2, 3, 4\};\ \text{B} = N = \{1, 2, 3, 4, \dots.\}$

$f(x) = x^3$

$x = 1;\ f(1) = 1^3 = 1$

$x = 2;\ f(2) = 2^3 = 8$

$x = 3;\ f(3) = 3^3 = 27$

$x = 4;\ f(4) = 4^3 = 64$

$(i)\ Range = \{1, 8, 27, 64\}$

$(ii) f\ is\ one - one\ functoin$

$(different\ elements\ have\ different\ images)$

$f\ is\ into\ function\ (Range \neq Co - domain)$

7. In each of the following cases state whether the function is bijective or not. justify your answer.

$\boldsymbol{(i)\ f: R \to R\ defined\ by\ f(x) = 2x + 1}$

$\boldsymbol{(ii)\ f: R \to R\ defined\ f(x) = 3 - 4x^2}$

$Given:\ \ f: R \to R\ defined\ by\ f(x) = 2x + 1$

$Let\ f(x_1) = f(x_2) \Longrightarrow\ \ 2x_1 + \cancel{1} = 2x_2 + \cancel{1}$

$2x_1 = 2x_2\ \ \Longrightarrow\ \ \cancel{2}x_1 = \cancel{2}x_2$

$x_1 = x_2$

$\boldsymbol{\therefore f\ is\ one\ to\ one\ function}$

$Let\ us\ take, y = 2x + 1\ [W.K.T\ y = f(x)]$

$$2x = y - 1 \Longrightarrow\ \ x = \frac{y-1}{2}$$

$$f(x) = 2x + 1 \Longrightarrow\ \ f(x) = \cancel{2}\left(\frac{y-1}{\cancel{2}}\right) + 1 \Longrightarrow\ \ f(x) = y - \cancel{1} + \cancel{1}$$

$f(x) = y$

$\boldsymbol{\therefore f\ is\ onto\ function}$

$\boldsymbol{\therefore f\ is\ one-one\ and\ onto\ \ \ \Longrightarrow f\ is\ bijective}$

$(ii)\ f: R \to R\ defined\ by\ \ \ f(x) = 3 - 4x^2$

$Let, f(x_1) = f(x_2) \Longrightarrow\ \ \cancel{3} - 4x_1^2 = \cancel{3} - 4x_2^2$

$\cancel{-4}x_1^2 = \cancel{-4}x_2^2 \Longrightarrow\ \ x_1^2 = x_2^2 \Longrightarrow\ \ x_1 = x_2\ \ (or) x_1 = -x_2$

$\boldsymbol{f\ is\ not\ one\ \ -one\ function}$

$Example: When\ x = -1, f(x) = f(-1) = 3 - 4(-1)^2 = 3 - 4(1)\ = 3 - 4 = -1$

$When\ x = 1, f(x) = f(1) = 3 - 4(1)^2 = 3 - 4(1) = 3 - 4 = -1$

$Two\ different\ elements\ in\ domain\ have\ same\ images\ in\ co-domain$

$Also, any\ even\ number\ in\ the\ co-domian\ is\ not\ image\ of\ any$
$elements\ x\ in\ the\ domain$

$\boldsymbol{\therefore f\ is\ not\ onto}$

$\boldsymbol{\therefore f\ is\ not\ objective}$

8. $\boldsymbol{Let\ A = \{-1, 1\} and\ B = \{0, 2\}. if\ the\ function\ \ f: A \to B\ defined\ by}$
$\boldsymbol{f(x) = ax + b}$ **is** $\boldsymbol{an\ onto\ function?\ Find\ `a'\ and\ `b'}$

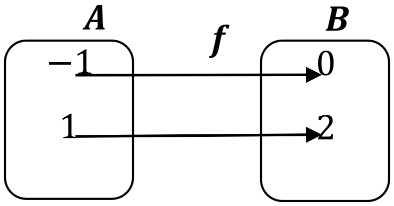
$Given : f(x) = ax + b\ \ and\ A = \{-1,1\}, B = \{0,2\}$

$(i)\ if\ \ x = -1\ and\ f(-1) = 0$

$f(-1) = a(-1) + b\ \Longrightarrow 0 = -a + b$

$-a + b = 0\ \ ...(1)$

$(ii)\ if\ \ x = 1\ and\ f(1) = 2$



$f(1) = a(1) + b \Rightarrow 2 = a + b$

$a + b = 2 \; \dots (2)$

Solve (1) & (2)

$(1) \Rightarrow -\cancel{a} + b = 0$

$(2) \Rightarrow \underline{a + b = 2}$

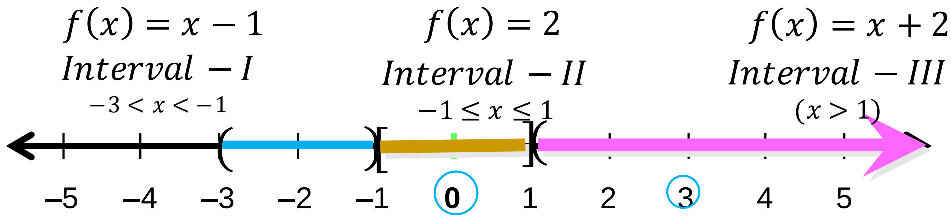
$\cancel{2}b = \cancel{2}1 \Rightarrow b = 1$

Sub $b = 1$ in eq (2) $a + b = 2$

$a + 1 = 2 \Rightarrow a = 2 - 1 \Rightarrow \boxed{a = 1}$

9. $\boldsymbol{If\ the\ function\ is\ defined\ by}\ f(x) = \begin{cases} x + 2 & if \quad x > 1 \\ 2 & if \quad -1 \leq x \leq 1 \\ x - 1 & if \quad -3 < x < -1 \end{cases}$

$\boldsymbol{find\ the\ values\ of}$ **(i)** $f(3)$ **(ii)** $f(0)$ **(iii)** $f(-1.5)$ **(iv)** $f(2) + f(-2)$



(i) $f(3)$

$x = 3$; it lies in Interval − III

$\therefore f(x) = x + 2$

$f(3) = 3 + 2 = 5$

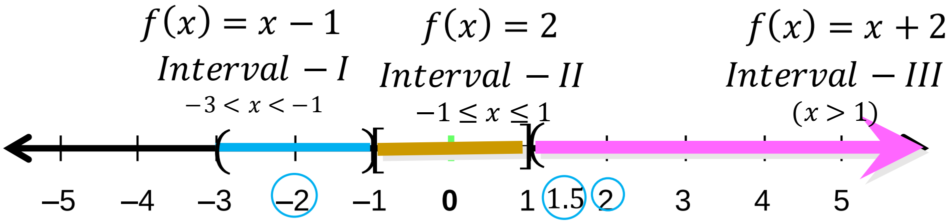
$\boxed{f(3) = 5}$

(ii) $f(0)$

$x = 0$; it lies in Interval − II

$\therefore f(x) = 2$

$\boxed{f(0) = 2}$



(iii) $f(1.5)$

$x = 1.5$; it lies in Interval − III

$\therefore f(x) = x + 2$

$f(1.5) = 1.5 + 2 = 3.5$

$\boxed{f(1.5) = 3.5}$

(iv) $f(2) + f(-2)$

To find $f(2)$

$x = 2$; it lies in Interval − III

$\therefore f(x) = x + 2$

$f(2) = 2 + 2 = 4$

$\boxed{f(2) = 4}$

To find $f(-2)$

$x = -2$; it lies in Interval − III

$\therefore f(x) = x - 1$

$f(-2) = -2 - 1 = -3$

$\boxed{f(-2) = -3}$

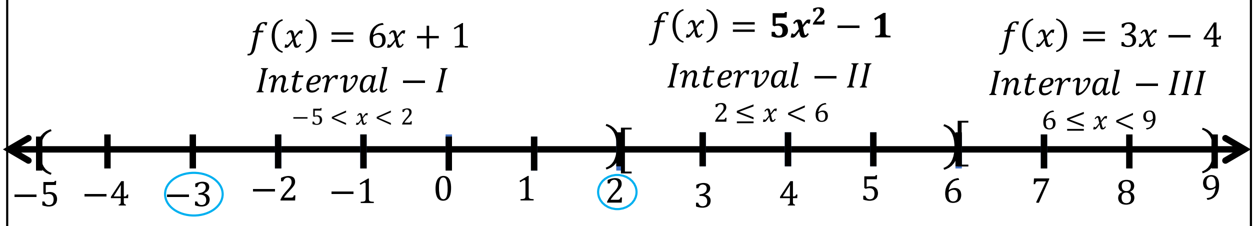
Now, $f(2) + f(-2) = 4 + (-3) = 4 - 3$

$\boldsymbol{f(2) + f(-2) = 1}$

10. A function $f:[-5,9] \to R$ is defined as follows:

$$f(x) = \begin{cases} 6x+1 & if \ -5 \le x < 2 \\ 5x^2-1 & if \quad 2 \le x < 6 \\ 3x-4 & if \quad 6 \le x < 9 \end{cases}$$

$find\ the\ values\ of$ $(i)\ f(-3)+f(2)$ $(ii)\ f(7)-f(1)$ $(iii)\ 2f(4)+f(8)$ $(iv)\ \dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$



$(i) f(-3)+f(2)$

$To\ find\ f(-3)$

$x = -3;\ \ it\ lies\ in\ Interval - I$

$\therefore\ f(x) = 6x+1$

$f(-3) = 6(-3)+1 = -18+1 = -17$

$\boxed{f(-3) = -17}$

$To\ find\ f(2)$

$x = 2; it\ lies\ in\ Interval - II$

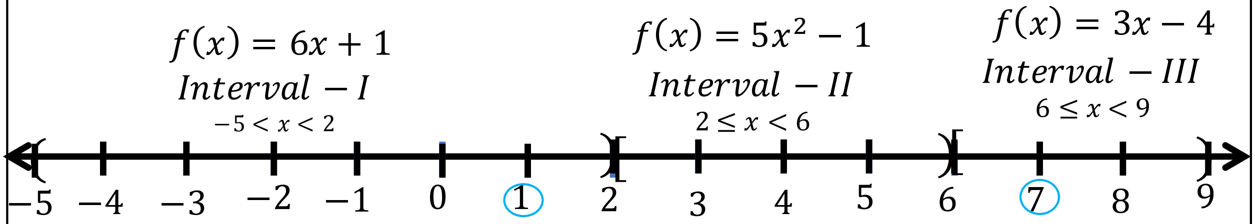
$\therefore\ f(x) = 5x^2 - 1$

$f(2) = 5(2)^2 - 1 = 5(4) - 1$

$f(2) = 20 - 1 \Longrightarrow \boxed{f(-3) = 19}$

$Now\ f(-3)+f(2)\ = -17+19$

$\boxed{f(-3)+f(2) = 2}$



$(ii)\ f(7)-f(1)$

$To\ find\ f(7)$

$x = 7; it\ lies\ in\ Interval - III$

$\therefore\ f(x) = 3x-4$

$f(7) = 3(7) - 4 = 21 - 4$

$\boxed{f(7) = 17}$

$To\ find\ f(1)$

$x = 1; it\ lies\ in\ Interval - I$

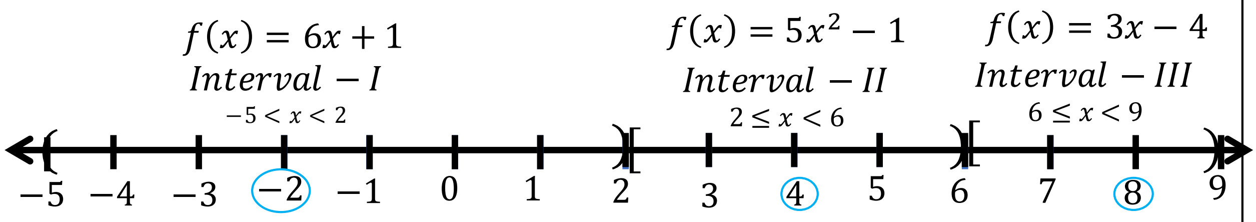
$\therefore\ f(x) = 6x+1$

$f(7) = 6(1) + 1\ = 6+1$

$\boxed{f(1) = 7}$

$Now\ f(7) - f(1) = 17 - 7$

$\boxed{f(7) - f(1) = 10}$



$(iii)\ 2f(4) + f(8)$

$To\ find\ f(4)$

$x = 4; it\ lies\ in\ Interval - II$

$\therefore f(x) = 5x^2 - 1$

$f(4) = 5(4)^2 - 1 = 5(16) - 1$

$= 80 - 1$

$\boxed{f(4) = 79}$

$To\ find\ f(8)$

$x = 8;\ it\ lies\ in\ Interval - III$

$\therefore f(x) = 3x - 4$

$f(8) = 3(8) - 4 = 24 - 4$

$f(8) = 20$

$Now\ 2f(4) + f(8) = 2(79) + 20$

$= 158 + 20$

$\boxed{2f(4) + f(8) = 178}$

$iv)\ \dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

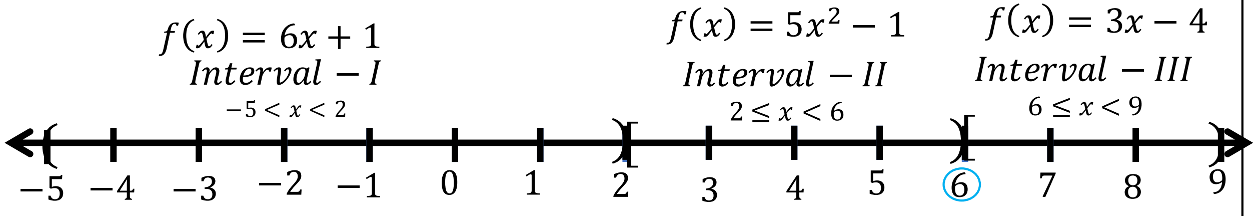
$To\ find\ f(-2)$

$x = -2;\ \ it\ lies\ in\ Interval - I$

$\therefore f(x) = 6x + 1$

$f(-2) = 6(-2) + 1 = -12 + 1$

$\boxed{f(-2) = -11}$



$To\ find\ f(6)$

$x = 6;\ it\ lies\ in\ Interval - III$

$\therefore f(x) = 3x - 4$

$f(6) = 3(6) - 4 = 18 - 4$

$\boxed{f(6) = 14}$

$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 + (-11)} = \frac{-22 - 14}{79 - 11} = \frac{\cancel{-36}^{\,9}}{\cancel{68}_{\,17}}$$

$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{-9}{17}$$

11. The distance S an object travels under the influence of gravity in time t seconds is given by $S(t)=\frac{1}{2}gt^2+at+b$ Where, (g is the acceleration due to gravity), a,b are constants. Check if the function $S(t)$ is one – one.

Given: $S(t)=\frac{1}{2}\,gt^2+at+b$

To verify $S(t)$ is one – one, Let us take $S(t_1)=S(t_2)$

$$\frac{1}{2}gt_1^2+at_1\cancel{+b}=\frac{1}{2}gt_2^2+at_2\cancel{+b}\implies\frac{1}{2}gt_1^2+at_1=\frac{1}{2}gt_2^2+at_2$$

$$\frac{1}{2}gt_1^2-\frac{1}{2}gt_2^2+at_1-at_2=0\implies\frac{1}{2}g(t_1^2-t_2^2)+a(t_1-t_2)=0$$

$$\frac{1}{2}g(t_1+t_2)(t_1-t_2)+a(t_1-t_2)=0\implies(t_1-t_2)\left[\frac{1}{2}g(t_1-t_2)+a\right]=0$$

$$t_1-t_2=0,\quad\because\left[\frac{1}{2}g(t_1-t_2)+a\right]\neq0$$
$$t_1=t_2$$

Hence it is one – one

12. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C)=F$ where $F=\frac{9C}{5}+32$. Find: (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C)=212$ (v) the temperature when the Celsius value is equal to the Fahrenheit value

Given: $F=\frac{9C}{5}+32$ and $t(C)=F$

(i) $t(0)$

Here $C=0$

$$F=\frac{9\times0}{5}+32=0+32$$

$$\boxed{F=32}$$

(ii) $t(28)$

where $C=28$

$$F=\frac{9(28)}{5}+32=\frac{252}{5}+32=50.4+32$$

$$\boxed{F=82.4}$$

(iii) $t(-10)$

Here $C=-10$

$$F=\frac{9(\overset{2}{\cancel{-10}})}{\cancel{5}}+32$$
$$=-18+32$$

$$\boxed{F=14}$$

(iv) when $t(C)=212$ $=F$

$$F=\frac{9C}{5}+32\implies212=\frac{9C}{5}+32$$

$$212-32=\frac{9C}{5}\implies\frac{9C}{5}=180$$

$$C=\overset{20}{\cancel{180}}\times\frac{5}{\cancel{9}}\implies\boxed{\therefore C=100}$$

(v) ***the temperature when the Celsius value is equal to the Fahrenheit value***

when $C = F$

$$F = \frac{9C}{5} + 32 \Rightarrow C = \frac{9C}{5} + 32$$

$$C - \frac{9C}{5} = 32 \Rightarrow \frac{5C - 9C}{5} = 32 \quad \Rightarrow \quad \frac{-4C}{5} = 32$$

$$-\cancel{4}C = \overset{8}{\cancel{32}} \times 5 \Rightarrow -C = 40 \quad \Rightarrow \boxed{C = -40°\text{C}}$$

$\boldsymbol{Example\ 1.20\ Find\ fog\ and\ gof\ when\ f(x) = 2x + 1\ and\ g(x) = x^2 - 2}$

$Given: f(x) = 2x + 1 \ \ and\ g(x) = x^2 - 2$

$To\ Prove: fog = gof$

$$fog = fog(x) = f(g(x))$$

$$= f(x^2 - 2) = 2(x^2 - 2) + 1 = 2x^2 - 4 + 1$$

$$\boldsymbol{fog = 2x^2 - 3} \quad \dots (\mathbf{1})$$

$$gof = gof(x) = g\big(f(x)\big)$$

$$= g(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x + 1 - 2$$

$$\boldsymbol{gof = 4x^2 + 4x - 1} \quad \dots (\mathbf{2})$$

$From\ (1)\ and\ (2)$

$$\boldsymbol{gof(x) \neq fog(x)}$$

$\boldsymbol{Example\ 1.21.\ Represent\ the\ function\ f(x) = \sqrt{2x^2 - 5x + 3}\ as\ a\ composition\ of\ two\ functions.}$

$Let\ \ f_2(x) = 2x^2 - 5x + 3\ \ and\ f_1(x) = \sqrt{x}$

$Then\ \ f(x) = \sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)}$

$$= f_1[f_2(x)]$$

$$f(x) = f_1 f_2(x)$$

$\boldsymbol{Example\ 1.22.\ If\ f(x) = 3x - 2, g(x) = 2x + k\ and\ if\ fog = gof, then\ find\ the\ value\ of\ k.}$

$Given\ : f(x) = 3x - 2, g(x) = 2x + k$

$$fog(x) = gof(x) \dots (1)$$

$$fog(x) = f\big(g(x)\big) = f(2x + k) = 3[2x + k] - 2 = 6x + 3k - 2$$

$$gof(x) = g\big(f(x)\big) = g(3x - 2) = 2[3x - 2] + k = 6x - 4 + k$$

$Substitute\ both\ the\ values\ in\ (1)$

$$\cancel{6}k + 3k - 2 = \cancel{6}k - 4 + k$$

$$3k - k = -4 + 2$$

$$\cancel{2}k = -\cancel{2} \Rightarrow \boxed{k = -1}$$

Example 1.23 Find k if $fof(k) = 5$ where $f(k) = 2k - 1$

$$Given: fof(k) = 5 \text{ } where \text{ } f(k) = 2k - 1$$

$$fof(k) = 5$$

$$f\big(f(k)\big) = 5 \Rightarrow f(2k-1) = 1$$

$$2(2k-1) - 1 = 5 \Rightarrow 4k - 2 - 1 = 5$$

$$4k - 3 = 5 \Rightarrow 4k = 5 + 3$$

$$4k = 8 \Rightarrow k = \frac{\cancel{8}^{\,2}}{\cancel{4}} \Rightarrow \boxed{k = 2}$$

Example 1.24. Find $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$,

Prove that $fo(goh) = (fog)oh$

$$Given: f(x) = 2x + 3, g(x) = 1 - 2x, h(x) = 3x$$

$$To\ Prove: fo(goh) = (fog)oh$$

$$(fog) = (fog)x$$

$$= f(g(x)) = f(1 - 2x)$$

$$= 2(1 - 2x) + 3 = 2 - 4x + 3$$

$$\boxed{(fog)x = 5 - 4x}$$

$$(fog)oh = fog(h(x))$$

$$= fog[3x] = 5 - 4(3x)$$

$$fog(h(x)) = 5 - 12x \quad \dots(1)$$

$$goh = goh(x)$$

$$= g(h(x)) = g(3x)$$

$$= 1 - 2(3x)$$

$$\boxed{goh(x) = 1 - 6x}$$

$$fo(goh) = fo(goh)(x)$$

$$= f[(goh)(x)] = f(1 - 6x)$$

$$= 2(1 - 6x) + 3 = 2 - 12x + 3$$

$$fo(goh)(x) = 5 - 12x \dots(2)$$

$$From\ (1)\ and\ (2)$$

$$(fog)oh = fo(goh) \text{ } . It\ is\ proved$$

Example 1.25. Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

$Given: f(x) = 3x + 1, g(x) = x + 3$

$Find:\ x$ if $gff(x) = fgg(x) \quad \dots (1)$

$$gff(x) = g[f[f(x)]\,]$$

$$= g[f(3x+1)] = g[3(3x+1)+1] = g[9x+3+1]$$

$$= g[9x+4] = 9x+4+3$$

$$gff(x) = 9x + 7 \quad \dots (2)$$

$$fgg(x) = f\big[g[g(x)]\big]$$

$$= f\big[g[x+3]\big] = f[x+3+3] = f[x+6]$$

$$= 3(x+6)+1 = 3x+18+1$$

$$fgg(x) = 3x + 19 \quad \dots (3)$$

$From\ (1),\ gff(x) = fgg(x)$

$$9x + 7 = 3x + 19 \implies 9x - 3x + 7 - 19 = 0$$

$$6x - 12 = 0 \implies \cancel{6}x = \cancel{12}^{2} \implies \boxed{x = 2}$$

1. *Using the functions f and g given below, find fog and gof.*

$(\boldsymbol{i})\boldsymbol{f(x) = x - 6. g(x) = x^2} \qquad (\boldsymbol{ii})\boldsymbol{f(x) = \frac{2}{x}, g(x) = 2x^2 - 1}$

$(\boldsymbol{iii})\boldsymbol{f(x) = \frac{x+6}{3}, g(x) = 3 - x} \qquad (\boldsymbol{iv})\boldsymbol{f(x) = 3 + x, g(x) = x - 4}$

$(\boldsymbol{v})\ \boldsymbol{f(x) = 4x^2 - 1, g(x) = 1 + x}$

$(i)\ f(x) = x - 6, g(x) = x^2$

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

$$fog = fog(x) = f[g(x)] = f(x^2)$$

$$\boxed{fog = x^2 - 6}$$

$$gof = gof(x) = g[f(x)] = g[x-6] = (x-6)^2$$

$$gof = x^2 - 12x + 36$$

$$\therefore fog \neq gof$$

$(\boldsymbol{ii})\ \boldsymbol{f(x) = \frac{2}{x}, g(x) = 2x^2 - 1}$

$$fog = fog(x) = f[g(x)] = f[2x^2 - 1]$$

$$\boxed{fog = \frac{2}{2x^2 - 1}}$$

$$gof = gof(x) = g[f(x)] = g\left[\frac{2}{x}\right] = 2\left(\frac{2}{x}\right)^2 - 1 = 2 \times \frac{4}{x^2} - 1$$

$$\boxed{gof = \frac{8}{x^2} - 1}$$

$$\boldsymbol{\therefore fog \neq gof}$$

$$\boldsymbol{(iii)\ f(x)=\frac{x+6}{3},g(x)=3-x}$$

$$fog = fog(x) = f[g(x)] = f(3-x) = \frac{3-x+6}{3}$$

$$\boxed{fog\ = \frac{9-x}{3}}$$

$$gof\ = gof(x)\ = g[f(x)] = g\left[\frac{x+6}{3}\right] = 3-\left(\frac{x+6}{3}\right) = \frac{9-x-6}{3}$$

$$\boxed{gof\ = \frac{3-x}{3}}$$

$$\therefore fog \neq gof$$

$$\boldsymbol{(iv)\ f(x)=3+x,g(x)=x-4}$$

$$fog = fog(x) = f[g(x)] = f(x-4) = 3+x-4$$

$$\boxed{fog\ = x-1}$$

$$gof\ = gof(x)\ = g[f(x)] = g[3+x] = 3+x-4$$

$$\boxed{gof\ = x-1}$$

$$\therefore fog = gof$$

$$\boldsymbol{(v)\ f(x)=4x^2-1,g(x)=1+x}$$

$$fog = fog(x) = f[g(x)] = f(1+x) = 4(1+x)^2-1$$

$$= 4(1+2x+x^2)-1 = 4+8x+4x^2-1$$

$$\boxed{fog\ = 4x^2+8x+3}$$

$$gof\ = gof(x)\ = g[f(x)] = g[4x^2-1] = 1+4x^2-1$$

$$\boxed{gof\ = 4x^2}$$

$$\boxed{(a+b)^2 = a^2+2ab+b^2}$$

$$\therefore fog \neq gof$$

2. Find the value of k, such that $fog = gof$.

$(i)\ f(x) = 3x+2.\ g(x) = 6x-k$

$(ii)\ f(x) = 2x-k, g(x) = 4x+5$

$$(i)\ f(x) = 3x+2, g(x) = 6x-k$$

$$fog = (fog)x = f[g(x)] = f(6x-k) = 3(6x-k)+2$$

$$fog\ = 18x-3k+2$$

$$gof\ = gof(x) = g[f(x)] = g[3x+2] = 6(3x+2)-k$$

$$gof\ = 18x+12-k$$

$$Now, fog = gof$$

$$\cancel{18x} - 3k + 2 = \cancel{18x} + 12 - k$$

$$-3k + k = 12 - 2$$

$$-2k = 10 \Longrightarrow \boxed{k = -5}$$

$(ii)\ \boldsymbol{f(x) = 2x - k, g(x) = 4x + 5}$

$$fog = (fog)x = f[g(x)] = f(4x + 5) = 2(4x + 5) - k$$

$$fog = 8x + 10 - k$$

$$gof = gof(x) = g[f(x)] = g[2x - k] = 4(2x - k) + 5$$

$$gof = 8x - 4k + 5$$

$$Now, fog = gof$$

$$\cancel{8}x + 10 - k = \cancel{8}x - 4k + 5 \Longrightarrow 10 - k = -4k + 5$$

$$-k + 4k = -10 + 5 \Longrightarrow 3k = -5$$

$$\boxed{k = -\frac{5}{3}}$$

3. $\boldsymbol{If\ f(x) = 2x - 1, g(x) = \frac{x+1}{2}. Show\ that\ fog = gof = x.}$

$$Given: f(x) = 2x - 1, g(x) = \frac{x+1}{2}$$

$$fog = (fog)x = f[g(x)] = f\left[\frac{x+1}{2}\right] = \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1 = x + \cancel{1} - \cancel{1}$$

$$\boldsymbol{fog = x}$$

$$gof = gof(x) = g[f(x)] = g[2x - 1] = \frac{2x - \cancel{1} + \cancel{1}}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$\boldsymbol{gof = x}$$

$$\therefore fog = gof \quad Hence\ proved$$

4. $\boldsymbol{(i)\ If\ f(x) = x^2 - 1, g(x) = x - 2\ find\ a, if\ gof(a) = 1}$
$\boldsymbol{(ii)\ Find\ k,}$ **if** $\boldsymbol{f(k) = 2k - 1\ and\ fof(k) = 5}$

$(i)\ If\ f(x) = x^2 - 1, g(x) = x - 2\ find\ a, if\ gof(a) = 1$

$$gof(a) = 1 \Longrightarrow g(f(a)) = 1$$

$$g(a^2 - 1) = 1 \Longrightarrow a^2 - 1 - 2 = 1 \Longrightarrow a^2 - 3 = 1$$

$$a^2 = 1 + 3 \Longrightarrow a^2 = 4 \Longrightarrow \boxed{a = \pm 2}$$

$\boldsymbol{(ii)\ Find\ k,}$ **if** $\boldsymbol{f(k) = 2k - 1\ and\ fof(k) = 5}$

$$fof(k) = 5 \Rightarrow f(f(k)) = 5$$

$$f(2k - 1) = 5 \Rightarrow 2(2k - 1) - 1 = 5 \Rightarrow 4k - 2 - 1 = 5$$

$$4k-3=5 \Rightarrow 4k=5+3 \Rightarrow 4k=8 \Rightarrow k=\frac{\overset{2}{\cancel{8}}}{\cancel{4}} \Rightarrow \boxed{k=2}$$

5. Let $A,B,C\subseteq N$ and a function $f:A\longrightarrow B$ be defined by $f(x)=2x+1$ and $g:B\longrightarrow C$ be defined by $g(x)=x^2$. Find the range of fog and gof.

Given: $f:A\to B,\ g:B\to C$ *where* $A,B,C\subseteq N$

$$f(x)=2x+1, g(x)=x^2$$

Range of fog: $fog=(fog)x=f[g(x)]=f(x^2)$

$$\boxed{fog=2x^2+1}$$

$\therefore$ *Range of* $fog=\{y\ \}$

Range of gof: $gof=(gof)x=g[f(x)]=g(2x+1)$

$gof=(2x+1)^2$ Where, $y=(2x+1)^2, x\in N$

$\therefore$ *Range of* $gof=\{y\}$ Where, $y=2x^2+1, x\in N$

6. Let $f(x)=x^2-1$. Find (i) fof (ii) $fofof$

Given: $f(x)=x^2-1$

(i) $fof=(fof)x=f[f(x)]$

$$=f(x^2-1)=(x^2-1)^2-1=x^4-2x^2+\cancel{1}-\cancel{1}$$

$$fof=x^4-2x^2$$

(ii) $fofof=(fofof)x=fof[f(x)]$

$$\boxed{(a-b)^2=a^2-2ab+b^2}$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$=fof(x^2-1)=(x^2-1)^4-2(x^2-1)^2$$

$$=(x^2-1)^2\ [(x^2-1)^2-2]$$

$$=(x^4-2x^2+1)[x^4-2x^2+1-2]$$

$$=(x^4-2x^2+1)(x^4-2x^2-1)$$

$$=(x^4-2x^2+1)(x^4-2x^2-1)$$

$$fofof=(x^4-2x^2)^2-1^2$$

7. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x^5$ and $g(x)=x^4$ then check if f,g are one − one and fog is one − one?

Given: $f(x)=x^5, g(x)=x^4$

Let, A be the domian

B be the co − domian

For every element $\in A$, *there is a unique image in B.*

Since, f is an odd function $\therefore$ *f is one − one function*

But $g(x)$ *is an even function*

$\therefore$ *Two elements of domain will have the same image in co − domian*

$\therefore$ *g is not one − one function*

$Given:\ f(x) = x^5, g(x) = x^4$

$$fog = (fog)x = f[g(x)] = f(x^4) = (x^4)^5$$

$$\boxed{fog = x^{20}}$$

(fog) *is an even function*

$\therefore$ *Two elements of domain will have the same image in co* $-$ *domian*

$\therefore (fog)$ *is not one* $-$ *one function*

8. Consider the functions $f(x), g(x), h(x)$ as given below. Show that $(fog)oh = fo(goh)$ in each case.

(i) $f(x) = x - 1, g(x) = 3x + 1$ and $h(x) = x^2$

(ii) $f(x) = x^2, g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4, g(x) = x^2$ and $h(x) = 3x - 5$

$(i)\ f(x) = x - 1, g(x) = 3x + 1$ *and* $h(x) = x^2$

$$fog = (fog)(x) = f[g(x)] = f(3x + 1) = 3x + \cancel{1} - \cancel{1}$$

$$\boxed{fog = 3x}$$

$$(fog)oh = (fog)oh(x) = (fog)(h(x)) = (fog)(x^2)$$

$$\boxed{(fog)oh = 3x^2}$$

$$goh = (goh)(x) = g[h(x)] = g(x^2)$$

$$goh = 3x^2 + 1$$

$$fo(goh) = fo(goh)(x) = f[(goh)(x)] = f(3x^2 + 1) = 3x^2 + \cancel{1} - \cancel{1}$$

$$\boxed{fo(goh) = 3x^2}$$

$\therefore (fog)oh = fo(goh)$ *Hence Proved*

(ii) $f(x) = x^2, g(x) = 2x$ and $h(x) = x + 4$

$$fog = (fog)(x) = f[g(x)] = f(2x) = (2x)^2$$

$$\boxed{fog = 4x^2}$$

$$(fog)oh = (fog)oh(x) = (fog)(h(x)) = (fog)(x + 4)$$

$$(fog)oh = 4(x + 4)^2$$

$$goh = (goh)(x) = g[h(x)] = g(x + 4)$$

$$\boxed{goh = 2(x + 4)}$$

$$fo(goh) = fo(goh)(x) = f[(goh)(x)] = f[2(x + 4)] = [2(x + 4)]^2$$

$$fo(goh) = 4(x + 4)^2$$

$\therefore (fog)oh = fo(goh)$ *Hence Proved*

$\boldsymbol{(iii)\ f(x) = x - 4, g(x) = x^2\ and\ h(x) = 3x - 5}$

$$fog = (fog)(x) = f[g(x)] = f(x^2)$$

$$fog = x^2 - 4$$

$$(fog)oh = (fog)oh(x) = (fog)(h(x)) = (fog)(3x - 5)$$

$$(fog)oh = (3x - 5)^2 - 4$$

$$goh = (goh)(x) = g[h(x)] = g(3x - 5)$$

$$goh = (3x - 5)^2$$

$$fo(goh) = fo(goh)(x) = f[(goh)(x)] = f[(3x - 5)^2]$$

$$fo(goh) = (3x - 5)^2 - 4$$

$$\therefore (fog)oh = fo(goh) \quad Hence\ Proved$$

9. $\boldsymbol{Let\ f = \{(-1,3), (0,-1), (2,-9)\}\ be\ a\ linear\ function\ from\ Z\ to\ Z.}$ $\boldsymbol{Find\ f(x)}$

$Given\ f = \{(-1,3), (0,-1), (2,-9)\}\ is\ a\ linear\ function\ from\ Z\ to\ Z.$

$To\ Find: f(x)$

$Let, y = ax + b \ldots (1)$

$When\ x = -1, y = 3 \Longrightarrow 3 = -a + b \ldots (2)$

$When\ x = 0, y = -1 \Longrightarrow -1 = 0 + b \ \ldots (3)$

$$\boxed{b = -1}$$

$Substitute, b = -1\ in\ (2)\ 3 = -a + b$

$3 = -a + (-1) \Longrightarrow 3 + 1 = -a$

$$4 = -a \Longrightarrow \boxed{a = -4}$$

$Substitute, a = -4\ and\ b = -1\ in\ (1)\ \ y = ax + b$

$$y = (-4)x + (-1)$$

$y = -4x - 1\ is\ the\ required\ \ Linear\ Function$

10. $\boldsymbol{In\ electrical\ circuit\ theory, a\ circuit\ C(t)\ is\ called\ a\ linear\ circuit\ if\ it\ satisfies\ the\ superposition\ principle\ given\ by\ C(at_1 + bt_2) = aC(t_1) + bC(t_2) where\ a, b\ are\ constants. Show\ that\ the\ circuit\ C(t) = 3t\ is\ linear.}$

$Given\ C(t) = 3t$

$To\ Prove: C(t)\ is\ \ linear$

$C(at_1) = 3at_1\ \ , C(bt_2) = 3bt^2$

$Adding, C(at_1) + C(bt_2) = 3at_1 + 3bt_2 \ = 3(at_1 + bt_2)$

$\therefore C(at_1) + C(bt_2) = 3(at_1 + bt_2)\ which\ is\ the\ principle\ of\ super\ position$

$\therefore C(t) = 3t\ is\ Linear\ Function$

NUMBER & SEQUENCE

Exercise 2.1

***Example 2.1**: **We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked**?*

34	=	5	×	6	+	4
Total number of cakes	=	*Number of cakes in each box*	×	*Number of box*	+	*Number of cakes left over*
↓		↓		↓		↓
Dividend a	=	*Divisor* b	×	*Quotient* q	+	*Remainder* r

5) 34 (6
30
―――
4

$$34 = 5 \times 6 + 4$$

***Example 2.2**: **Find the quotient and remainder when a is divided by b in the following cases** (i) $\boldsymbol{a = -12, b = 5}$ (ii) $\boldsymbol{a = 17, b = -3}$ (iii) $\boldsymbol{a = -19, b = -4}$*

(i) $\boldsymbol{a = -12, b = 5}$

By Euclid's division lemma

$$a = bq + r, \text{where } 0 \le r < |b|$$

$$-12 = 5q + r\ , \text{where } 0 \le r < |5|$$

$$-12 = 5(-3) + 3$$

Quotient : $q = -3$ *Remainder* : $r = 3$

5) 12 (3
15
―――
3

$a = -12,\ b = 5,\ q = -3,\ r = 3$

(ii) $\boldsymbol{a = 17, b = -3}$

By Euclid's division lemma

$$a = bq + r, \text{where } 0 \le r < |b|$$

$$17 = -3q + r\ , \text{where } 0 \le r < |-3|$$

$$17 = -3(-5) + 2$$

Quotient : $q = -5$ *Remainder* : $r = 2$

3) 17 (5
15
―――
2

$a = 17,\ b = -3,\ q = -5,\ r = 2$

(iii) $\boldsymbol{a = -19, b = -4}$

By Euclid's division lemma

$$a = bq + r, \text{where } 0 \le r < |b|$$

$$-19 = -4q + r, \text{where } 0 \le r < |-4|$$

4) 19 (5
20
―――
1

$a = -19,\ b = -4,\ q = 5,\ r = 1$

$$-19 = -4(5) + 1$$

Quotient : $q = 5$ *Remainder* : $r = 1$

Example 2.3: Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Let x be any odd integer. Since any odd integer is one more than an even integer $x = 2k + 1$, *for some integer k.*

$$x^2 = (2k + 1)^2$$
$$= (2k)^2 + 2(2k)(1) + 1^2$$
$$= 4k^2 + 4k + 1$$
$$= 4(k^2 + k) + 1 \quad \text{where } q = k^2 + k \text{ is some integer}$$
$$= 4q + 1$$

Example 2.4: If the Highest Common Factor of 210 and 55 is expressible in the form $5x - 325$, find x.

Using Euclid's Division Algorithm, let us find the HCF of given numbers

$$210 = 55 \times 3 + 45$$
$$55 = 45 \times 1 + 10$$
$$45 = 10 \times 4 + 5$$
$$10 = 5 \times 2 + 0$$

55) 210 (3
165
45

45) 55 (1
45
10

10) 45 (4
40
5

5) 10 (2
10
0

Since the remainder is zero, the process stops.
Since the divisor at this stage is 5

5 *is the Highest Common Factor (HCF) of* 210 *and* 55.

HCF is expressible in the form $5x - 325 = 5$

$$5x = 5 + 325$$
$$5x = 330 \implies x = \frac{330}{5}$$
$$x = 66$$

Example 2.5: Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Since the remainders are 4, 5 *respectively the required number is the HCF of the number*

$$445 - 4 = 441$$
$$572 - 5 = 567$$

Hence, determine the HCF of 441 *and* 567. *Using Euclid's Division Algorithm,*

$$567 = 441 \times 1 + 126$$
$$441 = 126 \times 3 + 63$$
$$126 = 63 \times 2 + 0$$

441) 567 (1
441
126

126) 441 (3
378
63

63) 126 (2
126
0

HCF of 441, 567 = 63 *and so the required number is* 63.

Example 2.6: Find the HCF of 396, 504, 636.

To find HCF of three given numbers, first we have to find HCF of the first two numbers.

To find HCF of 396 *and* 504

We start with the larger integer, that is, 504.

Using Euclid's Division Algorithm

$$504 = 396 \times 1 + 108$$

396) 504 (1
396
108

Since remainder 108 ≠ 0, *we apply theEuclid's Division Algorithm to* 396 *and* 108 *to obtain*

$$396 = 108 \times 3 + 72$$

108) 396 (3
324
72

Since remainder 72 ≠ 0, *we apply theEuclid's Division Algorithm to* 108 *and* 72 *to obtain*

$$108 = 72 \times 1 + 36$$

72) 108 (1
72
36

Since remainder 36 ≠ 0, *we apply theEuclid's Division Algorithm to* 72 *and* 36 *to obtain*

$$72 = 36 \times 2 + 0$$

36) 72 (2
72
0

Here the remainder is zero. Therefore HCF of 396, 504 = 36.

To find the HCF of 636 *and* 36. *Using Euclid's Division Algorithm*

$$636 = 36 \times 17 + 24$$

36) 636 (17
612
24

Since remainder 24 ≠ 0, *we apply theEuclid's Division Algorithm to* 36 *and* 24 *to obtain*

$$36 = 24 \times 1 + 12$$

24) 36 (1
24
12

Since remainder 12 ≠ 0, *we apply theEuclid's Division Algorithm to* 24 *and* 12 *to obtain*

$$24 = 12 \times 2 + 0$$

12) 24 (2
24
0

Here the remainder is zero. Therefore HCF of 636, 36 = 12.

Hence HCF of 396, 504 *and* 636 *is* 12

1. *Find all positive integers, when divided by 3 leaves remainder 2.*

Given

Divisor : $b = 3$ *Remainder* : $r = 2$

By Euclid's division lemma $a = bq + r$, *where* $0 \le r < |b|$

$$a = 3q + 2, \text{ where } q \ge 0$$

$$q = 0 \Longrightarrow a = 3(0) + 2$$

$$a = 2$$

$$q = 1 \Longrightarrow \; a = 3(1) + 2 = 3 + 2$$

$$a = 5$$

$$q = 2 \Longrightarrow \; a = 3(2) + 2 = 6 + 2$$

$$a = 8$$

$$q = 3 \Longrightarrow \; a = 3(3) + 2 = 9 + 2$$

$$a = 11$$

The positive integers are $2, 5, 8, 11, \ldots$

2. *A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.*

Total no. of flower pots : *Dividend* $a = 532$

Each row contains 21 flower pots : *Divisor* $b = 21$

By Euclid's division lemma $a = bq + r$

$$532 = 21(25) + 7$$

$$\begin{array}{l} 21)\; 532\; (25 \\ \;\;\;\;\;\; 42 \\ \hline \;\;\;\;\;\; 112 \\ \;\;\;\;\;\; 105 \\ \hline \;\;\;\;\;\;\;\; 7 \end{array}$$

The number of completed rows $= 25$

The number of flower pots are left over $= 7$

3. *Prove that the product of two consecutive positive integers is divisible by 2.*

Let the two consecutive positive integers be x *and* $x + 1$

Product of two consecutive positive integers $= x(x + 1) = x^2 + x$

case(i) : x *is even , let* $x = 2k$

$$x^2 + x = (2k)^2 + 2k$$

$$= 4k^2 + 2k$$

$$= 2k(2k + 1)$$ *Hence the product is divisible by 2*

case(ii) : x *is odd , let* $x = 2k + 1$

$$x^2 + x = (2k + 1)^2 + 2k + 1$$

$$= (2k)^2 + 2(2k)(1) + 1^2 + 2k + 1$$

$$= 4k^2 + 4k + 1 + 2k + 1 = 4k^2 + 6k + 2$$

$$= 2(2k^2 + 3k + 1)$$ *Hence the product is divisible by 2*

From the both the cases we can conclude that the product of two consecutive positive integers is divisible by 2.

4. *When the positive integers a, b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that a + b + c is divisible by 13.*

Given positive integers a, b, c *these are divisible by 13 they gives a remainder* $9, 7$ *and* 10

By Euclid's division lemma $a = bq + r$ *, there exist unique integers* x, y *and* z

if $q = x$ *and* $r = 9$

$a = 13x + 9$

$if\ \ q = y \ and\ r = 7$

$b = 13y + 7$

$if\ q = z\ and\ r = 10$

$c = 13z + 10$

$To\ find\ a + b + c$

$$\begin{aligned} a + b + c &= 13x + 9 + 13y + 7 + 13z + 10 \\ &= 13x + 13y + 13z + 26 \\ &= 13(x + y + z + 2) \end{aligned}$$

$Hence\ a + b + c\ is\ divisible\ by\ 13$

5. *Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.*

$Here\ the\ divisor\ is\ 4$

$Let\ x = 2n\ then\ x^2 = 4n^2$ $which\ is\ divisible\ by\ 4\ and\ leaves\ a\ remainder\ 0$

$Let\ x = 2n + 1\ then\ x^2 = (2n + 1)^2$

$$\begin{aligned} x^2 &= (2n)^2 + 2(2n)(1) + 1^2 \\ &= 4n^2 + 4n + 1 \\ &= 4(n^2 + n) + 1 \quad Assume : n^2 + n = k \\ &= 4k + 1 \end{aligned}$$

$x^2 = 4k + 1$ $which\ is\ divisible\ by\ 4\ and\ leaves\ a\ remainder\ 1$

6. *Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of*

(*i*) 340 *and* 412 (*ii*) 867 *and* 255

(*iii*) 10224 *and* 9648 (*iv*) 84, 90 *and* 120

(*i*) 340 *and* 412

$Let\ a = 412\ and\ b = 340, where\ a > b$

$By\ Euclid's\ division\ lemma$

$a = bq + r,\ where\ 0 \le r < |b|$

$412 = 340 \times 1 + 72$

340) 412 (1
340
72

$Since\ remainder\ 72 \neq 0, we\ apply\ the\ division\ lemma\ to\ 340\ and\ 72\ to\ obtain$

$340 = 72 \times 4 + 52$

72) 340 (4
288
52

$Since\ remainder\ 52 \neq 0, we\ apply\ the\ division\ lemma\ to\ 72\ and\ 52\ to\ obtain$

$72 = 52 \times 1 + 20$

52) 72 (1
52
20

$Since\ remainder\ 20 \neq 0, we\ apply\ the\ division\ lemma\ to\ 52\ and\ 20\ to\ obtain$

$52 = 20 \times 2 + 12$

20) 52 (2
40
12

Since remainder $12 \neq 0$, *we apply the division lemma to* 20 *and* 12 *to obtain*

$$20 = 12 \times 1 + 8$$

12) 20 (1
12
8

Since remainder $8 \neq 0$, *we apply the division lemma to* 12 *and* 8 *to obtain*

$$12 = 8 \times 1 + 4$$

8) 12 (1
8
4

Since remainder $4 \neq 0$, *we apply the division lemma to* 8 *and* 4 *to obtain*

$$8 = 4 \times 2 + 0$$

4) 8 (2
8
0

Hence HCF of 340 *and* 412 *is* 4

(*ii*) 867*and* 255

Let $a = 867$ *and* $b = 255$, *where* $a > b$
By Euclid's division lemma
$a = bq + r$, *where* $0 \leq r < |b|$

$$867 = 255 \times 3 + 102$$

255) 867 (3
765
102

102) 255 (2
204
51

Since remainder $102 \neq 0$, *we apply the division lemma to* 255 *and* 102 *to obtain*

$$255 = 102 \times 2 + 51$$

Since remainder $51 \neq 0$, *we apply the division lemma to* 102 *and* 51 *to obtain*

$$102 = 51 \times 2 + 0$$

51) 102 (2
102
0

Hence HCF of 867 *and* 255 *is* 51

(*iii*) 10224 *and* 9648

Let $a = 10244$ *and* $b = 9648$, *where* a
By Euclid's division lemma $> b$
$a = bq + r$, *where* $0 \leq r < b$

$$10224 = 9648 \times 1 + 576$$

9648) 10224 (1
9648
576

576) 9648 (16
576
3888
3456
432

Since remainder $576 \neq 0$, *we apply the division lemma to* 9648 *and* 576 *to obtain*

$$9648 = 576 \times 16 + 432$$

432) 576 (1
432
144

Since remainder $432 \neq 0$, *we apply the division lemma to* 576 *and* 432 *to obtain*

$$576 = 432 \times 1 + 144$$

Since remainder $144 \neq 0$, *we apply the division lemma to* 432 *and* 144 *to obtain*

$$432 = 144 \times 3 + 0$$

144) 432 (3
432
0

Hence HCF of 10224 *and* 9648 *is* 144

(*iii*) *Find the HCF of* 84, 90, 120.

To find HCF of three given numbers, first we have to find HCF of the first two numbers.

To find HCF of 84 *and* 90
Using Euclid's Division Algorithm

$$90 = 84 \times 1 + 6$$

84) 90 (1
84
6

Since remainder $6 \neq 0$, *we apply the division lemma to* 84 *and* 6 *to obtain*

$84 = 6 \times 14 + 0$

Here the remainder is zero. Therefore HCF of 84, 90 *is* 6.

6) 84 (14
6
24
24
0

To find the HCF of 120 *and* 6. *Using Euclid's Division Algorithm*

$120 = 6 \times 20 + 0$

Here the remainder is zero. Therefore HCF of 120, 6 *is* 6.

6) 120 (20
12
0

Hence HCF of 84, 90 *and* 120 *is* 6

7. *Find the largest number which divides* 1230 *and* 1926 *leaving remainder* 12 *in each case.*

When 1230 *and* 1926 *divided by a larger number and leaves a remainder* 12.

$1230 - 12 = 1218$

$1926 - 12 = 1914$

Hence, determine the HCF of 1218 *and* 1914. *Using Euclid's Division A logarithm*

$1914 = 1218 \times 1 + 696$

$1218 = 696 \times 1 + 522$

$696 = 522 \times 1 + 174$

$522 = 174 \times 3 + 0$

1218) 1914 (1
1218
696

696) 1218 (1
696
522

522) 696 (1
522
174

174) 522 (3
522
0

Hence 174 *is the larger number divides* 1230 *and* 1926 *and leaves a remainder* 12

8. *If d is the Highest Common Factor of* 32 *and* 60, *find x and y satisfying* $d = 32x + 60y$

$d = 32x + 60y$

Let $a = 60, b = 32$, *where* $a > b$

By Euclid's division lemma: $a = bq + r$, *where* $0 \leq r < b$

$60 = 32 \times 1 + 28 \ldots (1)$

$32 = 28 \times 1 + 4 \quad \ldots (2)$

$28 = 4 \times 7 + 0$

32) 60 (1
32
28

28) 32 (1
28
4

4) 28 (7
28
0

Hence HCF of 60 *and* 32 *is* 4

$\therefore d = 4$

$4 = 32x + 60y \ \ldots (3)$

From (1) $60 - 32(1) = 28$ *and* *From* (2) $32 - 28(1) = 4$

where $28 = 60 - 32(1)$

$32 - [60 - 32(1)] = 4$

$32 - 60 + 32 = 4 \implies 32(2) - 60 = 4$

sub $4 = 32(2) - 60$ *in* (3)

$32(2) - 60 = 32x + 60y$

$32(2) + 60(-1) = 32x + 60y$

$\therefore x = 2, y = -1$

9. *A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11*?

Let a be the positive integers

a *is the divisible by* 88 *and gives a remainder* 61

By Euclid's division lemma: $a = bq + r$, *where* $0 \leq r < b$

$a = 88q + 61$

$a - 61 = 88q$ *which is divisible by* 88.

To find: *Remainder when same number divided by* 11.

$$11)\ 61\ (5$$
$$\underline{55}$$
$$6$$

$a - 61$ *is also divisible by* 11

$a - 61 = 11q \Longrightarrow a = 11q + 61$

$a = 11q + 61$

$61 = 11 \times 5 + 6$ ∴ *Remainder is* 6

10. *Prove that two consecutive positive integers are always coprime*.

Let $n, n + 1$ *be the consecutives positive integers*

Two numbers are coprime if their highest commom factor (GCD) *is* 1

$G.C.D$ *of* n *and* $n + 1 = 1$

Assume they are not coprime

On contrary assume they are not coprime

$G.C.D$ *of* n *and* $n + 1 = k$, *where* $k > 1$

So k *divides both* n *and* $n + 1$. *In general* k *also divides* $n + 1 - n$

Hence k *divides* 1 *or* $k = 1$

But we assumed $k > 1$. *So by contradiction* n *and* $n + 1$ *are coprime*

Exercise 2. 2

Example 2.7: ***In the given factor tree, find the numbers m and n.***

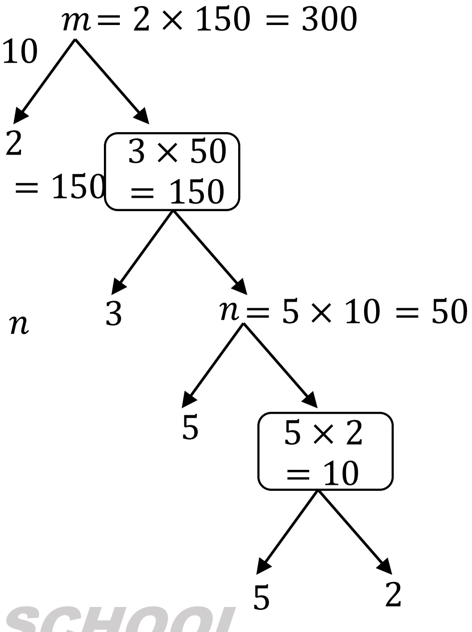
Value of the first box from bottom $= 5 \times 2 = 10$

Value of $n = 5 \times 10 = 50$

Value of the second box from bottom $= 3 \times 50 = 150$

Value of $m = 2 \times 150 = 300$

Thus, the required numbers are $m = 300$ *and* $n = 50$



Example 2.8: Can the number 6^n, n being a natural number end with the digit 5? Give reason for your answer.

Prime factorisation of $6^n = (2 \times 3)^n$

$$6^n = 2^n \times 3^n$$

2 is a factor of 6^n. So, 6^n is always even

But any number whose last digit is 5 is always odd.

Hence, 6^n cannot end with the digit 5

Example 2.9: Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your answer.

Yes, the given number is a composite number, because

$$7 \times 5 \times 3 \times 2 + 3 = 3 \times (7 \times 5 \times 2 + 1)$$
$$= 3 \times (70 + 1) = 3 \times (71)$$

Since the given number can be factorized in terms of two primes, it is a composite number.

Example 2.10: 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

$$\begin{array}{r|l} 2 & 800 \\ \hline 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

$$800 = 2^5 \times 5^2$$

$$a^b \times b^a = 800 \implies a^b \times b^a = 2^5 \times 5^2$$

This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

1. For what values of natural number n, 4^n can end with the digit 6?

Given : 4^n, where $n \in N$

$n = 1, 2, 3, 4, \ldots$

$$4^1 = 4, \quad 4^2 = 16, \; 4^3 = 64, \; 4^4 = 256, \; 4^5 = 1024, \; 4^6 = 4096$$

$\therefore$ 4^n end with the digit 6, only if n is even number.

2. Let m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

Given : $2^n \times 5^m$ where m and $n \in N$

$$n = 1, m = 1 \implies 2^1 \times 5^1 = 2 \times 5 = 10$$

$$n = 1, m = 2 \implies 2^1 \times 5^2 = 2 \times 25 = 50$$

$$n = 2, m = 3 \implies 2^2 \times 5^3 = 4 \times 125 = 500$$

2^n is a even number .

when it is multiplied by multiple of 5 it is end with 0.

3. *Find the HCF of* 252525 *and* 363636.

Given numbers are 252525, 363636

By using prime factorization

$$\begin{array}{r|l} 5 & 252525 \\ \hline 5 & 50505 \\ \hline 3 & 10101 \\ \hline 7 & 3367 \\ \hline & 481 \end{array} \qquad \begin{array}{r|l} 2 & 363636 \\ \hline 2 & 181818 \\ \hline 3 & 90909 \\ \hline 3 & 30303 \\ \hline 3 & 10101 \\ \hline 7 & 3367 \\ \hline & 481 \end{array}$$

$\therefore 252525 = 5 \times 5 \times 3 \times 7 \times 481$

$363636 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 481$

$\therefore HCF = 3 \times 7 \times 481 = 10{,}101$

4. *If* $13824 = 2^a \times 3^b$*, then find a and b.*

$$\begin{array}{r|l} 2 & 13824 \\ \hline 2 & 6912 \\ \hline 2 & 3456 \\ \hline 2 & 1728 \\ \hline 2 & 864 \\ \hline 2 & 432 \\ \hline 2 & 216 \\ \hline 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline & 3 \end{array}$$

Given: $2^a \times 3^b = 13824$

$2^a \times 3^b = 2^9 \times 3^2$

$\therefore a = 9, b = 2$

5. *If* $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ *where* p_1, p_2, p_3, p_4 *are primes in ascending order and* x_1, x_2, x_3, x_4 *are integers, find the value of* p_1, p_2, p_3, p_4 *and* x_1, x_2, x_3, x_4.

$$\begin{array}{r|l} 2 & 113400 \\ \hline 2 & 56700 \\ \hline 2 & 28350 \\ \hline 3 & 14175 \\ \hline 3 & 4725 \\ \hline 3 & 1575 \\ \hline 3 & 525 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

$\therefore \boxed{p_1}^{x_1} \times \boxed{p_2}^{x_2} \times \boxed{p_3}^{x_3} \times \boxed{p_4}^{x_4} = \boxed{2}^3 \times \boxed{3}^4 \times \boxed{5}^2 \times \boxed{7}^1$

$\therefore p_1 = 2,\ p_2 = 3,\ p_3 = 5,\ p_4 = 7$

and

$x_1 = 3,\ x_2 = 4,\ x_3 = 2,\ x_4 = 1$

6. *Find the LCM and HCF of* 408 *and* 170 *by applying the fundamental theorem of arithmetic.*

Given numbers are 408, 170.

$\therefore 408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

$\therefore H.C.F = 2 \times 17 = 34$

$L.C.M = 2^3 \times 17 \times 5 \times 3 = 2040$

$$\begin{array}{r|l} 2 & 408 \\ \hline 2 & 204 \\ \hline 2 & 102 \\ \hline 3 & 51 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

7. *Find the greatest number consisting of* 6 *digits which is exactly divisible by* 24, 15, 36?

Find L.C.M of 24, 15, 36.

$L.C.M = 5 \times 3^2 \times 2^3$

$= 5 \times 9 \times 8$

$= 360$

The greatest 6 *digit number is* 999999.

$\therefore$ *Required greatest number*

$= 999999 - 279(Remainder)$

$= 999720$

$$\begin{array}{r|l} 3 & 24,\ 15, 36 \\ \hline 2 & 8,\ \ 5,\ 12 \\ \hline 2 & 4,\ \ 5,\ \ 6 \\ \hline & 2,\ \ 5,\ \ 3 \end{array}$$

$$\begin{array}{l} 360)999999(277 \\ \quad\ \ 720 \\ \hline \quad\ \ 2799 \\ \quad\ \ 2520 \\ \hline \quad\ \ 2799 \\ \quad\ \ 2520 \\ \hline \quad\ \ \ 279 \end{array}$$

8. *What is the smallest number that when divided by three numbers such as* 35, 56, *and* 91 *leaves remainder* 7 *in each case*?

The required number is the L.C.M of (35, 56, 91) + *remainder* 7

$35 = 7 \times 5$

$56 = 7 \times 2 \times 2 \times 2$

$91 = 7 \times 13$

$\therefore L.C.M = 7 \times 5 \times 13 \times 8 = 3640$

$\therefore$ *The required number is* $3640 + 7 = 3647$

$$\begin{array}{r|l} 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

9. *Find the least number that is divisible by the first ten natural numbers*.

The first ten natural numbers. 1,2,3,4,5,6,7,8,9,10.

Numbers 10 *is divisible by* 2 *and* 5. *So strike out* 2 & 5.

Numbers 9 *is divisible by* 3. *So strike out* 3.

Numbers 8 *is divisible by* 2 *and* 4. *So strike out* 2 & 4.

To find LCM of 6,7,8,9,10

$6 = 3 \times 2$

$7 = \boxed{7}$

$8 = 2 \times 2 \times 2 = \boxed{2^3}$

$9 = 3 \times 3 = \boxed{3^2}$

$10 = \boxed{5} \times 2$

$$L.C.M = 2^3 \times 3^2 \times 5 \times 7$$

$$L.C.M = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$$

EXERCISE 2.3

Example 2.11: Find the remainders when 70004 and 778 is divided by 7.

Since 70000 *is divisible by* 7

$$70000 \equiv 0 (mod\ 7)$$

$$70000 + 4 \equiv 0 + 4\ (mod\ 7)$$

$$70004 \equiv 4 (mod\ 7)$$

Therefore, the remainder when 70004 *is divided by* 7 *is* 4.

Since 777 *is divisible by* 7

$$777 \equiv 0 (mod\ 7)$$

$$777 + 1 \equiv 0 + 1 (mod\ 7)$$

$$778 \equiv 1 (mod\ 7)$$

Therefore, the remainder when 778 *is divided by* 7 *is* 1.

Example 2.12: Determine the value of d such that $15 \equiv 3\ (mod\ d)$

$15 \equiv 3 (mod\ d)$

$15 - 3 = kd,$ *for some integer* k.

$12 = kd$. *Gives* d *divides* 12.

$\frac{12}{d} = k$. *The divisors of* 12 *are* $1, 2, 3, 4, 6, 12$.

But d *should be larger than* 3 *and also so the possible values for* d *are* $4, 6, 12$.

Example 2.13: Find the least positive value of x such that
$(i)\ 67 + x \equiv 1 (mod\ 4)$ $(ii)\ 98 \equiv (x + 4)(mod\ 5)$

$(i)\ 67 + x \equiv 1\ (mod\ 4)$

$67 + x - 1 = 4n,$ *for some integer* n

$66 + x = 4n$

$66 + x$ *is a multiple of* 4.

Therefore, the least positive value of x *must be* 2, *since* 68 *is the nearest multiple of* 4 *more than* 66.

$(ii)\ 98 \equiv (x + 4)(mod\ 5)$

$98 - (x + 4) = 5n,$ *for some integer* n.

$94 - x = 5n$

$94 - x$ *is a multiple of* 5.

Therefore, the least positive value of x *must be* 4

Since $94 - 4 = 90$ *is the nearest multiple of* 5 *less than* 94.

Example 2.14: Solve $8x \equiv 1 \ (mod\ 11)$

$8x \equiv 1\ (mod\ 11)$ *can be written as* $8x - 1 = 11k$, *for some integer* k.

$$8x - 1 = 11k \Longrightarrow 8x = 11k + 1$$

$$x = \frac{11k + 1}{8}$$

When we put $k = 5, 13, 21, 29, \ldots$ *then* $11k + 1$ *is divisible by* 8.

$$x = \frac{11 \times 5 + 1}{8} = \frac{56}{8} = 7$$

$$x = \frac{11 \times 13 + 1}{8} = \frac{143 + 1}{8} = \frac{144}{8} = 18$$

Therefore, the solutions are $7, 18, 29, 40, \ldots \ldots$

Example 2.15: Compute x, such that $10^4 \equiv x\ (mod\ 19)$

$$10^2 = 100 \equiv 5 (mod\ 19)$$

$$10^4 = (10^2)^2 \equiv 5^2\ (mod\ 19)$$

$$10^4 = 25\ (mod\ 19)$$

$$10^4 = 6\ (mod\ 19) \quad \left(Since\ 25 = 6\ (mod\ 19)\right)$$

Therefore, $x = 6$.

Example 2.16: Find the number of integer solutions of $3x \equiv 1\ (mod\ 15)$.

$3x \equiv 1\ (mod\ 15)$ *can be written as* $3x - 1 = 15\ k$ *for some integer* k

$$3x - 1 = 15k \Longrightarrow 3x = 15k + 1$$

$$x = \frac{15k + 1}{3} \quad \Longrightarrow \quad x = \frac{\overset{5}{\cancel{15}}k}{\cancel{3}} + \frac{1}{3}$$

$$x = 5k + \frac{1}{3}$$

Since $5k$ *is an integer*, $5k + \frac{1}{3}$ *cannot be an integer*.

So there is no integer solution.

Example 2.17: A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi ?

Starting time 22.30, *Travelling time* 32 *hours.*

Here we use modulo 24.

The reaching time is $22.30 + 32\ (mod\ 24) \equiv 54.30\ (mod\ 24)$

$\equiv 6.30\ (mod\ 24)$

24) 54.30 (2
48
6.30

Thus, he will reach Delhi on friday at 6.30 *hrs*

$(Since\ 32 = (1 \times 24) + 8)$

Thursday Friday

$Example\ 2.18$: $Kala\ and\ Vani\ are\ friends, Kala\ says,$ Today is my birthday $and\ she\ asks\ Vani,$ "$When\ will\ you\ celebrate\ your\ birthday$? Vani replies, $Today\ is\ Monday\ and\ I\ celebrated\ my\ birthday\ 75\ days\ ago$". $Find\ the\ day\ when\ Vani\ celebrated\ her\ birthday$.

$Let\ us\ assciate\ the\ numbers\ 0, 1, 2, 3, 4, 5, 6$

$to\ represent\ the\ weekdays\ from\ Sunday\ to\ Saturday\ respectively.$

$Vani\ says\ today\ is\ Monday. So\ the\ number\ for\ Monday\ is\ 1.$

$Since\ Vani's\ birthday\ was\ 75\ days\ ago, we\ have\ to\ subtract\ 75\ from\ 1$

$and\ take\ the\ modulo\ 7, since\ a\ week\ contain\ 7\ days.$

$-74 (mod\ 7) \equiv 3\ (mod\ 7)$

7) 74 (11
77
3

$(Since, -74 - 3 = -77\ is\ divisible\ by\ 7)$

$Thus, 1 - 75 \equiv 3\ (mod\ 7)$

$The\ day\ for\ the\ number\ 3\ is\ Wednesday.$

$Therefore, Vani's\ birthday\ must\ be\ on\ Wednesday.$

1. $Find\ the\ least\ positive\ value\ of\ x\ such\ that$

$(i)\ 71 \equiv x (mod\ 8) (ii)\ 78 + x \equiv 3\ (mod\ 5) (iii)\ 89 \equiv (x + 3)\ (mod\ 4)$

$(iv)\ 96 \equiv \frac{x}{7}\ (mod\ 5)\ (v)\ 5x \equiv 4\ (mod\ 6)$

$(i)\ 71 \equiv x\ (mod\ 8)$

$71 - x = 8n$

$\therefore 71 - x\ is\ a\ multiple\ of\ 8$

$(\because 71 - 7 = 64\ which\ is\ less\ than\ 71.$

$(\because 71 - 7 = 64\ is\ the\ nearest\ multiple\ of\ 8)$

$\therefore The\ least\ positive\ value\ of\ x\ is\ 7$

$(ii)\ 78 + x = 3\ (mod\ 5)$

$78 + x - 3 = 5n$

$75 + x\ is\ a\ multiple\ of\ 5$

$(\because 75 + 5 = 80, is\ the\ nearest\ multiple\ of\ 5\ above\ 75)$

$\therefore The\ least\ positive\ value\ of\ x\ is\ 5$

$(iii)\ 89 \equiv (x + 3)\ (mod\ 4)$

$89 - x - 3 = 4n$

$86 - x\ is\ a\ multiple\ of\ 4$

$\because 86 - 2 = 84\ is\ the\ nearest\ multiple\ of\ 4\ less\ than\ 86$

$\therefore The\ least\ positive\ value\ is\ 2$

$(iv)\ 96 \equiv \frac{x}{7} (mod\ 5)$

5) 96 (19
5
46
45
1

$96 \equiv \frac{x}{7} (mod\ 5)$

$By\ dividing\ 96\ by\ 5, we\ get\ 1\ as\ remainder$

$$\frac{x}{7} = 1 \Longrightarrow x = 7$$

$\therefore$ *The least positive value of* x *is* 7

(v) $\mathbf{5x \equiv 4\ (mod\ 6)}$

$5x - 4 = 6n$

$\therefore 5x - 4$ *is a multiple of* 6 $(\because 5(2) - 4 = 6$ *is a multiple of* $6)$

$\therefore$ *The least positive value of* x *is* 2

2. *If x is congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17*?

Given $x = 13\ (mod\ 17)$

$x - 13$ *is a multiple of* 17 $(\because 30 - 13 = 17$ *is a multiple of* $17)$

$\therefore$ *The least positive value of* x *is* 30

$\therefore 7x - 3 \equiv y\ (mod\ 17)$

$7(30) - 3 \equiv y\ (mod\ 17)$

$207 \equiv y\ (mod\ 17)$ $(\because 207 - 3 = 204$ *is divisible by* $17)$

$\therefore y = 3$

3. *Solve* $5x \equiv 4\ (mod\ 6)$

Given: $5x \equiv 4\ (mod\ 6)$

$$5x - 4 = 6n \Longrightarrow \frac{5x - 4}{6} = n$$

$$n = \frac{5x - 4}{6} \Longrightarrow n = \frac{(6 - 1)x - (6 - 2)}{6}$$

$$n = \frac{6x - 1x - 6 + 2}{6} \Longrightarrow n = \frac{6x - 6 - 1x + 2}{6}$$

$$n = \frac{6x - 6 - 1x + 2}{6} \Longrightarrow n = \frac{6(x - 1) - 1x + 2}{6} \Longrightarrow n = \frac{6(x - 1)}{6} - \frac{1x + 2}{6}$$

$\therefore x = 2, 8, 14, \dots\dots\dots$ *(by assumption)*

4. *Solve* $3x - 2 \equiv 0\ (mod\ 11)$

Given $3x - 2 \equiv 0\ (mod\ 11)$

$\therefore 3x - 2$ *is divisible by* 11

$\therefore$ *The possible value of* x *are* $8, 19, 30, \dots\dots$

5. *What is the time 100 hours after 7 a.m.*?

Formula:

$$\boxed{t + n = f\ (mod\ 24)}$$

$t \rightarrow$ *current time*

$n \rightarrow$ *no. of hrs.*

$f \rightarrow$ *future time*

$100+7=f(mod\ 24)$

$107-f$ is divisible by 24

$\therefore f=11$ so that $107-11=96$ is divisible by 24.

$\therefore$ The time is 11 A.M.

6. What is the time 15 hours before 11 p.m.?

Formula:

$$\boxed{t-n\equiv p\ (mod\ 24)}$$

$t\rightarrow$ Current time

$n\rightarrow$ no. of hrs.

$p\rightarrow$ past time

$$11-15=-4=-124+20$$
$$\equiv 20\ (mod\ 24)$$

$\therefore$ The time 15 hours in the past was 8 p.m.

7. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Today is Tuesday

Day after 45 days =?

When we divide 45 by 7, remainder is 3.

$\therefore$ The 3rd day from Tuesday is Friday

8. Prove that $2^n+6\times 9^n$ is always divisible by 7 for any positive integer n.

When $n=1,\ 2n+6\times 9n$

$=2^1+(6\times 9)^1=2+(54)=56$

56 divisible by 7.

9. Find the remainder when 2^{81} is divided by 17.

To find the remainder when 2^{81} is divided by 17.

$2^4=16=-1(mod\ 17)$

$2^{80}=(2^4)^{20}=(-1)^{20}=1$

$\therefore 2^{81}=2^{80}\times 2^1$
$=1\times 2=2$

10. *The duration of flight travel from chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.*

Formula:

$$\boxed{t + n \equiv f \ (mod\ 24)}$$

$t \rightarrow$ *present time* $n \rightarrow$ *no. of hours* $f \rightarrow$ *future time*

$$23.30 + 11 = f(mod\ 24)$$
$$34.30 = f\ (mod\ 24)$$
$$\therefore 34.30 - f \text{ is divisible by } 24$$
$$f = 10.30\ (a.m)$$

But the time difference between London & Chennai is 4.30 *hrs.*

$\therefore$ *Flight reaches London Airpot at*

$= 10.30\ hrs - 4.30\ hrs = 6\ AM$ *next day*

i. e. 6 *AM on Monday*

EXERCISE 2.4

Example 2.19: Find the next three terms of the sequences

$$(i)\ \mathbf{\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \ldots\ldots\ldots} \quad (ii)\ \mathbf{5, 2, -1, -4 \ldots\ldots} \quad (iii)\ \mathbf{1, 0.1, 0.01, \ldots\ldots}$$

$$\begin{tikzcd}(i)\ \frac{1}{2}, \arrow[r, bend right, "+4"'] & \frac{1}{6}, \arrow[r, bend right, "+4"'] & \frac{1}{10}, \arrow[r, bend right, "+4"'] & \frac{1}{14} \ldots\ldots\ldots\end{tikzcd}$$

Given sequence the numerators are same and the denominator is increased by 4.

So the next three terms are $a_5 = \frac{1}{14+4} = \frac{1}{18}$

$$a_6 = \frac{1}{18+4} = \frac{1}{22}$$

$$a_7 = \frac{1}{22+4} = \frac{1}{26}$$

(ii) **5, 2, −1, −4, …**

$$\begin{tikzcd}5, \arrow[r, bend right, "-3"'] & 2, \arrow[r, bend right, "-3"'] & -1, \arrow[r, bend right, "-3"'] & -4, \ldots\end{tikzcd}$$

Here each term is decreased by 3. *So the next three terms are*

$$a_5 = -4 - 3 = -7$$

$$a_6 = -7 - 3 = -10$$

$$a_7 = -10 - 3 = -13$$

$\boldsymbol{(iii)\ 1, 0.1, 0.01, \ldots}$

$$\begin{tikzcd} 1, \arrow[r, bend right, "\div 10"'] & 0.1, \arrow[r, bend right, "\div 10"'] & 0.01, \ldots \end{tikzcd}$$

Here each term is divided by 10. *Hence, the next three terms are*

$$a_4 = \frac{0.01}{10} = 0.001$$

$$a_5 = \frac{0.001}{10} = 0.0001$$

$$a_6 = \frac{0.0001}{10} = 0.00001$$

Example 2.20 : Find the general term for the following sequences

$\boldsymbol{(i)\ 3, 6, 9, \ldots\ (ii)\ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ (iii)\ 5, -25, 125, \ldots}$

$\boldsymbol{(i)\ 3, 6, 9, \ldots}$

Here the terms are multiples of 3. *So the general term is*

$$a_n = 3n$$

$\boldsymbol{(ii)\ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots}$

Numerator of n^{th} *term is* n, *and*

the denominator is one more than the numerator.

$$\text{Hence } a_n = \frac{n}{n+1}, n \in N$$

$\boldsymbol{(iii)\ 5, -25, 125, \ldots}$

The terms of the sequence have $+$ *and* $-$ *sign alternatively and also they are in powers of* 5.

So the general term $a_n = (-1)^{n+1}\, 5^n, n \in N$

Example 2.21 : The general term of a sequence is defined as

$$\boldsymbol{a_n = \begin{cases} n(n+3); & n \in N \text{ is odd} \\ n^2 + 1; & n \in N \text{ is even} \end{cases}}$$

Find the eleventh and eighteenth terms.

To find a_{11}

since 11 *is odd*

$n = 11$ *in* $a_n = n(n+3)$

$$a_{11} = 11(11+3) \Longrightarrow a_{11} = 11(14)$$

$$a_{11} = 154$$

To find a_{18}

since 18 is even

$n = 18$ in $a_n = n^2 + 1$

$$a_{18} = 18^2 + 1 \Longrightarrow a_{18} = 324 + 1$$

$$a_{18} = 325$$

Example 2.22 : Find the first five terms of the following sequence.

$$\boldsymbol{a_1 = 1, a_2 = 1, a_n = \frac{a_{n-1}}{a_{n-2}+3}; n \geq 3, n \in N}$$

The first two terms of this sequence are given

$a_1 = 1, a_2 = 1$

$$n = 3\ ;\ a_n = \frac{a_{n-1}}{a_{n-2}+3} \Longrightarrow a_3 = \frac{a_{3-1}}{a_{3-2}+3} \Longrightarrow a_3 = \frac{a_2}{a_1+3} \Longrightarrow a_3 = \frac{1}{1+3}$$

$$a_3 = \frac{1}{4}$$

$$n = 4;\ a_n = \frac{a_{n-1}}{a_{n-2}+3} \Longrightarrow a_4 = \frac{a_{4-1}}{a_{4-2}+3} \Longrightarrow a_3 = \frac{a_3}{a_2+3} \Longrightarrow a_4 = \frac{\frac{1}{4}}{4}$$

$$a_4 = \frac{\frac{1}{4}}{4} \Longrightarrow a_4 = \frac{1}{4} \times \frac{1}{4} \Longrightarrow a_4 = \frac{1}{16}$$

$$n = 5;\ a_n = \frac{a_{n-1}}{a_{n-2}+3} \Longrightarrow a_5 = \frac{a_{5-1}}{a_{5-2}+3} \Longrightarrow a_5 = \frac{a_4}{a_3+3} \Longrightarrow a_5 = \frac{\frac{1}{16}}{4}$$

$$a_5 = \frac{\frac{1}{16}}{4} \Longrightarrow a_5 = \frac{1}{16} \times \frac{1}{4} \Longrightarrow a_5 = \frac{1}{64}$$

Therefore, the first five terms of the sequence are $1, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

1. Find the next three terms of the following sequence.

(i) 8, 24, 72, ……… (ii) 5, 1, −3 …… (iii) $\boldsymbol{\frac{1}{4}, \frac{2}{9}, \frac{3}{16}}, \ldots\ldots$

(i)

$$\begin{tikzcd} 8, \arrow[r, bend right, "\times 3"'] & 24, \arrow[r, bend right, "\times 3"'] & 72, \ldots\ldots\ldots \end{tikzcd}$$

Here each term is multiplied by 3.

So the next three terms are $a_5 = 72 \times 3 = 216$

$a_6 = 216 \times 3 = 648$

$a_7 = 648 \times 3 = 1944$

(ii)
$$\begin{tikzcd} 5, \arrow[r, bend right, "-4"'] & 1, \arrow[r, bend right, "-4"'] & -3, \ldots\ldots\ldots \end{tikzcd}$$

$Here\ each\ term\ is\ decreased\ by\ 4.$

$So\ the\ next\ three\ terms\ are$ $a_5 = -3 - 4 = -7$

$$a_6 = -7 - 4 = -11$$

$$a_7 = -11 - 4 = -15$$

(iii)
$$\begin{tikzcd} \frac{1}{4}, \arrow[r, bend left, "+1"] & \frac{2}{9}, \arrow[r, bend left, "+1"] & \frac{3}{16}, \ldots\ldots\ldots \end{tikzcd}$$

$Each\ no.\ in\ numerator\ is\ increased\ by\ 1\ \&$
$all\ nos\ is\ denominator\ are\ consecutive\ square\ no's$

$So\ the\ next\ three\ terms\ are$ $a_5 = \frac{3+1}{5^2} = \frac{4}{25}$

$$a_6 = \frac{4+1}{6^2} = \frac{5}{36}$$

$$a_7 = \frac{5+1}{7^2} = \frac{6}{49}$$

2. Find the first four terms of the sequences whose n^{th} terms are given by

(i) $a_n = n^3 - 2$ (ii) $a_n = (-1)^{n+1}n(n+1)$ (iii) $a_n = 2n^2 - 6$

(i) $a_n = n^3 - 2$

$$n = 1\ ;\ a_n = n^3 - 2 \Rightarrow a_1 = 1^3 - 2 \Rightarrow a_1 = 1 - 2 \Rightarrow a_1 = -1$$

$$n = 2\ ;\ a_n = n^3 - 2 \Rightarrow a_2 = 2^3 - 2 \Rightarrow a_2 = 8 - 2 \Rightarrow a_2 = 6$$

$$n = 3\ ;\ a_n = n^3 - 2 \Rightarrow a_3 = 3^3 - 2 \Rightarrow a_3 = 27 - 2 \Rightarrow a_3 = 25$$

$$n = 4\ ;\ a_n = n^3 - 2 \Rightarrow a_4 = 4^3 - 2 \Rightarrow a_2 = 64 - 2 \Rightarrow a_4 = 62$$

$Therefore, the\ first\ four\ terms\ of\ the\ sequence\ are\ -1, 6, 25, 62, \ldots\ldots$

(ii) $a_n = (-1)^{n+1}n(n+1)$

$$n = 1\ ;\ a_n = (-1)^{n+1}n(n+1) \Rightarrow a_1 = (-1)^{1+1}.1(1+1)$$

$$\Rightarrow a_1 = (-1)^2.1(2) \Rightarrow a_1 = 1 \times 2 \Rightarrow a_1 = 2$$

$$n = 2\ ;\ a_n = (-1)^{n+1}n(n+1) \Rightarrow a_2 = (-1)^{2+1}.2(2+1)$$

$$\Rightarrow a_2 = (-1)^3.2(3) \Rightarrow a_2 = -1 \times 6 \Rightarrow a_2 = -6$$

$$n = 3\ ;\ a_n = (-1)^{n+1}n(n+1) \Rightarrow a_3 = (-1)^{3+1}.3(3+1)$$

$$\Rightarrow a_3 = (-1)^4.3(4) \Rightarrow a_3 = 1 \times 12 \Rightarrow a_3 = 12$$

$n = 4\ ;\ a_n = (-1)^{n+1}n(n+1) \Rightarrow a_4 = (-1)^{4+1}.4(4+1)$

$$\Rightarrow a_4 = (-1)^5.4(5) \Rightarrow a_4 = -1 \times 20 \Rightarrow a_4 = -20$$

$Therefore, the\ first\ four\ terms\ of\ the\ sequence\ are\ 2, -6, 12, -20, \dots \dots$

$(iii)\ \boldsymbol{a_n = 2n^2 - 6}$

$n = 1\ ;\ a_n = 2n^2 - 6 \Rightarrow a_1 = 2(1)^2 - 6 \Rightarrow a_1 = 2(1) - 6 \Rightarrow a_1 = 2 - 6$

$\Rightarrow a_1 = -4$

$n = 2\ ;\ a_n = 2n^2 - 6 \Rightarrow a_2 = 2(2)^2 - 6 \Rightarrow a_2 = 2(4) - 6 \Rightarrow a_2 = 8 - 6$

$\Rightarrow a_2 = 2$

$n = 3\ ;\ a_n = 2n^2 - 6 \Rightarrow a_3 = 2(3)^2 - 6 \Rightarrow a_3 = 2(9) - 6 \Rightarrow a_3 = 18 - 6$

$\Rightarrow a_3 = 12$

$n = 4\ ;\ a_n = 2n^2 - 6 \Rightarrow a_4 = 2(4)^2 - 6 \Rightarrow a_4 = 2(16) - 6 \Rightarrow a_4 = 32 - 6$

$\Rightarrow a_4 = 26$

$Therefore, the\ first\ four\ terms\ of\ the\ sequence\ are\ -4, 2, 12, 26, \dots \dots$

$\boldsymbol{3. Find\ the\ n^{th}\ term\ for\ the\ following\ sequences}$

$(i)\ \boldsymbol{2, 5, 10, 17, \dots}\ (ii)\ \boldsymbol{0, \frac{1}{2}, \frac{2}{3}, \dots}\quad (iii)\ \boldsymbol{3, 8, 13, 18, \dots}$

$(i)\ \boldsymbol{2, 5, 10, 17, \dots}$

$1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1, \dots \dots .$

$a_n = n^2 + 1$

$(ii)\ \boldsymbol{0, \frac{1}{2}, \frac{2}{3}, \dots}$

$$\frac{0}{1}, \frac{1}{2}, \frac{2}{3}, \dots \dots$$

$$\frac{1-1}{1}, \frac{2-1}{2}, \frac{3-1}{3}, \dots \dots$$

$$Hence\ \ a_n = \frac{n-1}{n}, n \in N$$

$(iii)\ \boldsymbol{3, 8, 13, 18, \dots}$

$5 - 2, 10 - 2, 15 - 2, 20 - 2, \dots$

$5(1) - 2, 5(2) - 2, 5(3) - 2, 5(4) - 2, \dots$

$So\ the\ general\ term\ a_n = 5n - 2\ , n \in N$

4. Find the indicated terms of the sequences whose n^{th} terms are given by

$$(i)\ a_n = \frac{5n}{n+2}; a_6 \text{ and } a_{13}\ (ii)\ a_n = -(n^2-4); a_4 \text{ and } a_{11}$$

$$(i)\ a_n = \frac{5n}{n+2}; a_6 \text{ and } a_{13}$$

$$n = 6\ ;\ a_n = \frac{5n}{n+2} \Rightarrow a_6 = \frac{5(6)}{6+2} \Rightarrow a_6 = \frac{30}{8} \Rightarrow a_6 = \frac{15}{4}$$

$$n = 13\ ;\ a_n = \frac{5n}{n+2} \Rightarrow a_{13} = \frac{5(13)}{13+2} \Rightarrow a_{13} = \frac{65}{15} \Rightarrow a_{13} = \frac{13}{3}$$

$$(ii)\ a_n = -(n^2-4); a_4 \text{ and } a_{11}$$

$$n = 4\ ;\ a_n = -(n^2-4) \Rightarrow a_4 = -(4^2-4) \Rightarrow a_4 = -(16-4) \Rightarrow a_4 = -12$$

$$n = 11\ ;\ a_n = -(n^2-4) \Rightarrow a_{11} = -(11^2-4)$$

$$\Rightarrow a_{11} = -(121-4) \Rightarrow a_{11} = -117$$

5. Find a_8 and a_{15} whose n^{th} term is $a_n = \begin{cases} \frac{n^2-1}{n+3}; n \text{ is even}, n \in N \\ \frac{n^2}{2n+1}; n \text{ is odd,} \end{cases}$

$$n = 8\ ; a_n = \frac{n^2-1}{n+3} \Rightarrow a_8 = \frac{8^2-1}{8+3} \Rightarrow a_8 = \frac{64-1}{8+3} \Rightarrow a_8 = \frac{63}{11}$$

$$n = 15;\ a_n = \frac{n^2}{2n+1} \Rightarrow a_{15} = \frac{15^2}{2(15)+1} \Rightarrow a_{15} = \frac{225}{30+1} \Rightarrow a_{15} = \frac{225}{31}$$

6. If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}; n \geq 3, n \in N$, then find the first six terms of the sequence.

The first two terms of this sequence are given

$a_1 = 1, a_2 = 1$

$$n = 3\ ;\ a_n = 2a_{n-1} + a_{n-2} \Rightarrow a_3 = 2a_{3-1} + a_{3-2} \Rightarrow a_3 = 2a_2 + a_1$$

$$\Rightarrow a_3 = 2 \times 1 + 1 \Rightarrow a_3 = 2 + 1 \Rightarrow a_3 = 3$$

$$n = 4\ ;\ a_n = 2a_{n-1} + a_{n-2} \Rightarrow a_4 = 2a_{4-1} + a_{4-2} \Rightarrow a_4 = 2a_3 + a_2$$

$$\Rightarrow a_4 = 2 \times 3 + 1 \Rightarrow a_4 = 6 + 1 \Rightarrow a_4 = 7$$

$$n = 5\ ;\ a_n = 2a_{n-1} + a_{n-2} \Rightarrow a_5 = 2a_{5-1} + a_{5-2} \Rightarrow a_5 = 2a_4 + a_3$$

$$\Rightarrow a_5 = 2 \times 7 + 3 \Rightarrow a_5 = 14 + 3 \Rightarrow a_5 = 17$$

$n = 6\ ;\ a_n = 2a_{n-1} + a_{n-2} \Rightarrow a_6 = 2a_{6-1} + a_{6-2} \Rightarrow a_6 = 2a_5 + a_4$

$\Rightarrow a_6 = 2 \times 17 + 7 \Rightarrow a_6 = 34 + 7 \Rightarrow a_6 = 41$

Therefore, the first six terms of the sequence are $1,1,3,7,17,41, \ldots$

EXERCISE 2.5

Example 2.23: Check whether the following sequence are in A.P or not?

(*i*) $\boldsymbol{x + 2, 2x + 3, 3x + 4, \ldots}$

$t_1 \quad t_2 \quad t_3$

$t_2 - t_1 = 2x + 3 - (x + 2)$

$= 2x + 3 - x - 2$

$t_2 - t_1 = x + 1$

$t_3 - t_2 = 3x + 4 - (2x + 3)$

$= 3x + 4 - 2x - 3$

$= x + 1$

$\therefore\ t_2 - t_1 = t_3 - t_2$

Hence $x + 2, 2x + 3, 3x + 4 \ldots.$ *are in A.P*

(*ii*) $\mathbf{2}, \ \mathbf{4}, \ \mathbf{8}, \ \mathbf{16}, \ldots$

$t_1 \quad t_2 \quad t_3 \quad t_4$

$t_2 - t_1 = 4 - 2 = 2$

$t_3 - t_2 = 8 - 4 = 4$

$\therefore t_2 - t_1 \neq t_3 - t_2$

Hence $2, 4, 8, 16, \ldots$ *are not in A.P*

(*iii*) $\boldsymbol{3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \ldots}$

$t_1 \quad t_2 \quad t_3 \quad t_4$

$t_2 - t_1 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$t_3 - t_2 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$

$\therefore t_2 - t_1 = t_3 - t_2$

Hence $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2} \ldots$ *are in A.P*

Example 2.24: Write an A.P. whose first term is 20 and common difference is 8.

First term: $a = 20$, *common difference* : $d = 8$

The A.P is $a, a + d, a + 2d, a + 3d, \ldots$

$$20,\ 20+8,\ 20+2(8),\ 20+3(8)\ \ldots$$

$$20, 20+8, 20+16, 20+24, \ldots$$

$$\therefore\ The\ A.P\ is\ \ 20, 28, 36, 44, \ldots$$

Example 2.25: Find the 15^{th}, 24^{th} and n^{th} term (general term) of A.P. is given by $3, 15, 27, 39, \ldots$

$The\ A.P\ is\ \ 3, 15, 27, 39, \ldots$

$a = t_1 = 3; t_2 = 15$

$d = t_2 - t_1 = 15 - 3 = 12$

$d = 12$

$To\ find\ t_{15}$

$t_n = a + (n-1)d \quad where\ n = 15.$

$t_{15} = 3 + (15-1)12$

$= 3 + (14)12 \ = 3 + 168 \ = 171$

$t_{15} = 171$

$To\ find\ t_{24}$

$t_n = a + (n-1)d \quad where\ n = 24.$

$t_{24} \ = 3 + (24-1)12 \ = 3 + (23)12$

$= 3 + 276 \ = 279$

$t_{24} = 279$

$To\ find\ t_n$

$t_n = a + (n-1)d$

$t_n = 3 + (n-1)12 \ = 3 + 12n - 12$

$t_n = 12n - 9$

Example 2.26: Find the number of terms in the A.P is $3, 6, 9, 12, \ldots\ldots 111$?

$a = 3; \quad d = 6 - 3 = 3; \quad l = 111$

$$n = \left[\frac{l-a}{d}\right] + 1 \ \Rightarrow \ n = \left[\frac{111-3}{3}\right] + 1$$

$$n = \left[\frac{\cancel{108}}{\cancel{3}}\right]^{36} + 1 \Rightarrow \ n = \ 36 + 1$$

$n = 37$

The A.P contains 37 terms.

Example 2.27: Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} is 17?

$$\boxed{t_n = a+(n-1)d}$$

Given : $t_7 = -1$ and $t_{16} = 17$

$t_7 = -1 \quad$ Here : $n = 7$

$t_7 = a + 6d \Rightarrow \quad a + 6d = -1 \ \dots (1)$

$t_{16} = 17 \quad$ Here : $n = 16$

$t_{16} = a + 15d \Rightarrow \quad a + 15d = 17 \dots (2)$

Solve (1) and (2)

$$\begin{array}{l} \cancel{a} + 6d \ = -1 \\ (-)\ \ (-) \qquad\ \ (-) \\ \cancel{a} + 15d = 17 \\ \hline \cancel{-}9d = \cancel{-}18 \end{array} \Rightarrow \quad d = \frac{\overset{2}{\cancel{18}}}{\cancel{9}} \Rightarrow \quad d = 2$$

Sub $d = 2$ in eq (1) $\ a + 6d = -1$

$a + 6(2) = -1 \Rightarrow \quad a + 12 = -1$

$a = -1 - 12 \Rightarrow \quad a = -13$

To find t_n

$t_n = a + (n-1)d$

$t_n = -13 + (n-1)2 \ = -13 + 2n - 2$

$t_n = 2n - 15$

Example 2.28: If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that (i) $x(m-n) + y(n-l) + z(l-m) = 0$

(ii) $(x-y)n + (y-z)l + (z-x)m = 0$

(i) Given : $t_l = x, \quad t_m = y, \quad t_n = z$

$t_n = a + (n-1)d$

$t_l = x \Rightarrow \quad a + (l-1)d = x \dots (1)$

$t_m = y \Rightarrow \quad a + (m-1)d = y \dots (2)$

$t_n = z \Rightarrow \quad a + (n-1)d = z \dots (3)$

$L.H.S = x(m-n) + y(n-l) + z(l-m)$

$= [a + (l-1)d](m-n) + [a + (m-1)d](n-l) + [a + (n-1)d](l-m)$

$= a(m-n) + (l-1)(m-n)d + a\underline{(n-l)} + (m-1)(n-l)d$
$\qquad + a\underline{(l-m)} + (n-1)(l-m)d$

$= a(m-n) + a(n-l) + a(l-m) + (l-1)(m-n)d + (m-1)(n-l)d$
$\qquad + (n-1)(l-m)d$

$$= a[\cancel{m} - \cancel{n} + \cancel{n} - \cancel{l} + \cancel{l} - \cancel{m}]$$

$$+ [(l-1)(m-n) + (m-1)(n-l) + (n-1)(l-m)]\ d$$

$$= a[0] + [\cancel{lm} - \cancel{m} - \cancel{ln} + \cancel{n} + \cancel{mn} - \cancel{n} - \cancel{ml} + \cancel{l} + \cancel{nl} - \cancel{l} - \cancel{nm} + \cancel{m}]d$$

$$= a(0) + d(0) \quad = 0$$

$(ii)\ (x-y)n + (y-z)l + (z-x)m = 0$

$t_l = x \implies a + (l-1)d = x \implies a + ld - d = x \ \ldots(1)$

$t_m = y \implies a + (m-1)d = y \implies a + md - d = y \ \ldots(2)$

$t_n = z \implies a + (n-1)d = z \implies a + nd - d = z \ \ldots(3)$

$Solve\ (1)\ and\ (2)$

$$\begin{array}{l} \cancel{a} + ld - \cancel{d} = x \\ (-)\ \ (-)\quad (+)\ \ (-) \\ \underline{\cancel{a} + md - \cancel{d} = y} \\ ld - md = x - y \end{array}$$

$Solve\ (2)\ and\ (3)$

$$\begin{array}{l} \cancel{a} + md - \cancel{d} = y \\ (-)\ \ (-)\quad (+)\ \ (-) \\ \underline{\cancel{a} + nd - \cancel{d} = z} \\ md - nd = y - z \end{array}$$

$Solve\ (3)\ and\ (1)$

$$\begin{array}{l} \cancel{a} + nd - \cancel{d} = z \\ (-)\ \ (-)\quad (+)\ \ (-) \\ \underline{\cancel{a} + ld - \cancel{d} = x} \\ nd - ld = z - x \end{array}$$

$$(x-y)n + (y-z)l + (z-x)m = (ld - md)n \ + (md - nd)l \ + (nd - ld)m$$

$$= \cancel{lnd} - \cancel{mnd} \ + \cancel{lmd} - \cancel{lnd} + \cancel{mnd} - \cancel{lmd} = 0$$

Example 2.29 : *In an A.P. sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers*?

Let the four consecutive terms of an A.P be $a-3d, a-d, a+d, a+3d$

Sum of four terms $= 28$

$$a - 3d + a - d + a + d + a + 3d = 28$$

$$4a = 28 \implies a = \frac{\cancel{28}^{\,7}}{\cancel{4}}$$

$$a = 7$$

Sum of their squares $= 276$

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$$

$$a^2 - 2(a)(3d) + (3d)^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2$$
$$+ a^2 - 2(a)(3d) + (3d)^2 = 276$$

$$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 - 6ad + 9d^2 = 276$$

$$4a^2 + 20d^2 = 276 \implies 4(7)^2 + 20d^2 = 276$$

where $a = 7$

$$4(49) + 20d^2 = 276 \implies 196 + 20d^2 = 276$$

$$20d^2 = 276 - 196 \implies 20d^2 = 80$$

$$d^2 = \frac{\cancel{80}^{\,4}}{\cancel{20}} \implies d^2 = 4 \implies d = \sqrt{4}$$

$$d = \pm 2$$

$If\ \ a = 7\ , d = 2$

$\therefore\ The\ four\ numbers\ are$

$$a - 3d, a - d, a + d, a + 3d$$

$$= 1, 5, 9, 13$$

$If\ \ a = 7\ , d = -2$

$Then\ the\ four\ numbers\ are\ 13, 9, 5, 1$

$a - 3d = 7 - 3(2) = 7 - 6$

$a - 3d = 1$

$a - d = 7 - 2$

$a - d = 5$

$a + d = 7 + 2$

$a + d = 10$

$a - 3d = 7 + 3(2) = 7 + 6$

$a - 3d = 13$

Example 2.30 : A mother divides Rs. 207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had Rs. 4623. Find the amount received by each child.?

$Let\ the\ three\ parts\ of\ the\ amount\ in\ A.P\ be\ a - d, a, a + d.$

$$Sum\ of\ the\ amount = Rs. 207$$

$$a - d + a + a + d = 207$$

$$3a = 207 \implies a = \frac{\cancel{207}}{\cancel{3}} \implies \boxed{a = 69}$$

$The\ product\ of\ the\ two\ least\ amounts = Rs. 4623$

$$(a - d)a = 4623\ , where\ \ a = 69$$

$$(69 - d)69 = 4623$$

$$69 - d = \frac{4623}{69} \implies 69 - d = 67 \implies -d = 67 - 69$$

$$-d = -2 \implies \boxed{d = 2}$$

$$\begin{array}{r} 69)4623(67 \\ 414\downarrow \\ \hline 483 \\ 483 \\ \hline 0 \\ \hline \end{array}$$

$When\ a = 69, and\ d = 2$

$The\ amounts\ received\ by\ each\ child\ are$

$$a - d, a, a + d.$$

$$= Rs. 67, Rs. 69, Rs. 71$$

$a - d = 69 - 2 = 67$

$a = 69$

$a + d = 69 + 2 = 71$

1. Check whether the following sequences are in A.P

(i) $a - 3, a - 5, a - 7, \ldots$

$$t_2 - t_1 = a - 5 - (a - 3)$$

$$= \cancel{a} - 5 - \cancel{a} + 3 = -2$$

$$t_3 - t_1 = a - 7 - (a - 5)$$

$$= \cancel{a} - 7 - \cancel{a} + 5 \quad = -2$$

$$t_2 - t_1 = t_3 - t_2$$

$\therefore a - 3, a - 5, -7, \ldots are\ in\ A.P$

$\boldsymbol{ii)\ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots}$

$$t_2 - t_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}$$

$$t_3 - t_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = -\frac{1}{12}$$

$$t_2 - t_1 \neq t_3 - t_2$$

$\therefore \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ *are not in A.P*

$\boldsymbol{iii)\ 9, 13, 17, 21, 25 \ldots}$

$t_1 = 9,\ t_2 = 13, t_3 = 17$

$t_2 - t_1 = 13 - 9 = 4$

$t_3 - t_2 = 17 - 13 = 4$

$t_2 - t_1 = t_3 - t_2$

$\therefore 9, 13, 17, 21, 25, \ldots$ *are in A.P.*

$\boldsymbol{iv) 1, -1, 1, -1, 1, -1, \ldots}$

$t_2 - t_1 = -1 - 1 = -2$

$t_3 - t_2 = 1 - (-1) = 1 + 1 = 2$

$t_2 - t_1 \neq t_3 - t_2$

$\therefore 1, -1, 1, -1, 1, -1, \ldots$ *are not in A.P*

$\boldsymbol{v)\ -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots}$

$$t_2 - t_1 = 0 - \left(\frac{-1}{3}\right) = \frac{1}{3}$$

$$t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$t_2 - t_1 = t_3 - t_2$$

$\therefore -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$ *are in A.P*

2. *First term a and common difference d are given below. Find the corresponding A.P.*

$\boldsymbol{i)\ a = 5, d = 6}$

The general form of an A.P is $a, a + d, a + 2d, a + 3d, \ldots$

$= 5,\ 5 + 6, 5 + 2(6), 5 + 3(6), \ldots$

$= 5, 5 + 6, 5 + 12, 5 + 18, \ldots$

$= 5, 11, 17, 23, \ldots$

$\therefore$ *A, P is* $5, 11, 17, 23, \ldots$

$\boldsymbol{ii)\ a = 7, d = -5}$

$The\ general\ form\ of\ an\ A.P\ is\ a, a+d, a+2d, a+3d, \ldots$

$= 7, 7+(-5), 7+2(-5), 7+3(-5), \ldots$

$= 7, 7-5,\ 7-10,\ 7-15, \ldots$

$= 7, 2, -3, -8, \ldots$

$\therefore A.P\ is\ 7, 2, -3, -8, \ldots$

$\boldsymbol{iii)\ a = \frac{3}{4}\ , d = \frac{1}{2}}$

$The\ general\ form\ of\ an\ A.P\ is\ a, a+d, a+2d, a+3d, \ldots$

$$= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+\cancel{2}\left(\frac{1}{\cancel{2}}\right), \frac{3}{4}+3\left(\frac{1}{2}\right), \ldots$$

$$= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+1\ , \frac{3}{4}+\frac{3}{2}, \ldots$$

$$= \frac{3}{4}, \frac{3+2}{4}, \frac{3+4}{4}\ , \frac{3+6}{4}, \ldots\ = \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$$

$\therefore A.P\ is \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

$\boldsymbol{3. Find\ the\ first\ term\ and\ common\ Difference\ of\ an\ A.P\ whose\ n^{th}\ terms\ given\ below}$:

$\boldsymbol{(i)\ t_n = -3 + 2n}$

$n = 1 \Rightarrow\ \ t_1 = -3 + 2(1)$

$= -3 + 2$

$t_1 = -1 = a$

$n = 2 \Rightarrow\ \ t_2 = -3 + 2(2)$

$= -3 + 4$

$t_2 = 1$

$d = t_2 - t_1$

$= 1 - (-1) = 1 + 1 = 2$

$\therefore a = -1, d = 2$

$\boldsymbol{(ii)\ \ t_n = 4 - 7n}$

$n = 1 \Rightarrow\ \ t_1 = 4 - 7(1)$

$= 4 - 7$

$t_1 = -3 = a$

$n = 2 \Rightarrow\ \ t_2 = 4 - 7(2)$

$= 4 - 14$

$t_2 = -10$

$d = t_2 - t_1$

$= -10 - (-3) = -10 + 3$

$d = -7$

$\therefore a = -3\,, d = -7$

$\boldsymbol{4. Find\ the\ 19^{th}\ term\ of\ an\ A.P\ is\ -11, -15, -19, \ldots}$

$A.P\ is\ -11, -15, -19, \ldots$

$a = -11 \quad d = t_2 - t_1$

$d = -15 - (-11) = -15 + 11$

$d = -4$

$To\ find\ t_{19}$

$t_n = a + (n-1)d \quad where\ n = 19.$

$t_{19} = a + 18d$

$\therefore t_{19} = -11 + (18)(-4)$

$= -11 - 72$

$t_{19} = -83$

$\boldsymbol{5. Which\ term\ of\ an\ A.P\ is\ 16, 11, 6, 1, \ldots\ is\ -54?}$

$a = 16, d = t_2 - t_1$

$d = 11 - 16$

$d = -5,\ l = -54.$

$n = \left(\frac{l-a}{d}\right) + 1 \quad \text{Where } a = 6, l = -54\ and\ d = -5$

$n = \left[\frac{l \quad - a}{d}\right] +1 \implies n = \left(\frac{-54 - 16}{-5}\right) + 1$

$n = \left(\frac{-\cancel{7}0}{-\cancel{5}}\right) + 1 \implies n = 14 + 1$

$n = 15$

$The\ 15^{th}\ term\ of\ the\ sequence\ is\ -54$

$\therefore t_{15} = -54$

6. $\boldsymbol{Find\ the\ middle\ term(s) of\ an\ A.P.\ 9, 15, 21, 27, \ldots 183.}$

$a = 9,\ d = t_2 - t_1$

$d = 15 - 9 \Rightarrow\ d = 6$

$l = 183$

$$n = \left(\frac{l-a}{d}\right) + 1 \quad \text{Where } a = 9, l = 183\ and\ d = 6$$

$$n = \frac{183-9}{6} + 1 \Rightarrow\ n = \frac{174}{6} + 1 \Rightarrow\ n = 29 + 1$$

$n = 30$

$if\ n\ is\ even\ there\ are\ two\ middle\ terms$

$$middle\ terms = \frac{n}{2}, \frac{n}{2} + 1$$

$$= \frac{30}{2}, \frac{30}{2} + 1\ = 15, 16$$

$15^{th}\ and\ 16^{th}\ terms\ are\ the\ middle\ terms$

$To\ find\ t_{15}$

$t_n = a + (n-1)d \quad where\ n = 15.$

$t_{15} = a + 14d\quad \text{Where } a = 9\ and\ d = 6$

$= 9 + (14)6 = 9 + 84\ = 93$

$t_{15} = 93$

$To\ find\ t_{16}$

$t_n = a + (n-1)d \quad where\ n = 16.$

$t_{15} = a + 15d\quad \text{Where } a = 9\ and\ d = 6$

$= 9 + (15)6 = 9 + 90\ = 99$

$t_{16} = 99$

$\therefore The\ middle\ terms\ are\ t_{15} = 93, t_{16} = 99.$

7. $\boldsymbol{If\ nine\ times\ ninth\ term\ is\ equal\ to\ the\ fifteen\ times\ fifteenth\ term, show\ that\ six\ times\ twenty\ fourth\ term\ is\ zero.}$

$Given:\ 9t_9 = 15t_{15} \quad To\ prove:\ 6t_{24} = 0$

$9t_9 = 15t_{15}$

$$9[a + (9-1)d] = 15[a + (15-1)d] \Rightarrow \overset{3}{\cancel{9}}[a + 8d] = \overset{5}{\cancel{15}}[a + 14d]$$

$$3[a + 8d] = 5[a + 14d]\ \Rightarrow\ 3a + 24d\ = 5a + 70d$$

$$3a - 5a = 70d - 24d \Rightarrow -\cancel{2}a = \overset{23}{\cancel{46}}d \Rightarrow a = -23d$$

$$\therefore 6t_{24} = 6[a + 23d]$$

$$= 6[-23d + 23d] = 6[0] = 0$$

$$\therefore 6t_{24} = 0$$

8. If $3 + k, 18 - k, 5k + 1$ are in A.P. then find k.

$$t_1 = 3 + k; t_2 = 18 - k; t_3 = 5k + 1$$

$$t_2 - t_1 = t_3 - t_2$$

$$18 - k - (3 + k) = 5k + 1 - (18 - k)$$

$$18 - k - 3 - k = 5k + 1 - 18 + k$$

$$15 - 2k = 6k - 17$$

$$-2k - 6k = -17 - 15$$

$$\cancel{-}8k = \cancel{-}32 \implies k = \frac{\cancel{32}}{\cancel{8}}$$

$$k = 4$$

9. Find x, y and z, given that the numbers $x, 10, y, 24, z$ are in A.P.

Given that $x, 10, y, 24, z$ are in A.P.

$$t_3 - t_2 = t_4 - t_3$$

$$y - 10 = 24 - y \implies y + y = 24 + 10$$

$$2y = 24 + 10 \implies y = \frac{10 + 24}{2} \implies y = \frac{\cancel{34}}{\cancel{2}}$$

$$y = 17$$

$$\therefore x, 10, 17, 24, z \text{ are in } A.P$$

$$t_1 = x, t_2 = 10, t_3 = 17, t_4 = 24, t_5 = z$$

$$d = t_3 - t_2 = 17 - 10$$

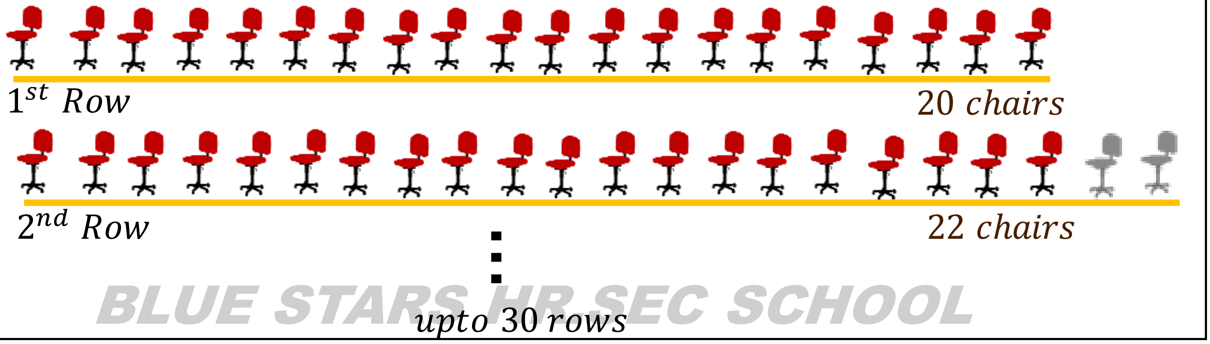
$$d = 7$$

$$\therefore z = 24 + 7 \implies z = 31$$

$$x = 10 - 7 \implies x = 3$$

$$\therefore x = 3, y = 17, z = 31$$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. how many seats are there in the last row?



$a = 20, d = 2, n = 30$

$To\ find\ t_{30}$

$$t_{30} = a + 29d$$
$$= 20 + 29(2) = 20 + 58$$
$$t_{30} = 78 \qquad \therefore The\ no.of\ seats\ in\ 30^{th}\ row = 78$$

11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms?

$Let\ the\ three\ consecutive\ terms\ in\ A.P.be\ a - d, a, a + d$

$Sum\ of\ three\ consecutive\ terms = 27$

$$a - d + a + a + d = 27 \Rightarrow \ 3a = 27$$

$$a = \frac{\cancel{27}^{9}}{\cancel{3}} \Rightarrow \ a = 9$$

$Product\ of\ three\ consecutive\ terms = 288$

$$(a - d) \times a \times (a + d) = 288 \Rightarrow \ a(a^2 - d^2) = 288 \ \ where\ a = 9$$

$$9(9^2 - d^2) = 288 \Rightarrow \ 81 - d^2 = \frac{\cancel{288}^{32}}{\cancel{9}} \Rightarrow \ 81 - d^2 = 32$$

$$-d^2 = 32 - 81 \Rightarrow \ -d^2 = -49 \Rightarrow \ d^2 = 49$$

$$\boxed{d = \pm 7}$$

$Ifa = 9, d = 7\ then\ the\ three\ terms\ are\ 2, 9, 13$

$a - d = 9 - 7 = 2$

$a = 9$

$a + d = 9 + 7 = 13$

$Ifa = 9, d = -7\ , then\ the\ three\ terms\ are\ 13, 9, 2$

12. The ratio of 6^{th} and 8^{th} term of an A.P. is 7:9. Find the ratio of 9^{th} to 13^{th} term ?

$Given\ t_6 : t_8 = 7 : 9 \quad To\ find : t_9 : t_{13}$

$$t_6 : t_8 = 7 : 9 \Rightarrow \frac{t_6}{t_8} = \frac{7}{9}$$

$$\frac{a + 5d}{a + 7d} = \frac{7}{9} \Rightarrow \frac{a + 5d}{a + 7d} = \frac{7}{9} \Rightarrow 9(a + 5d) = 7(a + 7d)$$

$$9a + 45d = 7a + 49d \Rightarrow \ 9a - 7a = 49d - 45d$$

$$\cancel{2}a = \cancel{4}d \Rightarrow \ a = 2d$$

$$t_9 : t_{13} = \frac{t_9}{t_{13}}$$

$$= \frac{a + 8d}{a + 12d} = \frac{a + 8d}{a + 12d} = \frac{2d + 8d}{2d + 12d} = \frac{\cancel{10d}}{\cancel{14d}} = \frac{5}{7}$$

$$\therefore t_9 : t_{13} = 5 : 7$$

13. *In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is $0^{\circ}C$ and the sum of the temperatures from Wednesday to Friday is $18^{\circ}C$. Find the temperature on each of the five days.*

Let the temperature from Monday to Friday respectively be

$$\begin{array}{ccccc} a, & a+d, & a+2d, & a+3d, & a+4d \\ M & Tu & W & Th & F \end{array}$$

$$\text{Sum of temperatures from Mon to Wed} = 0^{\circ}C$$

$$a + a + d + a + 2d = 0 \Rightarrow 3a + 3d = 0$$

$$\div\ 3$$

$$a + d = 0 \Rightarrow a = -d$$

$$\text{Sum of temperatures from Wed to Fri} = 18^{\circ}C$$

$$a + 2d + a + 3d + a + 4d = 18$$

$$3a + 9d = 18 \text{ where } a = -d$$

$$3(-d) + 9d = 18 \Rightarrow -3d + 9d = 18$$

$$\cancel{6}d = \cancel{18} \Rightarrow d = 3$$

$$\therefore a = -3; \quad d = 3$$

The temperature on each of the five days are

$$a, a+d, a+2d, a+3d, a+4d$$

$$= -3, -3+3, -3+2(3), -3+3(3), -3+4(3)$$

$$= -3, -3+3, -3+6, -3+9, -3+12$$

$$\boxed{\therefore \quad -3°C, 0°C, 3°C, 6°C, 9°C}$$

14. *Priya earned Rs. 15,000 in the first year. Thereafter her salary increased by Rs. 1500 per year. Her expences are Rs. 13,000 during the first year and the expences increased by Rs. 900 per year. How long will it take for her to save Rs. 20,000?*

	1^{st} year	2^{nd} year
Salary:	*Rs.* 15,000	*Rs.* 16,500
Expense:	*Rs.* 13,000	*Rs.* 13,900
Savings:	*Rs.* 2,000	*Rs.* 2,600

$\therefore$ *The yearly savings are in A.P*

$$Rs. 2{,}000,\ Rs. 2{,}600,\ Rs. 3{,}200, \ldots\ldots, Rs. 20{,}000$$

$$a = 2{,}000; d = t_2 - t_1 = 2{,}600 - 2{,}000$$

$$d = 600,\ l = 20{,}000$$

$$n = \left[\frac{l-a}{d}\right] + 1 \Rightarrow n = \left[\frac{20{,}000 - 2{,}000}{600}\right] + 1 \Rightarrow n = \left[\frac{\overset{30}{\cancel{18{,}000}}}{\cancel{600}}\right] + 1$$

$$n = 30 + 1 \Rightarrow n = 31$$

$\therefore$ *It will take* 31 *years to save Rs.* $20{,}000$

EXERCISE 2.6

***Example 2.31**: Find the sum of first 15 terms of the A.P*

$\mathbf{8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \ldots}$

$$a = 8,\ t_1 = 8,\ t_2 = 7\frac{1}{4},\ n = 15$$

$$d = t_2 - t_1$$

$$d = 7\frac{1}{4} - 8 = \frac{29}{4} - 8 = \frac{29-32}{4}$$

$$d = -\frac{3}{4}$$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{15} = \frac{15}{2}\left[2 \times 8 + (15-1)\left(\frac{-3}{4}\right)\right]$$

$$S_{15} = \frac{15}{2}\left[16 - (\overset{7}{\cancel{14}})\frac{3}{\underset{2}{\cancel{4}}}\right] \Rightarrow S_{15} = \frac{15}{2}\left[16 - \frac{21}{2}\right]$$

$$S_{15} = \frac{15}{2}\left[\frac{32-21}{2}\right] \Rightarrow S_{15} = \frac{15}{2} \times \frac{11}{2} \Rightarrow \boxed{S_{15} = \frac{165}{4}}$$

***Example 2.32** : Find the sum of* $\mathbf{0.40 + 0.43 + 0.46 + \cdots + 1}$

$$a = 0.40, \quad d = t_2 - t_1 \quad , l = 1$$

$$d = 0.43 - 0.40$$

$$d = 0.03$$

$$n = \left[\frac{l-a}{d}\right] + 1 \Rightarrow n = \left[\frac{1 - 0.40}{0.03}\right] + 1 \Rightarrow n = \left[\frac{0.60}{0.03}\right] + 1$$

$$n = \frac{0.60 \times 100}{0.03 \times 100} + 1 \Rightarrow n = \frac{60.00}{3.00} + 1 \Rightarrow n = \frac{\overset{20}{\cancel{60}}}{\cancel{3}} + 1$$

$$n = 20 + 1 \Rightarrow n = 21$$

$$S_n = \frac{n}{2}[a + l] \Rightarrow S_{21} = \frac{21}{2}[0.40 + 1] \Rightarrow S_{21} = \frac{21}{\cancel{2}}[\cancel{1.40}]$$

$$S_{21} = 21 \times 0.70 \Rightarrow S_{21} = 14.70$$

***Example 2.33**: How many terms of the series $1+5+9+\cdots$ must be taken so that their sum is 190?*

$$a=1,\quad d=t_2-t_1\quad ,S_n=190$$

$$d=5-1=4$$

$$d=4$$

$$S_n=\frac{n}{2}[2a+(n-1)d] \Rightarrow \quad 190=\frac{n}{2}[2\times1+(n-1)4]$$

$$\frac{n}{2}[2+(n-1)4]=190 \Rightarrow \quad n[2+4n-4]=190\times2$$

$$n[4n-2]=380 \Rightarrow \quad 4n^2-2n-380=0 \Rightarrow \quad 2n^2-n-190=0$$

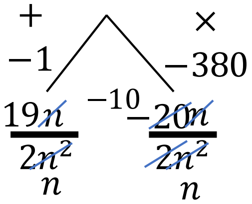
$\div$ *by* 2

$$(n-10)(2n+19)=0 \Rightarrow n-10=0,\ 2n+19=0$$

$$n=10,\ 2n=-19$$

$$n=\frac{-19}{2}\ \text{is impossible}.$$

$\therefore$ *No. of terms of given series* $n=10$



***Example 2.34**: The 13^{th} term of an A.P. is 3 and the sum of first 13 terms is 234. Find the common difference and the sum of first 21 terms.*

Given : $t_{13}=3, S_{13}=234$, *To find*: $a=?$ *and* $S_{21}=?$

$$t_{13}=3 \Rightarrow \ a+12d=3\ldots(1)$$

$$S_n=\frac{n}{2}[2a+(n-1)d] \quad \Rightarrow \ S_{13}=\frac{13}{2}[2a+(13-1)d]$$

$$234=\frac{13}{2}[2a+(13-1)d] \Rightarrow \quad 2a+(13-1)d=234\times\frac{2}{13}$$

$$2a+12d=\frac{\overset{18}{\cancel{234}}\times2}{\cancel{13}} \Rightarrow \quad 2a+12d=36\ldots(\mathbf{2})$$

Solve (1) *and* (2)

$$\begin{array}{r} a+\cancel{12d}=3 \\ (-)\quad(-)\quad(-) \\ 2a+\cancel{12d}=36 \\ \hline \cancel{-}a=\cancel{-}33 \end{array} \Rightarrow \quad a=33$$

Sub $a=33$ *in* (1) $a+12d=3$

$$33+12d=3 \Rightarrow \ 12d=3-33 \Rightarrow \ 12d=-30$$

$$d=\frac{\cancel{-30}}{\cancel{12}} \Rightarrow \quad d=-\frac{5}{2}$$

$To\ find: S_{21} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{Where } a = 33, n = 21 \text{ and d} = -\frac{5}{2}$$

$$S_{21} = \frac{21}{2}\left[2 \times 33 + (21-1)\left(\frac{-5}{2}\right)\right]$$

$$= \frac{21}{2}\left[66 + \overset{10}{\cancel{20}}\left(\frac{-5}{\cancel{2}}\right)\right] = \frac{21}{2}[66 - 50]$$

$$S_{21} = \frac{21}{\cancel{2}}[\overset{8}{\cancel{16}}] \Rightarrow \boxed{\therefore S_{21} = 168}$$

$\boldsymbol{Example\ 2.35: In\ an\ A.P.\ the\ sum\ of\ first\ n\ terms\ is\ S_n = \frac{5n^2}{2} + \frac{3n}{2}.}$
$\boldsymbol{Find\ the\ 17^{th}\ terms.}$

$$S_n = \frac{5n^2}{2} + \frac{3n}{2}$$

$To\ find\ S_{17}$

$Here: n = 17$

$$S_{17} = \frac{5 \times (17)^2}{2} + \frac{3 \times 17}{2} = \frac{5 \times 289}{2} + \frac{51}{2}$$

$$= \frac{1445}{2} + \frac{51}{2} = \frac{\cancel{1496}}{\cancel{2}} = 748$$

$$S_{17} = 748$$

$To\ find\ S_{16}$

$$S_{16} = \frac{5 \times (16)^2}{2} + \frac{3 \times 16}{2} = \frac{5 \times 256}{2} + \frac{48}{2} = \frac{\overset{640}{\cancel{1280}}}{\cancel{2}} + \frac{\overset{24}{\cancel{48}}}{\cancel{2}}$$

$$= 640 + 24$$

$$S_{16} = 664$$

$To\ find\ t_{17}$

$$t_{17} = S_{17} - S_{16}$$

$$= 748 - 664 = 84$$

$$\therefore t_{17} = 84$$

Example 2.36 : Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

$301 + 308 + 315 + \cdots + 595$

$a = 301, d = 7, l = 595$

$$n = \left(\frac{l-a}{d}\right) + 1 \implies n = \left(\frac{595-301}{7}\right) + 1$$

$$n = \left(\frac{\overset{42}{\cancel{294}}}{\cancel{7}}\right) + 1 \implies n = 42 + 1$$

$n = 43$

$$S_n = \frac{n}{2}(a + l) \quad Where\ n = 43, a = 301\ and\ l = 595$$

$$S_{43} = \frac{43}{2}(301 + 595) \implies S_{43} = \frac{43}{\cancel{2}}(\overset{448}{\cancel{896}})$$

$$S_{43} = 43 \times 448 \implies S_{43} = 19264$$

$$S_{43} = 19264$$

$7)300(42$
28
20
14
6

$7 - 6 = 1$

$1^{st}\ term = 300 + 1 = 301$

$2^{nd}\ term = 301 + 7 = 308$

$7)\ 600\ (85$
56
40
35
5

$last\ term\ 600 - 5 = 595$

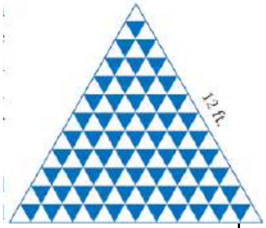
Example 2.37 : A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tile are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.

Since A mosaic is in the shape of an equilateral triangle, it's side = 12ft

A tile is in the shape of an equilateral triangle its side = 1ft = 12inch

Number of rows in the mosaic = 12

Number of white tiles in each row are 1,2,3,4 … 12. in A.P.



$Sum\ of\ white\ tiles = S_{12}$

$a = 1,\ d = 2 - 1$

$d = 1$

$$S_n = \frac{n}{2}[2a + (n-1)1] \implies S_{12} = \frac{\overset{6}{\cancel{12}}}{\cancel{2}}[2 \times 1 + (12-1)1]$$

$$S_{12} = 6[2 + 11] = 6[13]$$

$$= 78 \implies S_{12} = 78$$

Number of blue tiles in each row are 1, 2, 3, … …. . , 11. is also in A.P.

$Sum\ of\ blue\ tiles = S_{11}$

$a = 1, d = 1$

$$S_{11} = \frac{11}{2}[2 \times 1 + (11-1)1] = \frac{11}{2}[2 + 10]$$

$$= \frac{11}{\cancel{2}} \times \overset{6}{\cancel{12}}$$

$$S_{11} = 66$$

$Total\ no.\ of\ tiles\ in\ the\ mosaic = 78 + 66 = 144$

Example 2.38: The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to senthil's house is equal to the sum of numbers of the houses following senthil's house. Find senthil's house number?

Let Senthil's house number be x. Houses are numbered from $1, 2, \ldots, 49$.

$$\begin{bmatrix} \text{sum of numbers of the houses} \\ \text{prior to senthil's house} \end{bmatrix} = \begin{bmatrix} \text{sum of numbers of the houses} \\ \text{following to senthil's house} \end{bmatrix}$$

$$1 + 2 + 3 + \cdots + (x-1) = (x+1) + (x+2) + \cdots + 49.$$

$$1 + 2 + 3 + \cdots + (x-1) = [1 + 2 + 3 + \cdots + 49] - [1 + 2 + 3 + \cdots + x]$$

$$S_n = \frac{n}{2}[a + l]$$

$$\frac{(x-1)}{2}[1 + x - 1] = \frac{49}{2}[1 + 49] - \frac{x}{2}[1 + x]$$

$$\frac{x(x-1)}{2} = \frac{49 \times 50}{2} - \frac{x(1+x)}{2} \quad (\times)\ by\ 2$$

$$x^2 - x = 2450 - x - x^2 \Longrightarrow x^2 + x^2 = 2450$$

$$2x^2 = 2450 \Longrightarrow x^2 = 1225$$

$$x^2 = 35^2 \Longrightarrow x = 35$$

$\therefore$ Senthil's house number is 35

Example 2.39: The sum of first n, 2n and 3n terms of an A.P are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

$$S_1 = \frac{n}{2}[2a + (n-1)d],\ S_2 = \frac{2n}{2}[2a + (2n-1)d],\ S_3 = \frac{3n}{2}[2a + (3n-1)d]$$

$$S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left\{2[2a + (2n-1)d] - [2a + (n-1)d]\right\}$$

$$= \frac{n}{2}[4a + 2(2n-1)d - 2a - (n-1)d]$$

$$= \frac{n}{2}[4a + (4n-2)d - 2a - (n-1)d] \quad = \frac{n}{2}[2a + (4n-2)d - (n-1)d]$$

$$= \frac{n}{2}[2a + (4n - 2 - n + 1)d]$$

$$S_2 - S_1 = \frac{n}{2}[2a + (3n-1)d] \Rightarrow \; 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d] = S_3$$

$(\times)$ by 3 on both sides

$$\therefore S_3 = 3(S_2 - S_1)$$

1. Find the sum of the following

(i) 3, 7, 11, …. up to 40 terms.

A.P is 3,7,11, …. up to 40 terms.

$a = 3, d = 4, n = 40$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow \; S_{40} = \frac{40}{2}[(2 \times 3) + (40-1)4]$$

$$S_{40} = \frac{\cancel{40}}{\cancel{2}}[6 + 39(4)] \Rightarrow \; S_{40} = 20[6 + 156]$$

$$S_{40} = 20 \times 162 \Rightarrow \; S_{40} = 3240$$

$$S_{40} = 3240$$

(ii) 102, 97, 92, … up to 27 terms.

A.P is 102,97,92, … up to 27 terms

$a = 102, d = -5, n = 27$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow \; S_{27} = \frac{27}{2}[(2 \times 104) + (27-1)(-5)]$$

$$S_{27} = \frac{27}{2}[204 + 26(-5)] \Rightarrow \; S_{27} = \frac{27}{2}[204 - 130]$$

$$S_{27} = \frac{27}{\cancel{2}} \times \overset{37}{\cancel{74}} \Rightarrow \; S_{27} = 27 \times 37$$

$$S_{27} = 999$$

(iii) 6 + 13 + 20 + … + 97

$a = 6, d = 13 - 6 = 7, l = 97$

$$n = \frac{l-a}{d} + 1 \Rightarrow \; n = \frac{97-6}{7} + 1 = \frac{\overset{13}{\cancel{91}}}{\cancel{7}} + 1$$

$$n = 13 + 1 \Rightarrow \; n = 14$$

$$S_n = \frac{n}{2}[a + l] \Rightarrow \; S_{14} = \frac{\overset{7}{\cancel{14}}}{\cancel{2}}[6 + 97]$$

$$S_{14} = 7 \times 103 \Rightarrow \; \therefore S_{14} = 721$$

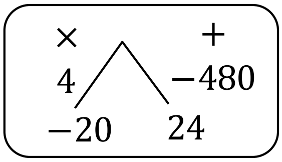
2. How many consecutive odd integers beginning with 5 will sum to 480?

$$Given: \ 5+7+9+\cdots n = 480.$$

$$a = 5, \quad d = 7-5 = 2, S_n = 480$$

$$S_n = \frac{n}{2}[2a+(n-1)d] \implies S_n = \frac{n}{2}[2\times 5+(n-1)2]$$

$$480 = \frac{n}{2}[2\times 5+(n-1)2] \implies 480 = \frac{n}{\cancel{2}}\times\cancel{2}[5+n-1]$$



$$n[5+n-1] = 480 \implies n(n+4) = 480$$

$$n^2+4n-480 = 0 \implies (n-20)(n+24) = 0$$

$$n-20 = 0, n+24 = 0$$

$$n = 20, \quad n = -24 \text{ is not possible}$$

$\therefore$ 20 *consecutive odd integers beginning with* 5 *will sum to* 480.

3. Find the sum of first 28 terms of an A.P. whose n^{th} term is $4n-3$.

$$Given: \ t_n = 4n-3$$

$$n = 1 \implies t_1 = 4(1)-3 = 4-3$$

$$t_1 = 1$$

$$n = 2 \implies t_2 = 4(2)-3 = 8-3$$

$$t_2 = 5$$

$$a = 1, \quad d = t_2 - t_1 \quad n = 28$$

$$d = 5-1 \implies d = 4$$

$$S_n = \frac{n}{2}[2a+(n-1)d] \implies S_{28} = \frac{28}{2}[2\times 1+(28-1)4]$$

$$S_{28} = \frac{28}{2}[2+(27)4] \implies S_{28} = \frac{\cancel{28}^{14}}{\cancel{2}}[2+108]$$

$$S_{28} = 14\times 110 \implies S_{28} = 1540$$

$$\therefore S_{28} = 1540$$

4. The sum of first n terms of a certain series is given as $2n^2-3n$. Show that the series is an A.P.?

$$Given: \ S_n = 2n^2-3n$$

$$n = 1 \implies S_1 = 2(1)^2 - 3(1) = 2-3$$

$$sum\ of\ first\ term = -1 = a = t_1$$

$$n = 2 \implies S_2 = 2(2)^2 - 3(2) = 2(4)-6 = 8-6$$

$$sum\ of\ first\ two\ terms = 2$$

$$t_1 + t_2 = 2 \implies -1 + t_2 = 2$$

$$t_2 = 2 + 1 \Rightarrow \ t_2 = 3$$

$$d = t_2 - t_1 \ = 3 - (-1) = 3 + 1$$

$$d = 4$$

$$\boldsymbol{A.P: a + (a + d) + (a + 2d) + (a + 3d), \ldots}$$

$$\therefore A.P. \, is \ \ -1 + 3 + 7 + 11 + \ \ldots$$

$$a = -1$$

$$a + d = -1 + 4 = 3$$

$$a + 2d = -1 + 2(4)$$

$$= -1 + 8 = 7$$

$$a + 3d = -1 + 3(4)$$

$$= -1 + 12 \ = 11$$

5. $\boldsymbol{The\ 104^{th}\ term\ and\ 4^{th}\ term\ of\ an\ A.P.\ are\ 125\ and\ 0.\ Find\ the\ sum\ of\ first\ 35\ terms.}$

$$Given: t_{104} = 125 \ , t_4 = 0, \ \ To\ find: S_{35}$$

$$t_{104} = 125 \Rightarrow \ a + 103d \ = 125 \ \ldots (1)$$

$$t_4 = 0 \Rightarrow \ a + 3d \ = 0 \ldots (2)$$

$$Solve \ (1) \ \ and \ \ (2)$$

$$\cancel{a} + 103d \ = 125$$

$$(-) \quad (-) \qquad (-)$$

$$\cancel{a} + 3d \ \ = \ \ 0$$

$$100d \ = 125 \ \ \Rightarrow \ \ \ d = \frac{\cancel{125}}{\cancel{100}} \ \ \Rightarrow \ \ \ d = \frac{5}{4}$$

$$Sub \ d = \frac{5}{4} \ in \ \ equ \ \ (2) \ a + 3d \ = 0$$

$$a + 3\left(\frac{5}{4}\right) = 0 \ \ \Rightarrow \ \ \ a + \frac{15}{4} = 0 \ \ \Rightarrow \ \ \ a = -\frac{15}{4}$$

$$S_n = \frac{n}{2}[2a + (n-1)d] \ \ \Rightarrow S_{35} = \frac{35}{2}\left[\cancel{2}\left(-\frac{15}{\cancel{4}}\right) + (35-1)\left(\frac{5}{4}\right)\right]$$

$$S_{35} = \frac{35}{2}\left[-\frac{15}{2} + (\overset{17}{\cancel{34}})\left(\frac{5}{\underset{2}{\cancel{4}}}\right)\right] \ \ \ \Rightarrow \ \ S_{35} = \frac{35}{2}\left[-\frac{15}{2} + (17)\left(\frac{5}{2}\right)\right]$$

$$S_{35} = \frac{35}{2}\left[-\frac{15}{2} + \frac{85}{2}\right] \Rightarrow \ S_{35} = \frac{35}{2}\left[-\frac{-15 + 85}{2}\right]$$

$$S_{35} = \frac{35}{\cancel{2}}\left[\frac{\overset{35}{\cancel{70}}}{2}\right] \Rightarrow \ S_{35} = \frac{1225}{2}$$

6. $\boldsymbol{Find\ the\ sum\ of\ all\ odd\ positive\ integers\ less\ than\ 450}$

$$1 + 3 + 5 + 7 + \ \ldots \ldots . + 449$$

$$a = 1, \ \ \ d = t_2 - t_1 \ = 3 - 1 = 2$$

$$d = 2$$

$$n = \frac{l - a}{d} + 1 \ \ \ \Rightarrow \ \ n = \frac{449 - 1}{2} + 1 \Rightarrow \ \ n = \frac{448}{2} + 1$$

$$n = 224 + 1 \Rightarrow \quad n = 225$$

$$S_n = \frac{n}{2}[a + l] \quad \Rightarrow \quad S_{225} = \frac{225}{2}[1 + 449]$$

$$S_{225} = \frac{225}{\cancel{2}}[\overset{225}{\cancel{450}}] \Rightarrow \quad S_{225} = 225 \times 225$$

$$S_{225} = 50{,}625$$

7. *Find the sum of all natural numbers between 602 and 902 which are not divisible by 4*

Sum of all natural numbers between 602 and 902

$$603 + 604 + \cdots + 901$$

$$a = 603, \quad d = t_2 - t_1 = 604 - 603, l = 901$$

$$d = 1$$

$$n = \frac{l - a}{d} + 1 \quad \Rightarrow \quad \therefore n = \frac{901 - 603}{1} + 1 \Rightarrow \quad n = 298 + 1$$

$$n = 299$$

$$S_n = \frac{n}{2}[a + l] \quad \Rightarrow \quad \therefore S_{229} = \frac{299}{2}[603 + 901]$$

$$S_{229} = \frac{299}{\cancel{2}}[\overset{752}{\cancel{1504}}] \Rightarrow \quad S_{229} = 229 \times 752$$

$$S_{229} = 224848$$

Sum of all natural numbers between 602 and 902 which are divisible by 4.

$$604 + 608 + 612 + \cdots + 900$$

$$a = 604, d = 4, l = 900$$

$$n = \frac{l - a}{d} + 1 \quad \Rightarrow \quad n = \frac{900 - 604}{4} + 1 \Rightarrow \quad n = \frac{296}{4} + 1$$

$$n = 74 + 1 \Rightarrow \quad \therefore n = 75$$

$$\therefore S_{75} = \frac{75}{2}[604 + 900] \Rightarrow \quad S_{75} = \frac{75}{\cancel{2}}[\overset{752}{\cancel{1504}}]$$

$$S_{75} = 75 \times 752 \Rightarrow \quad S_{75} = 56400$$

$\therefore$ *Sum of numbers which are not divisible by 4*

$$= 224848 - 56400 = 168448$$

$$\begin{array}{r} 15 \\ 4\overline{)602} \\ 60 \\ \hline 602 + 2 = 604 \end{array}$$

$$\begin{array}{r} 225 \\ 4\overline{)902} \\ 88 \\ \hline 22 \\ 20 \\ \hline (-) \quad 2 = 900 \end{array}$$

8. Raghu wish to buy a laptop. He can buy it by paying Rs. 40,000 cash or by giving it in 10 installments as Rs. 4800 in the first month, Rs. 4750 in the second month, Rs. 4700 in the third month and so on. If he pays the money in this fashion, find

(i) total amount paid in 10 installments.

(ii) how much extra amount that he has to pay than the cost?

$$Installment\ in\ 1st\ month = Rs.4800$$
$$Installment\ in\ 2nd\ month = Rs.4750$$
$$Installment\ in\ 3rd\ month = Rs.4700$$
$$4800 + 4750 + 4700 + \cdots is\ an\ A.P$$
$$a = 4800\ ,\ d = t_2 - t_1$$
$$d = 4750 - 4800\ ,\ d = -50$$

i) Total amount paid in 10 installments

$$S_n = \frac{n}{2}[2a + (n-1)d]\ ,n = 10$$

$$S_{10} = \frac{10}{2}[2 \times 4800 + (10-1)(-50)] \Rightarrow S_{10} = 5[9600 + 9(-50)]$$

$$S_{10} = 5[9600 - 450] \Rightarrow S_{10} = 5 \times 9150$$
$$= Rs.45,750\ /-$$

ii) Amount he paid extra in installments

$$= 10\ installments\ amount - cash\ amount$$
$$= 45,750 - 40,000\ = Rs.5,750/-$$

9. A man repays a loan of Rs. 65,000 by paying Rs. 400 in the first month and then increasing the payment by Rs. 300 every month. How long will it take for him to clear the loan?

$$Given : S_n = 65,000\ ,a = 400, d = 300$$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_n = \frac{n}{2}[2 \times 400 + (n-1)300]$$

$$65,000 = \frac{n}{2}[2 \times 400 + (n-1)300] \Rightarrow \frac{n}{2}[800 + (n-1)300] = 65,000$$

$$\frac{n}{2}[800 + 300n - 300] = 65,000 \Rightarrow \frac{n}{2}[500 + 300n] = 65,000$$

$$\frac{n}{\cancel{2}} \times \overset{50}{\cancel{100}}[5 + 3n] = 65,000 \Rightarrow 50n[5 + 3n] = 65000$$

$$+5 \qquad \times -3900$$
$$\frac{-65\cancel{n}}{3\cancel{n^2}}\ n \qquad 20\ \frac{\cancel{60n}}{\cancel{3n^2}}\ n$$

$$n[5 + 3n] = \frac{65000}{50} \Rightarrow n[5 + 3n] = 1300 \Rightarrow 5n + 3n^2 = 1300$$

$$3n^2 + 5n - 1300 = 0 \Rightarrow (3n + 65)(n - 20) = 0$$

$3n + 65 = 0, n - 20 = 0$

$3n = -65, \quad n = 20$

$n = \frac{-65}{3}$ *impossible*

$\therefore$ *It will take* 20 *monthes to clear his loan.*

10. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step. (i) How many bricks are required for the top most step? (ii) How many bricks are required to build the stair case?

Total no. of steps of staircase $= 30$

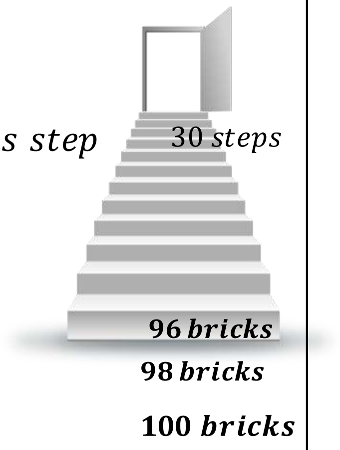
No. of bricks in bottom step $= 100$

Each successive steps are 2 *bricks less than previous step*

$\therefore 100 + 98 + 96 + 94 + \cdots$ *upto* 30 *steps*

$a = 100, d = t_2 - t_1, n = 30$

$d = 98 - 100 \Longrightarrow d = -2$



(i) *No. of bricks in the top most step*

$t_n = a + (n-1)d \Longrightarrow t_{30} = 100 + (30-1)(-2)$

$t_{30} = 100 + (29)(-2) \Longrightarrow t_{30} = 100 - 58$

$\therefore t_{30} = 42$

(ii) *Total no. of bricks are required to build the stair case*

$\therefore 100 + 98 + 96 + 94 + \cdots + 42$

$a = 100, d = -2, l = 42$ *and* $n = 30$

$$S_n = \frac{n}{2}[a + l] \Longrightarrow S_{30} = \frac{\cancel{30}^{15}}{\cancel{2}}[100 + 42]$$

$S_{30} = 15 \times 142 = 2130$

$\therefore S_{30} = 2130$

11. If $S_1, S_2, S_3, \ldots\ldots S_m$ are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3, ….., m and whose common differences are 1, 3, 5, … …(2m − 1) respectively, then show that

$$\boldsymbol{S_1 + S_2 + S_3 + \ldots, + S_m = \frac{1}{2}mn(mn+1).}$$

1^{st} $A.P. \Rightarrow a = 1, d = 1$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_1 = \frac{n}{2}[2(1) + (n-1)1] = \frac{n}{2}[2 + n - 1]$$

$$S_1 = \frac{n}{2}[n+1]$$

$2^{nd}\ A.P. \Rightarrow a = 2, d = 3$

$$S_2 \ = \frac{n}{2}[2(2) + (n-1)3] = \frac{n}{2}[4 + 3n - 3]$$

$$S_2 = \frac{n}{2}[3n+1]$$

$m^{th}\ A.P. \Rightarrow a = m, d = 2m - 1$

$$S_m \ = \frac{n}{2}[2(m) + (n-1)(2m-1)]$$

$$= \frac{n}{2}[\cancel{2m} + 2mn - \cancel{2m} - n + 1\,] = \frac{n}{2}[2mn - n + 1\,]$$

$$S_m = \frac{n}{2}[(2m-1)n + 1\,]$$

$$\therefore S_1 + S_2 + \ \dots, + S_m = \frac{n}{2}[n+1] + \frac{n}{2}[3n+1] + \cdots + \ \frac{n}{2}[(2m-1)n + 1\,]$$

$$= \frac{n}{2}[n + 1 + 3n + 1 + \cdots (2m-1)n + 1]$$

$$= \frac{n}{2}\ \ [(n + 3n + \cdots (2m-1)n)\ \ (1 + 1 + \cdots m\ terms)]$$
$$+$$

$$= \frac{n}{2}[n\left(1 + 3 + 5 + \cdots + (2m-1)\right) + m]$$

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

$$= \frac{n}{2}[n \times m^2 + m]\ = \frac{n}{2}[m(mn+1)]$$

$$\boxed{S_1 + S_2 + \ \dots, + S_m = \frac{1}{2}[mn(mn+1)]}$$

$$\mathbf{12.}\ \boldsymbol{Find\ the\ sum}\left[\underset{t_1}{\frac{\boldsymbol{a-b}}{\boldsymbol{a+b}}} + \underset{t_2}{\frac{\boldsymbol{3a-2b}}{\boldsymbol{a+b}}} + \frac{\boldsymbol{5a-3b}}{\boldsymbol{a+b}} + \cdots \boldsymbol{to\ 12\ terms}\right].$$

$$a = \frac{a-b}{a+b}, \quad n = 12$$

$$d = \ \frac{3a-2b}{a+b} - \ \frac{a-b}{a+b}$$

$$d = \frac{3a - 2b - (a-b)}{a+b} = \frac{3a - 2b - a + b}{a+b} \quad \Longrightarrow \quad d = \frac{2a-b}{a+b}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{\cancel{12}}{\cancel{2}}\left[2\left(\frac{a-b}{a+b}\right) + (12-1)\left(\frac{2a-b}{a+b}\right)\right]$$

$$= 6\left[2\left(\frac{a-b}{a+b}\right) + (11)\left(\frac{2a-b}{a+b}\right)\right]$$

$$= 6\left[\frac{2a-2b}{a+b} + \frac{22a-11b}{a+b}\right]$$

$$= 6\left[\frac{2a-2b+22a-11b}{a+b}\right]$$

$$\boxed{\therefore S_{12} = \frac{6}{a+b}[24a-13b]}$$

EXERCISE 2.7

Example 2.40: which of the following sequences form a G.P.?

(*i*) 7, 14, 21, 28, … … … …

$t_1 = 7, t_2 = 14, t_3 = 21, t_4 = 28$

$$\frac{t_2}{t_1} = \frac{\cancel{14}^{2}}{\cancel{7}} = 2$$

$$\frac{t_3}{t_2} = \frac{\cancel{21}^{3}}{\cancel{14}_{2}} = \frac{3}{2}$$

$$\frac{t_4}{t_3} = \frac{\cancel{28}^{4}}{\cancel{21}_{3}} = \frac{4}{3}$$

$$\therefore \frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$$

$\therefore 7, 14, 21, 28, \ldots$ *is not G.P.*

(*ii*) $\frac{1}{2}$, 1, 2, 4, … … .

$$t_1 = \frac{1}{2}, t_2 = 1, t_3 = 2, t_4 = 4$$

$$\frac{t_2}{t_1} = \frac{1}{\frac{1}{2}} = \frac{2}{1} = 2$$

$$\frac{t_3}{t_2} = \frac{2}{1} = 2$$

$$\frac{t_4}{t_3} = \frac{\cancel{4}}{\cancel{2}} = 2$$

$$\therefore \frac{t_2}{t_1}=\frac{t_3}{t_2}=\frac{t_4}{t_3}$$

$$\therefore \frac{1}{2},1,2,4,\ldots\, is\ G.P.$$

(iii) $\mathbf{5,25,50,75,\ldots\ldots}$

$$t_1=5,t_2=25,t_3=50,t_4=75$$

$$\frac{t_2}{t_1}=\frac{{}^{5}\cancel{25}}{\cancel{5}}=5$$

$$\frac{t_3}{t_2}=\frac{\cancel{50}^{2}}{\cancel{25}}=2$$

$$\frac{t_4}{t_3}=\frac{\cancel{75}^{3}}{\cancel{50}_{2}}=\frac{3}{2}$$

$$\therefore \frac{t_2}{t_1}\neq\frac{t_3}{t_2}\neq\frac{t_4}{t_3}$$

$$\therefore 5,25,50,75,\ldots is\ not\ G.P.$$

$\boldsymbol{Example\ 2.41}$: $\boldsymbol{Find\ the\ geometric\ progression\ whose\ first\ term\ and\ common\ ratios\ are\ given\ by}$

$(\boldsymbol{i})$ $\boldsymbol{a=-7,r=6}$

$The\ general\ form\ of\ G.P\ is\ a,ar,ar^2,\ldots..$

$$-7,-7(6),-7(6)^2,\ldots.$$

$$-7,-7(6),-7(36),\ldots.$$

$$\therefore G.P.is-7,-42,-252,\ldots.$$

$(\boldsymbol{ii})$ $\boldsymbol{a=256,r=0.5}$

$$256,\ 256(0.5),256(0.5)^2,\ldots$$

$$256,256(0.5),256(0.25),\ldots\ldots.$$

$$256,\overset{128}{\cancel{256}}\left(\frac{1}{\cancel{2}}\right),\overset{64}{\cancel{256}}\left(\frac{1}{\cancel{4}}\right),\ldots$$

$$\therefore G.P.is\ \ 256,128,64,\ldots$$

$\boldsymbol{Example\ 2.42}$: $\boldsymbol{Find\ the\ 8^{th}\ term\ of\ the\ G.P.9,3,1,\ldots}$

$G.P\ is\ \ 9,3,1,\ldots$

$$a=9,\ r=\frac{t_2}{t_1}=\frac{\cancel{3}}{\cancel{9}}=\frac{1}{3}\ ,n=8$$

$$t_n=ar^{n-1}$$

$$t_8 = 9\left(\frac{1}{3}\right)^{8-1} = 9\left(\frac{1}{3}\right)^{7}$$

$$= \cancel{9}\left(\frac{1}{\cancel{243}}\right)_{27}$$

$$t_8 = \frac{1}{27}$$

Example 2.43: In a G.P. the 4th term is $\frac{8}{9}$ and the 7th term is $\frac{64}{243}$. Find the GP.

$$t_4 = \frac{8}{9} \implies ar^{4-1} = \frac{8}{9} \implies ar^3 = \frac{8}{9} \quad \dots(1) \qquad \boxed{t_n = ar^{n-1}}$$

$$t_7 = \frac{64}{243} \implies ar^{7-1} = \frac{64}{243} \implies ar^6 = \frac{64}{243} \dots(2)$$

$$Sub\ \ r = \frac{2}{3}\ \ in\ (1)$$

$$a\left(\frac{2}{3}\right)^3 = \frac{8}{9} \implies a\left(\frac{8}{27}\right) = \frac{8}{9} \implies a = \frac{\cancel{8}}{\cancel{9}} \times \frac{\cancel{27}^{\,3}}{\cancel{8}} \implies a = 3$$

$$The\ G.P\ is\ \ a, ar, ar^2, \dots.$$

$$3,\ 3\left(\frac{2}{3}\right), 3\left(\frac{2}{3}\right)^2, \dots$$

$$3, \cancel{3}\left(\frac{2}{\cancel{3}}\right), \cancel{3}\left(\frac{4}{\cancel{9}}\right)_3, \dots \qquad \therefore G.P. is\ \ 3, 2, \frac{4}{3}, \dots$$

Example 2.44: The product of three consecutive terms of a G.P. is 343 and their sum is $\frac{91}{3}$. Find the three terms?

$$The\ three\ consecutive\ terms\ of\ G.P.\ be\ \frac{a}{r}, a, ar.$$

$$Their\ product = 343$$

$$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 343 \implies a^3 = 7^3 \implies a = 7$$

$$Their\ Sum = \frac{91}{3}$$

$$\frac{a}{r} + a + ar = \frac{91}{3} \implies a\left(\frac{1}{r} + r + r^2\right) = \frac{91}{3}$$

$$\cancel{7}\left(\frac{1 + r + r^2}{r}\right) = \frac{\cancel{91}^{\,13}}{3} \implies \frac{1 + r + r^2}{r} = \frac{13}{3} \implies 3 + 3r + 3r^2 = 13r$$

$$3r^2+3r-13r+3=0 \Rightarrow 3r^2-10r+3=0$$

$$3r^2-9r-r+3=0 \Rightarrow 3r(r-3)-1(r-3)=0$$

$$(r-3)(3r-1)=0 \Rightarrow r-3=0\,,3r-1=0$$

$$r=3, 3r=1$$

$$r=\frac{1}{3}$$

The three terms are $\frac{a}{r}, a, ar.$

If $a=7, r=3$

$$\frac{7}{3}, 7, 7(3) \Rightarrow \frac{7}{3}, 7{,}21$$

If $a=7, r=\frac{1}{3}$

$$\frac{7}{\frac{1}{3}}, 7, 7\left(\frac{1}{3}\right) \Rightarrow 7(3), 7, 7\left(\frac{1}{3}\right) \Rightarrow\ 21, 7, \frac{7}{3}$$

Example 2.45: The present value of a machine is Rs. 40,000 and its value depreciates each year by 10% . Find the estimated value of the machine in the 6th year.

The present value of machine is Rs. 40,000.

Since it is depreciated at the rate of 10% after one year the value of the machine is 90% of the initial value.

i.e. The value of the machine end of the 1st year $= Rs. 40{,}000 \times \frac{90}{100}$

The value of the machine end of the 2nd year $= Rs. 40{,}000 \times \left(\frac{90}{100}\right)^2$

Continuing this way ,the value of the machine depreciates in the following way as

$$40000, 40000 \times \frac{90}{100}, 40000 \times \left(\frac{90}{100}\right)^2, \ldots\ldots\ldots \text{is a } G.P.$$

$$a=40000\ ,\ r=\frac{90}{100}=\frac{9}{10}, n=6$$

$$t_n = ar^{n-1}$$

$$t_6 = 40000\left(\frac{9}{10}\right)^{6-1} = 40000\left(\frac{9}{10}\right)^5 = 4\cancel{0000} \times \frac{9}{\cancel{10}} \times \frac{9}{\cancel{10}} \times \frac{9}{\cancel{10}} \times \frac{9}{\cancel{10}} \times \frac{9}{10}$$

$$= \frac{4\times 9\times 9\times 9\times 9\times 9}{10} = \frac{236196}{10}$$

$$t_6 = 23619.6$$

$$\text{The value of the machine in the } 6^{th} \text{ year is Rs.} 23{,}619.60.$$

1. Which of the following sequences are in G.P

(i) $3, 9, 27, 81, \ldots\ldots$

$$t_1 = 3\ ,\ t_2 = 9\ ,\ t_3 = 27\ ,\ t_4 = 81$$

$$\frac{t_2}{t_1} = \frac{\cancel{9}}{\cancel{3}} = 3$$

$$\frac{t_3}{t_2} = \frac{\cancel{27}}{\cancel{9}} = 3$$

$$\frac{t_4}{t_3} = \frac{\cancel{81}}{\cancel{27}} = 3$$

$$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$$

$\therefore 3, 9, 27, 81, \ldots\ldots$ is a G.P.

(ii) $4, 44, 444, \ldots\ldots$

$$t_1 = 4\ ,\ t_2 = 44\ , t_3 = 444$$

$$\frac{t_2}{t_1} = \frac{\cancel{44}}{\cancel{4}} = 11$$

$$\frac{t_3}{t_2} = \frac{\cancel{444}}{\cancel{44}} = \frac{111}{11}$$

$$\therefore \frac{t_2}{t_1} \neq \frac{t_3}{t_2}$$

$\therefore 4, 44, 444, \ldots$ is not a G.P

(iii) $0.5, 0.05, 0.005, \ldots\ldots.$

$$t_1 = 0.5, \quad t_2 = 0.05\ ,\ t_3 = 0.005$$

$$\frac{t_2}{t_1} = \frac{0.05}{0.5} = \frac{0.05 \times 100}{0.5 \times 100} = \frac{5}{50.0} = \frac{\cancel{5}}{\cancel{50}} = \frac{1}{10}$$

$$\frac{t_3}{t_2} = \frac{0.005}{0.05} = \frac{0.005 \times 1000}{0.05 \times 1000} = \frac{5}{50} = \frac{\cancel{5}}{\cancel{50}} = \frac{1}{10}$$

$$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$\therefore 0.5, 0.05, 0.005, \ldots$ is a G.P

$\boldsymbol{iv)\ \frac{1}{3},\frac{1}{6},\frac{1}{12}}\ ,\ldots\ldots.$

$$t_1=\frac{1}{3}\ ,\ t_2=\frac{1}{6}\ ,t_3=\frac{1}{12}$$

$$\frac{t_2}{t_1}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{\underset{2}{\cancel{6}}}\times\frac{\cancel{3}}{1}=\frac{1}{2}$$

$$\frac{t_3}{t_2}=\frac{\frac{1}{12}}{\frac{1}{6}}=\frac{1}{\underset{2}{\cancel{12}}}\times\frac{\cancel{6}}{1}=\frac{1}{2}$$

$$\therefore\frac{t_2}{t_1}=\frac{t_3}{t_2}$$

$$\therefore\ \frac{1}{3},\frac{1}{6},\frac{1}{12}\ ,\ldots\ldots is\ a\ G.P.$$

$\boldsymbol{(v)\ 1,-5,25,-125},\ldots\ldots.$

$$t_1=1,\ t_2=-5,\ \ t_3=25,\ t_4=25$$

$$\frac{t_2}{t_1}=\frac{-5}{1}=-5$$

$$\frac{t_3}{t_2}=\frac{-\cancel{25}}{\cancel{5}}=-5$$

$$\frac{t_4}{t_3}=\frac{\cancel{-}125}{\cancel{-}25}=-5$$

$$\therefore\frac{t_2}{t_1}=\frac{t_3}{t_2}=\frac{t_4}{t_3}$$

$$\therefore 1,-5,25,-125,\ldots is\ a\ G.P.$$

$\boldsymbol{(vi)\ 120,60,30,18},\ldots\ldots.$

$$t_1=120,\ t_2=60,\ t_3=30,\ t_4=30$$

$$\frac{t_2}{t_1}=\frac{\cancel{60}}{\cancel{120}_2}=\frac{1}{2}$$

$$\frac{t_3}{t_2}=\frac{\cancel{30}}{\cancel{60}_2}=\frac{1}{2}$$

$$\frac{t_4}{t_3}=\frac{\cancel{18}^3}{\cancel{30}_5}=\frac{3}{5}\neq\frac{1}{2}$$

$$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$$

$\therefore$ 120, 60, 30, 18, …, $is\ not\ a\ G.P$

$\boldsymbol{vii)}$ $\mathbf{16, 4, 1, \frac{1}{4}}, \dots\dots$

$$t_1 = 16, t_2 = 4, t_3 = 1, t_4 = \frac{1}{4}$$

$$\frac{t_2}{t_1} = \frac{\cancel{4}}{\cancel{16}_4} = \frac{1}{4}$$

$$\frac{t_3}{t_2} = \frac{1}{4}$$

$$\frac{t_2}{t_1} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$$

$$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$$

$\therefore 16, 4, 1, \frac{1}{4}, \dots\dots$ $is\ a\ G.P.$

2. *Write the first three terms of the G.P. whose first term and the common ratio are given belows*:

$\mathbf{(i)\ a = 6, r = 3}$

$The\ first\ three\ terms\ of\ G.P.\ are\ \ a, ar, ar^2.$

$The\ first\ three\ terms\ of\ the\ GP\ are$ $6, 6(3), 6(3)^2$

$$6, 6(3), 6(9) \Rightarrow \therefore 6, 18, 54$$

$\mathbf{(ii)\ a = \sqrt{2}, r = \sqrt{2}}$

$The\ first\ three\ terms\ of\ the\ GP\ are$ $\sqrt{2}, \sqrt{2}(\sqrt{2}), \sqrt{2}(\sqrt{2})^2$

$\therefore \sqrt{2}, 2, 2\sqrt{2}$

$\mathbf{(iii)\ a = 1000, r = \frac{2}{5}}$

$The\ first\ 3\ terms\ of\ the\ GP\ are$ $1000,\ 1000\left(\frac{2}{5}\right),\ 1000\left(\frac{2}{5}\right)^2$

$$1000, 1000\left(\frac{2}{5}\right), 1000\left(\frac{4}{25}\right) \Rightarrow \therefore 1000, 400, 160$$

3. In a G.P. 729, 243, 81, …… find t_7?

$$Given : \underset{t_1}{729}, \underset{t_2}{243}, \underset{t_3}{81}, \ldots\ldots are\ in\ G.P.$$

$$a = 729$$

$$r = \frac{t_3}{t_2} = \frac{\cancel{81}^{9}}{\underset{27}{\cancel{243}}} = \frac{\cancel{9}}{\cancel{27}_3} = \frac{1}{3}$$

$$t_n = ar^{n-1}$$

$$\therefore t_7 = 729 \times \left(\frac{1}{3}\right)^{7-1}$$

$$= 729 \times \left(\frac{1}{3}\right)^{6} = 729 \times \frac{1}{3^6} = 729 \times \frac{1}{729}$$

$$\therefore t_7 = 1$$

4. Find x so that $x+6, x+12, x+15$ are consecutive terms of a G.P.

$$Given : The\ consecutive\ terms\ are\ x+6, x+12, x+15$$

$$\frac{t_2}{t_1} = \frac{t_3}{t_2} \implies \frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$(x+12)^2 = (x+6)(x+15)$$

$$x^2 + 2(x)(12) + 12^2 = x^2 + 15x + 6x + 90$$

$$\cancel{x^2} + 24x + 144 = \cancel{x^2} + 21x + 90$$

$$24x + 144 = 21x + 90 \implies 24x - 21x = 90 - 144$$

$$3x = -54 \implies x = \frac{-\cancel{54}}{\cancel{3}}$$

$$\therefore x = -18$$

5. Find the number of terms in the following G.P.

i) 4 , 8 , 16 , …… , 8192?

$$\boxed{t_n = ar^{n-1}}$$

$$a = 4,\ \ r = \frac{t_2}{t_1} = \frac{8}{4} = 2,\ t_n = 8192$$

$$r = 2$$

$$t_n = 8192 \implies ar^{n-1} = 8192$$

$$4 \times 2^{n-1} = 8192 \implies 2^{n-1} = \frac{\cancel{8192}\ 2048}{\cancel{4}}$$

$$2^{n-1} = 2048 \implies 2^{n-1} = 2^{11}$$

$$n - 1 = 11 \implies n = 11 + 1$$

$$n = 11 + 1 \implies \therefore n = 12$$

$$\begin{array}{r|l} 2 & 2048 \\ 2 & 1024 \\ 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

(ii) $\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots\ldots,\frac{1}{2187}$?

$$a=\frac{1}{3},\ r=\frac{\frac{1}{9}}{\frac{1}{3}}=\frac{1}{\cancel{9}_{3}}\times\frac{\cancel{3}}{1}=\frac{1}{3}$$

$$t_n=\frac{1}{2187}\ \Rightarrow\ ar^{n-1}=\frac{1}{2187}$$

$$\frac{1}{3}\times\left(\frac{1}{3}\right)^{n-1}=\frac{1}{2187}\ \Rightarrow\ \left(\frac{1}{3}\right)^{n-1}=\frac{\cancel{3}}{\underset{729}{\cancel{2187}}}\ \Rightarrow\ \left(\frac{1}{3}\right)^{n-1}=\frac{1}{729}$$

$$\left(\frac{1}{3}\right)^{n-1}=\left(\frac{1}{3}\right)^{6}\ \Rightarrow\ n-1=6$$

$$n=6+1\Rightarrow\therefore n=7$$

6. *In a G.P. the 9^{th} term is 32805 and 6^{th} term is 1215. Find the 12^{th} term.*

$Given: t_9=32805,\ t_6=1215,\ t_{12}=?$

$$t_n=ar^{n-1}$$

$t_9=32805\Rightarrow a\times r^{9-1}=32805$

$a\times r^{8}=32805\ \ldots(1)$

$t_6=1215\Rightarrow a\times r^{6-1}=1215$

$ar^{5}=1215\ \ldots(2)$

$$\begin{array}{r|l} & \ \ 27 \\ 1215 & 32805 \\ & 2430 \\ \hline & \ \ 8505 \\ & \ \ 8505 \\ \hline & \ \ \ \ \ \ 0 \end{array}$$

$$(1)\div(2)\Rightarrow\ \frac{\cancel{a}r^{8}\ r^{3}}{\cancel{a}r^{5}}=\frac{\cancel{32805}}{\cancel{1215}}$$

$$r^3=27\Rightarrow r^3=3^3\Rightarrow r=3$$

$Sub\ r=3, in\ (2)\ ar^5=1215$

$$a\times3^5=1215\Rightarrow a\times243=1215\Rightarrow a=\frac{1215}{243}\Rightarrow a=5$$

$$\therefore\ t_{12}=5\times3^{12-1}=5\times3^{11}$$

7. *Find the 10^{th} term of the G.P. whose 8^{th} term is 768 and the common ratio is 2.*

$Given:\ t_8=768, r=2$

$t_8=768\Rightarrow a.r^{8-1}=768$

$a\times2^7=768\ \Rightarrow a\times128=768$

$$\boxed{\begin{aligned}2^7&=2^4\times2^3\\&=16\times8\\&=128\end{aligned}}$$

$$a=\frac{\cancel{768}}{\cancel{128}} \Longrightarrow \quad a=6$$

$$\begin{array}{r} 6 \\ 128\,\overline{)\,768} \\ \underline{768} \\ \underline{0} \end{array}$$

To find t_{10}

$$t_n=ar^{n-1} \text{ where } a=6, r=2$$

$$\therefore t_{10}=6\times 2^{10-1}$$

$$=6\times 2^9=6\times 512$$

$$t_{10}=3072$$

$$\begin{aligned} 2^9 &= 2^4\times 2^4\times 2^1 \\ &= 16\times 16\times 2 \\ &= 256\times 2 \\ &= 512 \end{aligned}$$

8. *If a, b, c are in A.P. then show that* $\underset{t_1}{3^a}, \underset{t_2}{3^b}, \underset{t_3}{3^c}$ ***are in G.P.?***

Given: a, b, c *are in A.P.*

$$t_2-t_1=t_3-t_2$$

$$b-a=c-b \Longrightarrow \quad b+b=c+a$$

$$2b=a+c \quad \ldots (1)$$

To show : $\overset{t_1}{3^a}, \overset{t_2}{3^b}, \overset{t_3}{3^c}$ *are in G.P.*

$$\frac{t_2}{t_1}=\frac{t_3}{t_2} \Longrightarrow \frac{3^b}{3^a}=\frac{3^c}{3^b}$$

$$\left(3^b\right)^2=3^a.3^c \Longrightarrow \quad 3^{2b}=3^{a+c}$$

$$L.H.S=3^{2b} \text{ where : } 2b=a+c$$

$$=3^{a+c}=R.H.S$$

$$\therefore 3^a, 3^b, 3^c \text{ are in } G.P.$$

9. *In a G.P. the product of three consecutive terms is 27 and the sum of the product of two termstaken at a time is* $\frac{57}{2}$**. *Find three terms.***

Let the 3 consecutive terms of G.P. be $\frac{a}{r}, a, ar$

Product of three terms $=27$

$$\frac{a}{r}\times a\times ar=27 \quad \Longrightarrow a^3=27$$

$$\therefore a=3$$

Sum of product of terms taken 2 at a time $=\frac{57}{2}$

$$\frac{a}{r}\times a+a\times ar+ar\times\frac{a}{r}=\frac{57}{2} \quad \Longrightarrow \frac{a^2}{r}+a^2r+a^2=\frac{57}{2}$$

$$a^2\left[\frac{1}{r}+r+1\right]=\frac{57}{2} \quad \Longrightarrow 3^2\left[\frac{1}{r}+r+1\right]=\frac{57}{2} \quad \Longrightarrow \quad 9\left[\frac{1+r^2+r}{r}\right]=\frac{57}{2}$$

$$\frac{1+r^2+r}{r} = \frac{\cancel{57}^{\,19}}{{}_{3}\cancel{9}\times 2} \Longrightarrow \frac{1+r^2+r}{r} = \frac{19}{6} \Longrightarrow 6r^2+6r+6 = 19r$$

$$6r^2-13r+6=0$$

$$6r^2-9r-4r+6=0 \Longrightarrow 3r(2r-3)-2(2r-3)=0$$

$$(2r-3)(3r-2)=0 \Longrightarrow 2r-3=0,\ 3r-2=0$$

$$2r=3, 3r=2$$

$$\boxed{r=\frac{3}{2}\quad ,r=\frac{2}{3}}$$

$$\boxed{\cancel{3}\times\frac{2}{\cancel{3}}=2}$$

$$if\ a=3, r=\frac{3}{2} then\ the\ three\ terms\ are \frac{a}{r}, a, ar$$

$$=\frac{3}{\frac{3}{2}}, 3, 3\left(\frac{3}{2}\right), \ldots. = 2, 3, \frac{9}{2}, \ldots$$

$$if\ a=3, r=\frac{2}{3} then\ the\ three\ terms\ are\ \frac{9}{2}, 3, 2$$

10. *A man joined a company as Assistant Manager. The company gave him a starting salary of* Rs.60,000 *and agreed to increase his salary* 5% *annually. what will be his salary after* 5 *years*?

$Given$: $Initial\ salary = \text{R.S. } 60{,}000$

$Annual\ increment = 5\ \%$

$n = 5\ years$

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 60{,}000\left(1+\frac{5}{100}\right)^5 = 60{,}000 \times \left(\frac{105}{100}\right)^5$$

$$= 60{,}000 \times (1.05)^5$$

$Taking\ log\ on\ both\ side$

$$logA = \log[60{,}000 \times (1.05)^5]$$

$$= \log 60{,}000 + 5\log(1.05)]$$

$$= 4.7782 + 0.1060$$

$$= 5.8842$$

$$A = antilog\ 5.8842$$

$$= 76{,}000$$

$$\begin{array}{rl} \log 60{,}000 = & 4.7782 \\ 5\log(1.05) = & \underline{0.1060} \\ & \underline{5.8842} \end{array}$$

$$Antilog\ 5.8842 = 76{,}000$$

$\therefore Rs. 76{,}600\ will\ be\ his\ salary\ after\ 5years$

11. Sivamani is attending an interview for a job and the company gave two offers to him Offer A: Rs. 20,000 to start with followed by a guranteed annual increase of 6% for the first 5 years Offer B: Rs. 22,000 to start with followed by a guranteed annual increase of 3% for the first 3 years. What is his salary in the 4^{th} year with respect to the offers A and B?

Offer A: $P = Rs. 20{,}000$, $r = 6\%$

$n = 3 (in\ the\ 4^{th} year)$

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 20{,}000\left(1+\frac{6}{100}\right)^3 = 20{,}000\left(\frac{106}{100}\right)^3$$

$$= 20{,}000(1.06)^3$$

Taking log on both side

$$logA = \log[20{,}000 \times (1.06)^3]$$

$$= \log 20{,}000 + 3\log(1.06) = 4.3010 + 0.0759$$

$$= 4.3769$$

$$A = antilog\ 4.3769 = 23{,}820$$

$\therefore$ *His salary is in the* 4^{th} *year* $= Rs. 23{,}820.$

$\log 20{,}000 = 4.3010$

$3\log(1.06) = 0.0759$

4.3769

$antilog\ 4.3769 = 23820$

Offer B: $P = Rs. 22{,}000,$ $r = 3\%$

$n = 3$ (in the 4^{th} year)

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 22{,}000\left(1+\frac{3}{100}\right)^3 = 22{,}000\left(\frac{103}{100}\right)^3$$

$$= 22{,}000 \times (1.03)^3$$

Taking log on both side

$$logA = \log[22{,}000 \times (1.03)^3]$$

$$= \log 22{,}000 + 3\log(1.03)$$

$$= 4.3424 + 0.0384$$

$$= 4.3808$$

$$A = antilog\ 4.3808$$

$$= 24040$$

$\log 22{,}000 = 4.3424$

$3\log(1.03) = 0.0384$

$= 4.3808$

$antilog\ 4.3808 = 24040$

$\therefore$ *His salary is in the* 4^{th} *year* $= Rs. 24{,}040.$

12. If a, b, c are three consecutive terms of A.P and x, y, z are three consecutive term of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

Given: a, b, c are three consecutive terms of A.P. $a, a+d, a+2d$

$$a = a, b = a + d, c = a + 2d$$

$$b - c = a + d - (a + 2d)$$

$$= \cancel{a} + d - \cancel{a} - 2d \implies b - c = -d$$

$$c - a = \cancel{a} + 2d - \cancel{a} \implies c - a = 2d$$

$$a - b = a - (a + d)$$

$$= \cancel{a} - \cancel{a} - d \implies a - b = -d$$

x, y, z are three consecutive terms of G.P. $x, xr, xr^2, \ldots$

To Prove: $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

$$L.H.S = x^{b-c} \times y^{c-a} \times z^{a-b}$$

$$= (x)^{-d} \times (xr)^{2d} \times (xr^2)^{-d} = x^{-d} \times x^{2d} \times r^{2d} \times x^{-d} \times r^{-2d}$$

$$= x^{-d+2d-d} \times r^{2d-2d} = x^0 \times r^0 = 1 \times 1$$

$$= 1 = R.H.S. \qquad \textit{Hence proved}$$

EXERCISE 2.8

Example 2.46: Find the sum of 8 terms of the G.P. $1, -3, 9, -27, \ldots..$

Given: G.P. is $1, -3, 9, -27, \ldots..$

$$a = 1,\ t_1 = 1, t_2 = -3$$

$$r = \frac{t_2}{t_1} = \frac{-3}{1} \implies r = -3 < 1$$

$$\text{if } r < 1, S_n = \frac{a(1 - r^n)}{1 - r} \qquad \text{where } n = 8$$

$$S_8 = \frac{1(1 - (-3)^8)}{1 - (-3)} = \frac{1 - 6561}{1 + 3}$$

$$= \frac{\cancel{-6560}}{\cancel{4}} = -1640$$

$$\therefore S_{18} = -1640$$

Example 2.47: Find the first term of G.P. in which $S_6 = 4095$ and $r = 4$

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

Given: $S_6 = 4095, r = 4 > 1$

$$S_6 = 4095$$

$$\frac{a(r^n - 1)}{r - 1} = 4095 \qquad \text{where } n = 6 \text{ and } r = 4$$

$$\frac{a(4^6 - 1)}{4 - 1} = 4095 \implies \frac{a(4096 - 1)}{4 - 1} = 4095$$

$$\frac{a(4095)}{3} = 4095 \Longrightarrow a = \cancel{4095} \times \frac{3}{\cancel{4095}}$$

Here first term $a = 3$

Example 2.48: *How many terms of the series* $1+4+16+\cdots$ *make the sum* 1365?

Given: $1+4+16+\cdots$

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

$$a = 1,\ r = \frac{4}{1} \Rightarrow r = 4 > 1$$

$$S_n = 1365 \Rightarrow \frac{1(4^n-1)}{4-1} = 1365 \Rightarrow \frac{1(4^n-1)}{3} = 1365$$

$$4^n - 1 = 1365 \times 3 \Rightarrow 4^n = 4095 + 1$$

$$4^n = 4096$$

$$4^n = 4^6 \Rightarrow \therefore n = 6$$

$$\begin{array}{r|r} 4 & 4096 \\ \hline 4 & 1024 \\ \hline 4 & 256 \\ \hline 4 & 64 \\ \hline 4 & 16 \\ \hline 4 & 4 \\ \hline & 1 \end{array}$$

Example 2.49: *Find the sum of* $\mathbf{3 + 1 + \frac{1}{3} + \cdots\infty}$

$$a = 3,\ r = \frac{t_2}{t_1} = \frac{1}{3}$$

$$r = \frac{1}{3}$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$$S_\infty = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{3-1}{3}} = \frac{3}{\frac{2}{3}}$$

$$= 3 \times \frac{3}{2} = \frac{9}{2}$$

$$\therefore S_\infty = \frac{9}{2}$$

Example 2.50: *Find the rational form of the number* 0.6666 …

$$= 0.6 + 0.06 + 0.006 + 0.0006 + \cdots \textit{ is a G.P.}$$

$$a = 0.6$$

$$r = \frac{t_2}{t_1} = \frac{0.06}{0.6} = \frac{0.06 \times 100}{0.6 \times 100} = \frac{\cancel{6}}{\cancel{6}0} = \frac{\cancel{1}}{\cancel{1}0} = 0.1$$

$$r = 0.1$$

The sum of infinite terms in G.P. is $S_\infty = \dfrac{a}{1-r}$

$$S_\infty = \frac{0.6}{1-0.1} = \frac{0.6}{0.9}$$

$$= \frac{0.6 \times 10}{0.9 \times 10} = \frac{\cancel{6}}{\cancel{9}} = \frac{2}{3}$$

Thus the rational number equivalent of $0.6666 \ldots$ *is* $\frac{2}{3}$

Example 2.51: *Find the sum of n terms of the series* $\mathbf{5 + 55 + 555 + \cdots}$

$$S_n = 5 + 55 + 555 + \cdots \text{to } n \text{ terms}$$

$$= 5\,(1 + 11 + 111 + \cdots \text{to } n \text{ terms})$$

$$= \frac{5}{9}(9 + 99 + 999 + \cdots \text{to } n \text{ terms})$$

$$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + \cdots \text{to } n \text{ terms}]$$

$$= \frac{5}{9}[\underbrace{10 + 100 + 1000 + \cdots n\text{terms}}_{} + (-1 - 1 - 1 \ldots \text{to } n \text{ terms})]$$

Which are in G.P $a = 10$, $r = \frac{100}{10} = 10 > 1$

$$= \frac{5}{9}\left[\frac{a(r^n - 1)}{r - 1} + (-n)\right] = \frac{5}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$S_n = \frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

$$= \frac{50(10^n - 1)}{81} - \frac{5n}{9}$$

Example 2.52: *Find the least positive integer n such that*
$\mathbf{1 + 6 + 6^2 + \cdots + 6^n > 5000}$**.**

$$1 + 6 + 6^2 + \cdots + 6^n \implies S_n > 5000$$

$$a = 1,\ r = \frac{6}{1}$$

$$r = 6 > 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, r > 1$$

$$S_n = \frac{1(6^n - 1)}{6 - 1} \implies S_n = \frac{6^n - 1}{5}$$

$6^2 = 36$
$6^3 = 36 \times 6 = 216$
$6^4 = 216 \times 6 = 1296$
$6^5 = 1296 \times 6 = 7776$
$6^6 = 7776 \times 6$
$= 46656$

$$S_n > 5000 \implies \frac{6^n - 1}{5} > 5000 \implies 6^n - 1 > 25000$$

$6^n > 25001$ *Since,* $6^5 = 7776$

$6^6 = 46656 \implies \therefore n = 6$

The least positive value of n is 6 such that $1 + 6 + 62 + \cdots + 6^n > 5000$

Example 2.53: A person saved money every year. half as much as he could in the previous year. If he had totally saved Rs.7875 in 6 years then how much did he in the first year?

Total amount saved in 6 years is $S_6 = 7875$

Since he saved half as much money as every year he saved in the previous year.

$$r = \frac{1}{2} < 1$$

$$S_6 = 7875$$

$$S_n = \frac{a(1-r^n)}{1-r}, if\ r < 1 \quad where\ n = 6\ and\ r = 4$$

$$S_6 = \frac{a\left[1-\left(\frac{1}{2}\right)^6\right]}{1-\frac{1}{2}} \Longrightarrow 7875 = \frac{a\left[1-\left(\frac{1}{2}\right)^6\right]}{1-\frac{1}{2}}$$

$$\frac{a\left(1-\frac{1}{64}\right)}{\frac{1}{2}} = 7875 \Longrightarrow \frac{a\left(\frac{63}{64}\right)}{\frac{1}{2}} = 7875$$

$$a \times \frac{63}{\cancel{64}} \times \frac{\cancel{2}}{1} = 7875 \Longrightarrow a \times \frac{63}{32} = 7875$$

$$a = \frac{\overset{\cancel{875}\ 125}{\cancel{7875}} \times 32}{\underset{\cancel{7}}{\cancel{63}}} \Longrightarrow a = 125 \times 32$$

$$a = 4000$$

The amount saved in the first year is Rs.4000

1. Find the sum of first n terms of the G.P. (i) $\mathbf{5, -3, \frac{9}{5}, -\frac{27}{25}}$

$$a = 5, r = \frac{t_2}{t_1} \Longrightarrow r = \frac{-3}{5} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}, r < 1$$

$$\therefore S_n = 5 \times \left[\frac{1-\left(-3/5\right)^n}{1-\left(-3/5\right)}\right] \Longrightarrow S_n = 5 \times \left[\frac{1-\left(-3/5\right)^n}{1+\frac{3}{5}}\right]$$

$$S_n = 5 \times \left[\frac{1-\left(-3/5\right)^n}{\frac{8}{5}}\right] \Longrightarrow S_n = 5 \times \frac{5}{8}\left[1-\left(-3/5\right)^n\right]$$

$$S_n = \frac{25}{8}\left[1-\left(-3/5\right)^n\right]$$

ii) Find the sum of first n terms of the G.P. is 256, 64, 16, … … …

$$a = 256,\ r = \frac{t_2}{t_1} \Rightarrow r = \frac{64}{256} \Rightarrow r = \frac{1}{4} < 1$$

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}\ , if\ r < 1}$$

$$\therefore S_n = 256 \times \left[\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}\right] \Rightarrow S_n = 256 \times \frac{1-\left(\frac{1}{4}\right)^n}{\frac{4-1}{4}}$$

$$S_n = 256 \times \frac{1-\left(\frac{1}{4}\right)^n}{\frac{3}{4}} \Rightarrow S_n = 256 \times \frac{4}{3}\left[1-\left(\frac{1}{4}\right)^n\right]$$

$$S_n = \frac{1024}{3}\left[1-\left(\frac{1}{4}\right)^n\right]$$

2. Find the sum of first six terms of the G.P. 5, 15, 45, …

$$a = 5, r = \frac{t_2}{t_1} \Rightarrow r = \frac{15}{5} \Rightarrow r = 3 > 1$$

$$n = 6$$

$$S_n = \frac{a(r^n-1)}{1-r}, if\ r > 1$$

$$\therefore S_6 = \frac{5(3^6-1)}{3-1} \Rightarrow S_6 = \frac{5(729-1)}{3-1} \Rightarrow S_6 = \frac{5 \times \overset{364}{\cancel{728}}}{\cancel{2}}$$

$$S_6 = 5 \times 364$$

$$\therefore S_6 = 1820$$

3. Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872

$Given: r = 5 > 1\ , S_6 = 46872$

$$S_n = \frac{a(r^n-1)}{1-r}, if\ r > 1$$

$$n = 6$$

$$\frac{a(5^6-1)}{5-1} = 46872 \Rightarrow \frac{a(5^6-1)}{4} = 46872$$

$$a(5^6-1) = 46872 \times 4 \Rightarrow a(15625-1) = 46872 \times 4$$

$$a(15624) = 46872 \times 4 \Rightarrow a = \frac{46872 \times \cancel{4}}{\cancel{15624}_{\,3906}}$$

$$\therefore a = 12$$

$$\boxed{\begin{aligned}&\underbrace{5\times5}\times\underbrace{5\times5}\times\underbrace{5\times5}\\ &= \underbrace{25\times25}\times25\\ &= 625\times25 = 15625\end{aligned}}$$

$$\begin{array}{r|l} & 12 \\ \hline 3906 & 46872 \\ & 3906 \\ \hline & 7812 \\ & 7812 \\ \hline & 0 \end{array}$$

4. Find the sum to infinity of (i) $9 + 3 + 1 + \dots\dots$ is a geometric series

$$a = 3, r = \frac{t_2}{t_1} \Rightarrow r = \frac{3}{9} \Rightarrow r = \frac{1}{3} < 1$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$$\therefore S_\infty = \frac{9}{1-\frac{1}{3}} \Rightarrow S_\infty = \frac{9}{1-\frac{1}{3}}$$

$$S_\infty = \frac{9}{\frac{3-1}{3}} \Rightarrow S_\infty = \frac{9}{\frac{2}{3}} \Rightarrow S_\infty = 9 \times \frac{3}{2}$$

$$\boxed{S_\infty = \frac{27}{2}}$$

(ii) $21 + 14 + \frac{28}{3} + \dots\dots\dots$ is a geometric series

$$a = 21, r = \frac{t_2}{t_1} \Rightarrow r = \frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{21}}} \Rightarrow r = \frac{2}{3} < 1$$

$$S_\infty = \frac{a}{1-r} \Rightarrow S_\infty = \frac{21}{1-\frac{2}{3}} \Rightarrow S_\infty = \frac{21}{\frac{3-2}{3}}$$

$$S_\infty = \frac{21}{\frac{1}{3}} \Rightarrow S_\infty = \frac{21 \times 3}{1}$$

$$\boxed{S_\infty = 63}$$

5. Find the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{2}$ then find the common ratio

Given: $a = 8, S_\infty = \frac{32}{3}, r = ?$

$$S_\infty = \frac{a}{1-r} \Rightarrow S_\infty = \frac{32}{3}$$

$$\frac{8}{1-r} = \frac{32}{3} \Rightarrow 32(1-r) = 24$$

$$32 - 32r = 24 \Rightarrow -32r = 24 - 32$$

$$\cancel{-}32r = \cancel{-}8 \Rightarrow r = \frac{\cancel{8}}{\underset{4}{\cancel{32}}} \Rightarrow \therefore r = \frac{1}{4}$$

6. *Find the sum to n terms of the series*: (*i*) 0.4 + 0.04 + 0.004 + ⋯ *to n terms*

$$0.4 + 0.44 + 0.444 + \cdots to\ n\ terms$$

$$= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ \ldots n\ terms \quad = 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ \ldots n\ terms\right]$$

$$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ \ldots n\ terms\right]$$

$$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ \ldots n\ terms\right]$$

$$= \frac{4}{9}\left[(1 + 1 + 1 + \cdots + n\ terms) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots + n\ terms\right)\right]$$

$$= \frac{4}{9}\left[(1 + 1 + 1 + \cdots + n\ terms) - \frac{1}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \cdots + n\ terms\right)\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{10} \times \frac{1 \times \left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}\right] \qquad a = 1, r = \frac{1}{10} < 1 \qquad \boxed{S_n = \frac{a(1 - r^n)}{1 - r}}$$

$$= \frac{4}{9}\left[n - \frac{1}{10} \times \frac{\left(1 - \left(\frac{1}{10}\right)^n\right)}{\frac{10 - 1}{10}}\right] \quad = \frac{4}{9}\left[n - \frac{1}{10} \times \frac{\left(1 - \left(\frac{1}{10}\right)^n\right)}{\frac{9}{10}}\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{\cancel{10}} \times \frac{\cancel{10}}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right] \quad = \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right]$$

ii*) 3 + 33 + 333 + ⋯ *upto n terms

$$S_n = 3 + 33 + 333 + \cdots to\ n\ terms$$

$$= 3\ (1 + 11 + 111 + \cdots to\ n\ terms)$$

$$= \frac{3}{9}\ (9 + 99 + 999 + \cdots to\ n\ terms)$$

$$= \frac{1}{3}[(10 - 1) + \ (100 - 1) + (1000 - 1) + \cdots to\ n\ terms]$$

$$= \frac{1}{3}[\underbrace{10 + 100 + 1000 + \cdots nterms}_{} + (-1 - 1 - 1 \ldots to\ n\ terms)]$$

Which are in G.P$a = 10,\ r = \frac{100}{10} = 10 > 1$ $\boxed{S_n = \frac{a(r^n - 1)}{r - 1}, if\ \ r > 1}$

$$= \frac{1}{3}\left[\frac{a(r^n - 1)}{r - 1} + (-n)\right] = \frac{1}{3}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$S_n = \frac{1}{3}\left[\frac{10(10^n - 1)}{9} - n\right] \Rightarrow S_n = \frac{10}{27}(10^n - 1) - \frac{n}{3}$$

7. *Find the sum of the Geometric series* $3 + 6 + 12 + \dots\dots + 1536$

$Given:\ G.P.\ is\ 3 + 6 + 12 + \dots + 1536$

$$a = 3, r = \frac{t_2}{t_1} \Rightarrow r = \frac{6}{3} \Rightarrow r = 2 > 1$$

$$t_n = 1536 \Rightarrow ar^{n-1} = 1536$$

$$3 \times 2^{n-1} = 1536 \Rightarrow 2^{n-1} = \frac{\overset{512}{\cancel{1536}}}{\cancel{3}}$$

$$2^{n-1} = 512 \Rightarrow 2^{n-1} = 2^9$$

$$n - 1 = 9 \Rightarrow n = 9 + 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, if\ r > 1$$

$$\therefore S_{10} = \frac{3(2^{10} - 1)}{2 - 1} \Rightarrow S_{10} = \frac{3(1024 - 1)}{1}$$

$$S_{10} = 3 \times 1023 \Rightarrow \therefore S_{10} = 3069$$

8. *Kumar writes a letter to four of his friends. He asks each one of them to copy the letter to four different person with the instruction that they continuethe process similarly. Assuming that the process is unaltered and its cost Rs. 2 to mail one letter. Find the amount spent on postage when 8th set of the letters is mailed.*

The numbers of mails delivered are $4, 4 \times 4, 4 \times 4 \times 4, \dots$

$4, 16, 64, \dots\dots\dots.8^{th}\ set\ of\ letters$

Each mail cost Rs. 2

$\therefore The\ total\ cost\ is\ (4 \times 2) + (16 \times 2) + (64 \times 2) + \dots\dots..8^{th} set$

$8 + 32 + 28 + \dots\dots 8^{th}\ set$ is a G. P.

$$a = 8, r = \frac{t_2}{t_1} \Rightarrow r = \frac{32}{8} \Rightarrow r = 4 > 1$$

$$n = 8$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, if\ r > 1 \Rightarrow S_8 = \frac{8(4^8 - 1)}{4 - 1}$$

$$S_8 = \frac{8(4^8 - 1)}{4 - 1} \Rightarrow S_8 = \frac{8(65536 - 1)}{3}$$

$$S_8 = \frac{8 \times \overset{21845}{\cancel{65535}}}{\cancel{3}} \Rightarrow S_8 = 8 \times 21845$$

$$S_8 = 174760$$

Amount spent on postage Rs. 174760.

$$= \frac{19 \times \overset{10}{\cancel{20}} \times \overset{13}{\cancel{39}}}{\cancel{6}_{\cancel{3}}} = 19 \times 10 \times 13$$

$$= 2470$$

(ii) $\mathbf{5^2 + 10^2 + 15^2 + \ \dots . +105^2}$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \ 5^2 + 10^2 + 15^2 + \ \dots . +105^2 = 5^2 \ (1^2 + 2^2 + 3^2 + \ \dots \dots . +21^2)$$

$$= \ 25 \left(\frac{21\ (21+1)(2 \times 21 + 1)}{6} \right) \ = \ 25 \left(\frac{\overset{7}{\cancel{21}} \times \overset{11}{\cancel{22}} \times 43}{\cancel{6}_{\cancel{3}}} \right)$$

$$= 25 \times 7 \times 11 \times 43$$

$$= 82775$$

$iii)$ $\mathbf{15^2 + 16^2 + 17^2 + \ \dots + 28^2}$

$$= (1^2 + 2^2 + 3^2 + \ \dots + 28^2) - (1^2 + 2^2 + 3^2 + \ \dots + 14^2)$$

$$\boxed{1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$= \left(\frac{28\ (28+1)(2 \times 28 + 1)}{6} \right) - \left(\frac{14\ (14+1)(2 \times 14 + 1)}{6} \right)$$

$$= \left(\frac{\overset{14}{\cancel{28}} \times 29 \times \overset{19}{\cancel{57}}}{\cancel{6}_{\cancel{3}}} \right) - \left(\frac{\overset{7}{\cancel{14}} \times \overset{5}{\cancel{15}} \times 29}{\cancel{6}_{\cancel{3}}} \right) = 14 \times 29 \times 19 - 7 \times 5 \times 29$$

$$= 7714 - 1015 \ = 6699$$

Example $\mathbf{2.57}$: Find the sum of $\mathbf{(i) 1^3 + 2^3 + 3^3 + \ \dots + 16^3}$
$\mathbf{(ii) 9^3 + 10^3 + \ \dots + 21^3}$

$$\boxed{1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2}$$

(i) $\mathbf{1^3 + 2^3 + 3^3 + \ \dots + 16^3}$

$$= \left(\frac{16\ (16+1)}{2} \right)^2 = \left(\frac{\overset{8}{\cancel{16}} \times 17}{\cancel{2}} \right)^2$$

$$= (8 \times 17)^2 \ = (136)^2 \ = 18496$$

$\boldsymbol{(ii)\ 9^3 + 10^3 + \ \ldots + 21^3}$

$$9^3 + 10^3 + \ \ldots + 21^3 = (1^3+2^3 + \ \ldots + 21^3) - (1^3+2^3 + \ \ldots + 8^3)$$

$$\boxed{1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$$= \left(\frac{21\ (21+1)}{2}\right)^2 - \left(\frac{8\ (8+1)}{2}\right)^2$$

$$= \left(\frac{21\ \times \overset{11}{\cancel{22}}}{\cancel{2}}\right)^2 - \left(\frac{\overset{4}{\cancel{8}} \times 9}{\cancel{2}}\right)^2 = \left(21\times 11\right)^2 - \left(4\times 9\right)^2$$

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

$$= (231)^2 - (36)^2$$

$$= (231+36)(231-36) = 267 \times 195$$

$$= 52065$$

Example 2.58: ***If*** $\boldsymbol{1+2+3+\cdots+n = 666}$ ***then find*** $\boldsymbol{n}$.

$$\boxed{1+2+3+\cdots+n = \frac{n(n+1)}{2}}$$

$Given: 1+2+3+\cdots+n = 666$

$$\frac{n(n+1)}{2} = 666 \Longrightarrow n(n+1) = 2\times 666$$

$$n^2 + n = 1332 \Longrightarrow n^2 + n - 1332 = 0$$

$$\begin{array}{ccc} + & & \times \\ 1 & & -1332 \\ -36 & & 37 \end{array}$$

$$\begin{array}{r|l} 2 & 1332 \\ \hline 2 & 666 \\ \hline 3 & 333 \\ \hline 3 & 111 \\ \hline & 37 \end{array}$$

$$(n+37)(n-36) = 0 \Longrightarrow n + 37 = 0, n - 36 = 0$$

$n = -37$ *is not natural number.*

Hence $n = 36$

1. *Find the sum of the following series* $\boldsymbol{(i)\ 1+2+3+\ \ldots\ldots+60}$

$\boldsymbol{(ii)\ 3+6+9+\ \ldots\ldots+96}$ $\boldsymbol{(iii)\ 51+52+53+\ \ldots\ldots\ldots+92}$

$\boldsymbol{(iv)\ 1+4+6+9+16+\ \ldots+225}$ $\boldsymbol{(v)\ 6^2+7^2+8^2+\ \ldots+21^2}$

$\boldsymbol{(vi)\ 10^3+11^3+12^3+\ \ldots+20^3}$ $\boldsymbol{(vii)\ 1+3+5+\ \ldots\ldots+71}$

$\boldsymbol{i)\ 1+2+3+\ \ldots\ldots+60}$

$$\boxed{1+2+3+\cdots+n = \frac{n(n+1)}{2}}$$

$$= \frac{60\ (60+1)}{2} = \frac{\overset{30}{\cancel{60}} \times 61}{\cancel{2}}$$

$$= 30\times 61 = 1830$$

$$\begin{array}{r} 30 \times 61 \\ \hline 30 \\ 180 \\ \hline 1830 \\ \hline \end{array}$$

ii) $3+6+9+\dots\dots+96$

$$\boxed{1+2+3+\cdots+n=\frac{n(n+1)}{2}}$$

$3+6+9+\dots\dots+96=3(1+2+3+\dots\dots+32)$

$$=3\times\left(\frac{32\,(32+1)}{2}\right)$$

$$=\frac{3\times\overset{16}{\cancel{32}}\times 33}{\cancel{2}}=3\times 16\times 33$$

$$=1584$$

$$\begin{array}{r} {}^{2}\\ 48\times 33\\ \hline 144\\ 144\\ \hline 1584\\ \hline \end{array}$$

iii) $51+52+53+\dots\dots\dots+92$

$51+52+53+\dots\dots\dots+92=(1+2+3+\dots+92)-(1+2+3+\dots+50)$

$$=\frac{92\,(92+1)}{2}-\frac{50\,(50+1)}{2}$$

$$\boxed{1+2+3+\cdots+n=\frac{n(n+1)}{2}.}$$

$$=\frac{\overset{46}{\cancel{92}}\times 93}{\cancel{2}}-\frac{\overset{25}{\cancel{50}}\times 51}{\cancel{2}}=(46\times 93)-(25\times 51)$$

$$=4278-1275=3003$$

$$\begin{array}{r} {}^{5}\\ 46\times 93\\ \hline 138\\ 414\\ \hline 278\\ \hline \end{array}\qquad\begin{array}{r} {}^{2}\\ 25\times 51\\ \hline 25\\ 125\\ \hline 1275\\ \hline \end{array}$$

iv) $1+4+9+16+\dots\dots\dots+225$

$$=1^2+2^2+3^2+\dots+15^2$$

$$\boxed{\sum_{k=1}^{n}K^2=\frac{n(n+1)(2n+1)}{6}}$$

$$=\frac{15\quad(15+1)\quad(2\times 15+1)}{6}=\frac{\overset{5}{\cancel{15}}\times\overset{8}{\cancel{16}}\times 31}{\cancel{6}\,\cancel{3}}$$

$$=5\times 8\times 31=40\times 31=1240$$

v) $6^2+7^2+8^2+\dots+21^2$

$$\boxed{1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}}$$

$6^2+7^2+8^2+\dots+21^2$

$$=(1^2+2^2+3^2+\dots+21^2)-(1^2+2^2+3^2+\dots+5^2)$$

$$=\frac{21\quad(21+1)\quad(2\times 21+1)}{6}-\frac{5\,(5+1)\,(2\times 5+1)}{6}$$

$$=\frac{\overset{7}{\cancel{21}}\times\overset{11}{\cancel{22}}\times 43}{\cancel{6}\,\cancel{3}}-\frac{5\times\cancel{6}\times 11}{\cancel{6}}$$

$$=(7\times 11\times 43)-(5\times 11)=77\times 43-55$$

$$=3311-55=3256$$

$$\begin{array}{r} {}^{2}\\ 43\times 77\\ \hline 301\\ 301\\ \hline 3311\\ \hline \end{array}$$

vi) $\mathbf{10^3 + 11^3 + 12^3 + \dots + 20^3}$

$$= (1^3 + 2^3 + \dots\dots + 20^3) - (1^3 + 2^3 + \dots\dots + 9^3)$$

$$= \left(\frac{20\ (20+1)}{2}\right)^2 - \left(\frac{9\ (9+1)}{2}\right)^2$$

$$\boxed{1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$$= \left(\frac{\overset{10}{\cancel{20}} \times 21}{\cancel{2}}\right)^2 - \left(\frac{9 \times \overset{5}{\cancel{10}}}{\cancel{2}}\right)^2$$

$$= (210)^2 - (45)^2$$

$$= 44100 - 2025$$

$$= 42075$$

21 × 21
21
42
441

2
45 × 45
1 225
180
2025

vii) $\mathbf{1 + 3 + 5 + \dots\dots + 71}$

$$\boxed{1 + 3 + 5 + \dots\dots\dots + l = \left(\frac{l+a}{d}\right)^2}$$

Take $a = 1, d = 3 - 1 = 2, l = 71$

$$1 + 3 + 5 + \dots\dots\dots + 71 = \left(\frac{71+1}{2}\right)^2$$

$$= \left(\frac{\overset{36}{\cancel{72}}}{\cancel{2}}\right)^2 = (36)^2$$

$\therefore 1 + 3 + 5 + \dots + 71 = 1296$

36 × 36
216
108
1296

3

2. If $\mathbf{1 + 2 + 3 + \dots\dots\dots + k = 325}$**, then find** $\mathbf{1^3 + 2^3 + 3^3 + \dots\dots\dots + k^3}$

Given : $1 + 2 + 3 + \dots + k = 325$

$$\frac{k(k+1)}{2} = 325$$

$$1^3 + 2^3 + 3^3 + \dots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$$

$$= (325)^2$$

$\therefore 1^3 + 2^3 + 3^3 + \dots + k^3 = 105625$

325 × 325
1625
1 650
975
105625

3. If $\mathbf{1^3 + 2^3 + 3^3 + \dots\dots\dots + k^3 = 44100}$**, then find**
$\mathbf{1 + 2 + 3 + \dots\dots + k}$.

Given: $1^3 + 2^3 + 3^3 + \dots\dots + k^3 = 44100$

$$\left(\frac{k(k+1)}{2}\right)^2 = 44100 \Rightarrow \frac{k(k+1)}{2} = \sqrt{44100}$$

$$\frac{k(k+1)}{2} = 210$$

$$\boxed{1 + 2 + 3 + \dots\dots + k = \frac{k(k+1)}{2}}$$

$\therefore 1 + 2 + 3 + \dots\dots\dots + k = 210$

2	44100
2	22050
5	11025
5	2205
3	441
3	147
7	49
	7

$$\sqrt{44100} = \sqrt{\underbrace{2\times 2}\times \underbrace{5\times 5}\times \underbrace{3\times 3}\times \underbrace{7\times 7}}$$

$$= 2 \times 5 \times 3 \times 7$$

$$= 10 \times 21 = 210$$

4. *How many terms of the series $1^3 + 2^3 + 3^3 + \cdots$ should be taken to get the sum 14400?*

Given: $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + k^3 = 14400$

$$\left(\frac{k(k+1)}{2}\right)^2 = 14400$$

$$\frac{k(k+1)}{2} = 120 \Rightarrow k^2 + k = 240$$

$$k^2 + k - 240 = 0 \Rightarrow (k+16)(k-15) = 0$$

$$k - 15 = 0,\ k + 16 = 0$$

$$k = 15, k = -16 \quad But\ k \neq -16$$

$$\therefore k = 15$$

2	14400
2	7200
2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
	5

$$\sqrt{14400} = \sqrt{\underbrace{2\times 2}\times \underbrace{2\times 2}\times \underbrace{2\times 2}\times \underbrace{3\times 3}\times \underbrace{5\times 5}}$$

$$= 2 \times 2 \times 2 \times 3 \times 5$$

$$= 8 \times 15 = 120$$

5. *The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the values of n.*

Given: *sum of the squares of first "n" natural numbers* $= 285$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = 285$$

$$\frac{n(n+1)(2n+1)}{6} = 285 \quad \ldots(1)$$

Sum of their cubes $= 2025$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = 2025$$

$$\left(\frac{n(n+1)}{2}\right)^2 = 2025 \Rightarrow \frac{n(n+1)}{2} = \sqrt{2025}$$

$$\frac{n(n+1)}{2} = 45 \Rightarrow n(n+1) = 90$$

$$sub\ n(n+1) = 90\ in\ (1)\quad \frac{n(n+1)(2n+1)}{6} = 285$$

$$\frac{\overset{45}{\cancel{90}} \times (2n+1)}{\cancel{6}\ 3} = 285 \Rightarrow \overset{15}{\cancel{45}} \times \frac{2n+1}{\cancel{3}} = 285$$

$$2n+1=\frac{285}{15} \Rightarrow 2n+1=19$$

$$2n=19-1 \Rightarrow \cancel{2}n=\cancel{1}8$$

$$\boxed{\therefore n=9}$$

$$\begin{array}{l} 15)\ 285\ (19 \\ \quad\ \ 15 \\ \hline \quad\ \ 135 \\ \quad\ \ 135 \\ \hline \quad\quad 0 \end{array}$$

6. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, … … , 24 cm. how much area can be decorated with these colour papers.

Given : Sides of 15 *square Colour papers are* 10 cm, 11 cm, 12 cm, … . . 24 cm

$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$\therefore It's\ area = 10^2+11^2+12^2+\ \ldots..+24^2$$

$$=(1^2+2^2+3^2+\ \ldots\ldots\ldots+24^2)-(1^2+2^2+3^2+\ \ldots\ldots\ldots+9^2)$$

$$=\frac{24(24+1)(2\times24+1)}{6}-\frac{9(9+1)(2\times9+1)}{6}$$

$$=\frac{\overset{4}{\cancel{24}}\times25\times49}{\cancel{6}}-\frac{\overset{3}{\cancel{9}}\times\overset{5}{\cancel{10}}\times19}{\cancel{6}\,{}_{\cancel{2}}}$$

$$=4\times25\times49\ -3\times5\times19$$

$$=4900-285=4615\ cm^2$$

7. Find the sum of the series to $(2^3-1^3)+(4^3-3^3)+(6^3-5^3)+\ \ldots+$ up to n terms

(i) n terms **(ii) 8 terms**

$$Let\ \ S_n=(2^3-1^3)+(4^3-3^3)+(6^3-5^3)+\ \ldots\ldots\ldots+upto\ n\ terms$$

$$=(2^3+4^3+6^3+\ \ldots\ldots\ldots n\ terms)-(1^3+3^3+5^3+\ \ldots\ldots+n\ terms)$$

$$=\sum_1^n(2n)^3-\sum_1^n(2n-1)^3$$

$$\boxed{\begin{array}{l}\left[\left(\because a^3-b^3=(a-b)(a^2+ab+b^2)\right)\right] \\ \left[\left(\because (a-b)^2=(a^2-2ab+b^2)\right)\right]\end{array}}$$

$$=\sum_1^n[(2n)^3-(2n-1)^3]$$

$$=\sum_1^n[(\cancel{2n}-\cancel{2n}+1)(4n^2+2n(2n-1)+(2n-1)^2]$$

$$=\sum_1^n[4n^2+4n^2-2n+4n^2-4n+1]\ \ =\sum_1^n[12n^2-6n+1]$$

$$= 12\sum n^2 - 6\sum n + \sum 1$$

$$= 12\left[\frac{n(n+1)(2n+1)}{6}\right] - 6\left[\frac{n(n+1)}{2}\right] + n$$

$$= n(n+1)\left[\frac{\overset{2}{\cancel{12}}(2n+1)}{\cancel{6}} - \frac{\overset{3}{\cancel{6}}}{\cancel{2}}\right] + n$$

$$= n(n+1)[2(2n+1) - 3] + n$$

$$= (n^2 + n)[4n + 2 - 3] + n$$

$$= (n^2 + n)[4n - 1] + n$$

$$= 4n^3 + 4n^2 - n^2 - n + n$$

$$= 4n^3 + 3n^2$$

$$\boxed{\therefore S_n = 4n^3 + 3n^2}$$

$ii)$ when $n = 8$

$$S_8 = 4(8^3) + 3(8^2)$$

$$= 4(512) + 3(64)$$

$$= 2048 + 192 = 2240$$

ALGEBRA

EXERCISE 3.1

Simultaneous Linear Equations in Three Variables

Example 3.1 The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

Let the present age of father be x years andthe present age of son be y years

$$\text{Age of father} = 6 \times \text{ son's age}$$

$$x = 6y \; \ldots (1)$$

$$\text{After 6years father's age } x+6 \text{ and son's } y+6$$

$$\text{After 6years age of father} = 4 \times \text{ son's age}$$

$$x + 6 = 4(y+6) \cdots (2)$$

$$sub\ x = 6y \text{ in } (2)\ \ 6y + 6 = 4(y+6)$$

$$6y + 6 = 4y + 24 \implies 6y - 4y = 24 - 6$$

$$2y = 18 \implies y = \frac{18}{2} \implies y = 9$$

$$sub\ y = 9 \text{ in } (1)\ \ x = 6(9) \implies x = 54$$

$$\therefore \text{son's age} = 9 \text{ years and father's age} = 54 \text{ years.}$$

Example 3.2 Solve $2x - 3y = 6,\ x + y = 1$

$$2x - 3y = 6 \ldots (1) \text{ and } x + y = 1 \ \ldots (2)$$

$$\text{Solve } (1) \text{ and } (2)$$

$$\begin{array}{lll} (1) & \implies & \cancel{2x} - 3y = 6 \\ & & (-)\ \ (-)\ \ \ \ (-) \\ (2)\times 2 & \implies & \cancel{2x} + 2y = 2 \\ \hline & & -5y = 4 \implies y = -\frac{4}{5} \end{array}$$

$$\text{Sub } y = -\frac{4}{5} \text{ in } (2)\ x + y = 1$$

$$x - \frac{4}{5} = 1 \implies x = 1 + \frac{4}{5} \implies x = \frac{5+4}{5} \implies x = \frac{9}{5}$$

$$\therefore x = \frac{9}{5}, y = -\frac{4}{5}$$

Example 3.3 Solve the following system of linear equations in three variables
$3x - 2y + z = 2, 2x + 3y - z = 5, x + y + z = 6.$

$$3x - 2y + z = 2 \ldots (1)\quad 2x + 3y - z = 5 \ldots (2)\quad x + y + z = 6 \ldots (3)$$

$$\text{Solve } (1) \text{ and } (2)$$

$$\begin{array}{l} 3x - 2y \cancel{+z} = 2 \\ 2x + 3y \cancel{-z} = 5 \\ \hline 5x + y = 7 \ldots (4) \end{array}$$

$$\text{Solve } (2) \text{ and } (3)$$

$$\begin{array}{l} 2x + 3y \cancel{-z} = 5 \\ x + y \cancel{+z} = 6 \\ \hline 3x + 4y = 11 \ldots (5) \end{array}$$

$$\text{Solve } (4) \text{ and } (5)$$

$$\begin{array}{lll} (4)\times 4 & \implies & 20x + \cancel{4y} = 28 \\ & & (-)\ \ \ (-)\ \ \ \ \ (-) \\ (5) & \implies & 3x + \cancel{4y} = 11 \\ \hline & & 17x = 17 \end{array}$$

$x = \frac{17}{17} \Rightarrow x = 1$

Sub $x = 1$ in (4) $5x + y = 7$

$5(1) + y = 7 \Rightarrow 5 + y = 7$

$y = 7 - 5 \Rightarrow y = 2$

Sub $x = 1, y = 2$ in (3) $x + y + z = 6$

$1 + 2 + z = 6 \Rightarrow 3 + z = 6$

$z = 6 - 3 \Rightarrow z = 3$

$\therefore x = 1, y = 2, z = 3$

Example 3.4 In an interschool atheletic meet, with 24 individual events, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

Let the number of I, II and III place finishers be x , y and z respectively.

Total number of events $= 24$; *Total number of points* $= 56$.

Hence, the linear equations in three variables are

$x + y + z = 24 \ldots (1)$

$5x + 3y + z = 56 \ldots (2)$

$x + y = z \ldots (3)$

Sub (3) *in* (1) $x + y + z = 24$

$z + z = 24$

$2z = 24 \Rightarrow z = \frac{24}{2} \Rightarrow z = 12$

sub $z = 12$ *in* (3) $x + y = z$

$x + y = 12 \ldots (4)$

Sub $z = 12$ *in* (2) $5x + 3y + 12 = 56$

$5x + 3y = 56 - 12 \Rightarrow 5x + 3y = 44 \ldots (5)$

Solve (5) *and* (4)

$$\begin{array}{lcl} (5) & \Rightarrow & 5x + \cancel{3y} = 44 \\ & & \overset{(-)}{}\quad \overset{(-)}{} \quad \overset{(-)}{} \\ (4) \times 3 & \Rightarrow & \underline{3x + \cancel{3y} = 36} \\ & & 2x = 8 \Rightarrow x = \frac{8}{2} \Rightarrow x = 4 \end{array}$$

Sub $x = 4, z = 12$ *in* (3) $x + y = z$

$4 + y = 12 \Rightarrow y = 12 - 4 \Rightarrow y = 8$

$\therefore$ *Number of first place finishers is* 4

Number of second place finishers is 8

Number of third place finishers is 12

System of Linear Equations in Three Variables

The system may have only one solution, infinitely many solutions or no solution

(*i*) *A system of linear equations is called inconsistent if it has no solutions.*

If you obtain a false equation such as $0 = 1$, *in any of the steps then the system has no solution.*

(*ii*) *A system of linear equations is called consistent if it has infinitely solutions.*

If you do not obtain a false solution, but obtain an identity, such as $0 = 0$ *then the system has infinitely many solutions.*

(*iii*) *A system of linear equations is called consistent if it has unique solutions.*

***Example* 3.5 Solve** $\boldsymbol{x + 2y - z = 5; x - y + z = -2; -5x - 4y + z = -11}$

Let $x + 2y - z = 5 \ldots (1)$, $x - y + z = -2 \ldots (2)$, $-5x - 4y + z = -11 \ldots (3)$

Solve (1) *and* (2)

$$\begin{array}{r} x + 2y - \cancel{z} = 5 \\ x - y + \cancel{z} = -2 \\ \hline 2x + y = 3 \ldots (4) \end{array}$$

Solve (2) *and* (3)

$$\begin{array}{r} x - y + \cancel{z} = -2 \\ (+) \quad (+) \quad (-) \qquad (+) \\ -5x - 4y + \cancel{z} = -11 \\ \hline 6x + 3y = 9 \\ \div 3 \\ 2x + y = 3 \ldots (5) \end{array}$$

Solve (4) *and* (5)

$$\begin{array}{r} \cancel{2x} + \cancel{y} = \cancel{3} \\ (-) \quad (-) \quad (-) \\ \cancel{2x} + \cancel{y} = \cancel{3} \\ \hline 0 = 0 \end{array}$$

Here we arrive at an identity $0 = 0$

Hence the system has an consistence and infinite number of solutions.

Example* 3.6 *Solve $\boldsymbol{3x + y - 3z = 1; -2x - y + 2z = 1; -x - y + z = 2}$

Let $3x + y - 3z = 1 \ldots (1)$, $-2x - y + 2z = 1 \ldots (2)$, $-x - y + z = 2 \ldots (3)$

Solve (1) and (2)

$$\begin{array}{r} 3x + \cancel{y} - 3z = 1 \\ -2x - \cancel{y} + 2z = 1 \\ \hline x - z = 2 \ldots (4) \end{array}$$

Solve (1) and (3)

$$\begin{array}{r} 3x + \cancel{y} - 3z = 1 \\ -x - \cancel{y} + z = 2 \\ \hline 2x - 2z = 3 \ldots (5) \end{array}$$

Solve (4) and (5)

$$\begin{array}{lr} (4) \times 2 \Longrightarrow & 2x - \cancel{2z} = 4 \\ & (-) \quad (+) \quad (-) \\ (5) \Longrightarrow & \cancel{2x} - 2z = 3 \\ \hline & 0 = 1 \end{array}$$

Here we arrive at a contradiction as $0 \neq 1$

This means that the system is inconsistent and has no solution.

Example* 3.7 *Solve $\boldsymbol{\frac{x}{2} - 1 = \frac{y}{6} + 1 = \frac{z}{7} + 2; \frac{y}{3} + \frac{z}{2} = 13}$

$$\frac{x}{2} - 1 = \frac{y}{6} + 1$$

$$\frac{x}{2} - \frac{y}{6} = 1 + 1 \Rightarrow \frac{3x - y}{6} = 2$$

$$\frac{3x-y}{12}=1 \Longrightarrow 3x-y=12 \ \dots(1)$$

$$\frac{x}{2}-1=\frac{z}{7}+2$$

$$\frac{x}{2}-\frac{z}{7}=1+2 \Longrightarrow \frac{7x-2z}{14}=3 \Longrightarrow 7x-2z=42 \ \dots(2)$$

$$\frac{y}{3}+\frac{z}{2}=13 \Longrightarrow \frac{2y+3z}{6}=13 \Longrightarrow 2y+3z=78 \ \dots(3)$$

Eliminating z from (2) *and* (3)

$$\begin{array}{lcr}
(2)\times 3 & \Longrightarrow & 21x+0y-\cancel{6z}=126 \\
(3)\times 2 & \Longrightarrow & 0x+4y+\cancel{6z}=156 \\
\hline
 & & 21x+\cancel{4y}=282 \\
(1)\times 4 & \Longrightarrow & 12x-\cancel{4y}=48 \\
\hline
 & & 33x=330 \Longrightarrow x=\dfrac{330}{33} \Longrightarrow x=10
\end{array}$$

Sub $x=10$ in (1) $3(10)-y=12$

$$30-y=12 \Longrightarrow 30-12=y \Longrightarrow 18=y \Longrightarrow y=18$$

Sub $x=10$ in (2) $7(10)-2z=42$

$$70-2z=42 \Longrightarrow -2z=42-70 \Longrightarrow -2z=-28$$

$$z=\frac{-28}{-2} \Longrightarrow z=14$$

$$\therefore x=10, y=18, z=14$$

Example 3.8 Solve : $\boldsymbol{\frac{1}{2x}+\frac{1}{4y}-\frac{1}{3z}=\frac{1}{4};\ \frac{1}{x}=\frac{1}{3y};\ \frac{1}{x}-\frac{1}{5y}+\frac{4}{z}=2\frac{2}{15}}$

Let $\frac{1}{x}=p,\ \frac{1}{y}=q,\ \frac{1}{z}=r$

The given equations are written as $\frac{p}{2}+\frac{q}{4}-\frac{r}{3}=\frac{1}{4},\ p=\frac{q}{3}$

$$p-\frac{q}{5}+4r=2\frac{2}{15} \Longrightarrow p-\frac{q}{5}+4r=\frac{32}{15}$$

By simplifying we get,

$$\frac{6p+3q-4r}{\cancel{12}\,3}=\frac{1}{\cancel{4}} \Longrightarrow \frac{6p+3q-4r}{3}=1 \Longrightarrow 6p+3q-4r=1 \ \dots(1)$$

$$3p=q \ \dots(2)$$

$$p-\frac{q}{5}+4r=\frac{32}{15} \Longrightarrow \frac{5p-q+20r}{\cancel{5}}=\frac{32}{\cancel{15}\,3}$$

$$5p-q+20r=\frac{32}{3} \Longrightarrow 3(5p-q+20r)=32$$

$$15p-3q+60r=32 \ \dots(3)$$

$Sub\ (2)\ in\ (1)\ and\ (3)\ 6p + 3(3p) - 4r = 3$

$6p + 9p - 4r = 3 \implies 15p - 4r = 3 \ldots (4)$

$15p - 3(3p) + 60r = 32 \implies 15p - 9p + 60r = 32$

$$\underset{\div 2}{6p + 60r = 32} \implies 3p + 30r = 16 \ldots (5)$$

$Solve\ (4)\ and\ (5)$

$$\begin{array}{lcl} (4) & \implies & 15p - 4r = 3 \\ (5) \times 5 & \implies & \underline{\overset{(-)}{15p} \overset{(-)}{+ 150r} \overset{(-)}{=} 80} \\ & & -154r = -77 \end{array}$$

$$r = \frac{-77}{-154} \implies r = \frac{1}{2}$$

Sub r $= \frac{1}{2}$ in (4) $15p - 4r = 3$

$$15p - \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right) = 3 \implies 15p - 2 = 3 \implies 15p = 3 + 2 \implies 15p = 5$$

$$p = \frac{\cancel{5}}{\cancel{15}_3} \implies p = \frac{1}{3}$$

Sub $p = \frac{1}{3}$ in (2) $3p = q$

$$\cancel{3} \times \frac{1}{\cancel{3}} = q \implies q = 1$$

$$\frac{1}{x} = p\ , \frac{1}{y} = q\ , \frac{1}{z} = r \implies \frac{1}{x} = \frac{1}{3}\ , \frac{1}{y} = 1\ , \frac{1}{z} = \frac{1}{2}$$

$\therefore x = 3, y = 1, z = 2$

Example 3.9 The sum of thrice the first number, second number and twice the third number is 5. If thrice the second number is subtracted from the sum of first number and thrice the third we get 2. If the third number is subtracted from the sum of twice the first, thrice the second, we get 1. Find the numbers.

$Let\ the\ three\ numbers\ be\ x, y, z$

From the given data we get the following equations,

$3x + y + 2z = 5 \ldots (1),\ x - 3y + 3z = 2 \ldots (2),\ 2x + 3y - z = 1 \ldots (3)$

Solve (1) and (2)

$$\begin{array}{lcl} (1) & \implies & 3x + y + 2z = 5 \\ (2) \times 3 & \implies & \underline{\overset{(-)}{3x} \overset{(+)}{- 9y} \overset{(-)}{+ 9z} \overset{(-)}{=} 6} \\ & & 10y - 7x = -1 \ldots (4) \end{array}$$

Solve (1) and (3),

$$\begin{array}{lll}(1)\times 2 & \implies & \cancel{6x}+2y+4z=10\\ (3)\times 3 & \implies & \overset{(-)}{\cancel{6x}}\overset{(-)}{+}9y\overset{(+)}{-}3z=\overset{(-)}{3}\\ \hline & & -7y+7z=7\ \ldots(5)\end{array}$$

Solve (4) and (5)

$$\begin{array}{lll}(4) & \implies & 10y-\cancel{7x}=-1\\ (5) & \implies & \overset{(+)}{-7y}\overset{(-)}{+}\cancel{7z}=\overset{(-)}{7}\\ \hline & & 3y=6\implies y=\dfrac{6}{3}\\ & & y=2\end{array}$$

Sub $y=2$ in (5) $-7(2)+7z=7$

$-14+7z=7\implies 7z=7+14\implies 7z=21$

$$z=\frac{21}{7}\implies z=3$$

Sub $y=2$ and $z=3$ in (1) $3x+2+2(3)=5$

$3x+2+6=5\implies 3x+8=5$

$3x=5-8$

$$3x=-3\implies x=\frac{-3}{3}\implies x=-1$$

$\therefore x=-1, y=2, z=3$

1. ***Solve the following system of linear equations in three variables***
(*i*) $x+y+z=5$; $2x-y+z=9$; $x-2y+3z=16$

Let $x+y+z=5\ \ldots(1)$, $2x-y+z=9\ \ldots(2)$, $x-2y+3z=16\ \ldots(3)$

Solve (1) and (2)

$$\begin{array}{l}x+\cancel{y}+z=5\\ 2x-\cancel{y}+z=9\\ \hline 3x+2z=14\ \ldots(4)\end{array}$$

Solve (2) and (3)

$$\begin{array}{lll}(2)\times 2 & \implies & 4x-\cancel{2y}+2z=18\\ (3) & \implies & \overset{(-)}{x}\overset{(+)}{-}\cancel{2y}\overset{(-)}{+}3z=\overset{(-)}{16}\\ \hline & & 3x-z=2\ \ldots(5)\end{array}$$

Solve (4) and (5)

$$\begin{array}{lll}(4) & \implies & \cancel{3x}+2z=14\\ (5) & \implies & \overset{(-)}{\cancel{3x}}\overset{(+)}{-}z=\overset{(-)}{2}\\ \hline & & 3z=12\implies z=\dfrac{12}{3}\implies z=4\end{array}$$

Sub $z=4$ in (4) $3x+2z=14$

$3x+2(4)=14\implies 3x+8=14\implies 3x=14-8\implies 3x=6$

$$x=\frac{6}{3}\implies x=2$$

Sub $x=2$ and $z=4$ in (1) $x+y+z=5$

$2+y+4=5$

$6+y=5\implies y=5-6\implies y=-1$

$\therefore x=2, y=-1, z=4$

$$\textbf{(ii)}\ \frac{1}{x}-\frac{2}{y}+4=0;\ \frac{1}{y}-\frac{1}{z}+1=0;\ \frac{2}{z}+\frac{3}{x}=14$$

$$\frac{1}{x}-\frac{2}{y}+4=0\ \ldots(1)\qquad \frac{1}{y}-\frac{1}{z}+1=0\ \ldots(2)\qquad \frac{2}{z}+\frac{3}{x}=14\ \ldots(3)$$

Let $\frac{1}{x}=a\,,\frac{1}{y}=b\,,\frac{1}{z}=c$

The given equations are written as

$$a-2b+4=0 \Longrightarrow a-2b=-4\ \ldots(1)$$

$$b-c+1=0 \Longrightarrow b-c=-1\ \ldots(2)$$

$$2c+3a=14 \Longrightarrow 3a+2c=14\ \ldots(3)$$

Solve (1) and (2)

$$(1) \Longrightarrow a-\cancel{2b}+0c=-4$$

$$(2)\times 2 \Longrightarrow 0a+\cancel{2b}-2c=-2$$

$$a-2c=-6\ \ldots(4)$$

Solve (3) and (4)

$$(3) \Longrightarrow 3a+\cancel{2c}=14$$

$$(4) \Longrightarrow a-\cancel{2c}=-6$$

$$4a=8 \Longrightarrow a=\frac{8}{4}$$

$$a=2$$

Sub $a=2$ in (1) $2-2b=-4$

$$-2b=-4-2 \Longrightarrow -2b=-6$$

$$b=\frac{-6}{-2} \Longrightarrow b=3$$

Sub $b=3$ in (2) $3-c=-1$

$$-c=-1-3 \Longrightarrow -c=-4 \Longrightarrow c=4$$

$$a=\frac{1}{x}=2 \Longrightarrow x=\frac{1}{2}$$

$$b=\frac{1}{y}=3 \Longrightarrow y=\frac{1}{3}$$

$$c=\frac{1}{z}=4 \Longrightarrow z=\frac{1}{4}$$

$$\therefore x=\frac{1}{2},y=\frac{1}{3},z=\frac{1}{4}$$

$$\textbf{(iii)}\ x+20=\frac{3y}{2}+10=2z+5=110-(y+z)$$

$$x+20=\frac{3y}{2}+10$$

$$x = \frac{3y}{2} + 10 - 20 \Longrightarrow \ x = \frac{3y}{2} - 10 \ \ldots (1)$$

$$2z + 5 = 110 - (y + z)$$

$$2z + 5 = 110 - y - z$$

$$y = 110 - z - 5 - 2z$$

$$y = 105 - 3z \ \ldots (2)$$

Sub (2) in (1)

$$x = \frac{3(105 - 3z\,)}{2} - 10 \ \Rightarrow x = \frac{315 - 9z}{2} - 10$$

$$x = \frac{315 - 9z - 20}{2} \ \ \Longrightarrow \ \ 2x = 315 - 9z - 20$$

$$\therefore 315 - 9z - 20 = 4z - 30 \ \Longrightarrow \ 315 - 20 + 30 = 4z + 9z$$

$$325 = 13z \ \Rightarrow z = \frac{325}{13} \ \Rightarrow z = 25$$

$$x + 20 = 2z + 5 \Rightarrow x + 20 = 2(25) + 5$$

$$x + 20 = 50 + 5 \Longrightarrow \ x = 55 - 20$$

$$x = 35$$

Sub $z = 25$ in (2) $y = 105 - 3(25)$

$$y = 105 - 75 \ \Longrightarrow \ y = 30$$

$$\therefore x = 35, y = 30, z = 25$$

The system has unique solutions.

2. *Discuss the nature of solutions of the following system of equations* (*i*) $\boldsymbol{x + 2y - z = 6; -3x - 2y + 5z = -12; x - 2z = 3}$

$$x + 2y - z = 6 \ldots (1)\,, -3x - 2y + 5z = -12 \ldots (2)\,, x - 2z = 3 \ldots (3)$$

Solve (1) and (2)

$$\begin{array}{lll} (1) & \Longrightarrow & x + \cancel{2y} - z = 6 \\ (2) & \Longrightarrow & -3x - \cancel{2y} + 5z = -12 \\ \hline & & -2x + 4z = -6 \\ & & \div 2 \\ & & -x + 2z = -3 \quad \ldots (4) \end{array}$$

Solve (3) and (4)

$$\begin{array}{lll} (3) & \Longrightarrow & \cancel{x} - \cancel{2z} = \cancel{3} \\ (4) & \Longrightarrow & \cancel{-x} + \cancel{2z} = \cancel{-3} \\ \hline & & 0 = 0 \end{array}$$

Here we arrive at an identity $0 = 0$

Hence the system has an infinite number of solutions.

$$\boldsymbol{(ii)\ 2y + z = 3(-x + 1);\ -x + 3y - z = -4;\ 3x + 2y + z = -\frac{1}{2}}$$

$$2y + z = 3(-x + 1) \Longrightarrow 2y + z = -3x + 3 \Longrightarrow 3x + 2y + z = 3 \ldots (1)$$

$$-x + 3y - z = -4 \ldots (2)$$

$$3x + 2y + z = -\frac{1}{2} \Longrightarrow 2 \times 3x + 2 \times 2y + 2 \times z = 2 \times -\frac{1}{2}$$

$$6x + 4y + 2z = -1 \ldots (3)$$

Solve (1) and (2)

$$\begin{array}{lll} (1) & \Longrightarrow & 3x + 2y + \cancel{z} = 3 \\ (2) & \Longrightarrow & \underline{-x + 3y - \cancel{z} = -4} \\ & & 2x + 5y = -1 \ldots (4) \end{array}$$

Solve (2) and (3)

$$\begin{array}{lll} (2) \times 2 & \Longrightarrow & -2x + 6y - \cancel{2z} = -8 \\ (3) & \Longrightarrow & \underline{6x + 4y + \cancel{2z} = -1} \\ & & 4x + 10y = -9 \ldots (5) \end{array}$$

Solve (4) and (5)

$$\begin{array}{lll} (4) \times 2 & \Longrightarrow & \cancel{4x} + \cancel{10y} = -2 \\ (5) & \Longrightarrow & \underline{\overset{(-)}{\cancel{4x}} + \overset{(-)}{\cancel{10y}} = \overset{(+)}{-9}} \\ & & 0 = 7 \end{array}$$

This is contradiction. The system of equation is inconsistent and has no solution.

$$\boldsymbol{(iii)\ \frac{y + z}{4} = \frac{z + x}{3} = \frac{x + y}{2};\ x + y + z = 27}$$

$$\frac{y + z}{4} = \frac{z + x}{3} \Longrightarrow 3(y + z) = 4(z + x) \Longrightarrow 3y + 3z = 4z + 4x$$

$$0 = 4z + 4x - 3y - 3z \Longrightarrow 4x - 3y + z = 0 \ldots (1)$$

$$\frac{z + x}{3} = \frac{x + y}{2} \Longrightarrow 2(z + x) = 3(x + y) \Longrightarrow 2z + 2x = 3x + 3y$$

$$0 = 3x + 3y - 2z - 2x \Longrightarrow x + 3y - 2z = 0 \ldots (2)$$

$$x + y + z = 27 \ldots (3)$$

Solve (1) and (2)

$$\begin{array}{lll} (1) & \Longrightarrow & 4x - \cancel{3y} + z = 0 \\ (2) & \Longrightarrow & \underline{x + \cancel{3y} - 2z = 0} \\ & & 5x - z = 0 \ \ldots (4) \end{array}$$

Solve (2) and (3)

$$\begin{array}{lll} (3) \times 3 & \Longrightarrow & 3x + \cancel{3y} + 3z = 81 \\ (2) & \Longrightarrow & \underline{\overset{(-)}{x} + \overset{(-)}{\cancel{3y}} \overset{(+)}{- 2z} = \overset{(-)}{0}} \\ & & 2x + 5z = 81 \ \ldots (5) \end{array}$$

Solve (4) and (5)

$$\begin{array}{lll} (5) & \Longrightarrow & 2x + \cancel{5z} = 81 \\ (4) \times 5 & \Longrightarrow & \underline{25x - \cancel{5z} = 0} \\ & & 27x = 81 \Longrightarrow x = \dfrac{81}{27} \Longrightarrow x = 3 \end{array}$$

Sub $x = 3$ in (4) $5(3) - z = 0$

$$15 - z = 0 \Longrightarrow -z = -15 \Longrightarrow z = 15$$

Sub $x = 3, z = 15$ in (3)

$$3 + y + 15 = 27$$

$$y + 18 = 27 \Longrightarrow y = 27 - 18 \Longrightarrow y = 9$$

$$\therefore x = 3, y = 9, z = 15$$

3. Vani, her father and her grand father have an average age of 53. One – half of her grand father's age plus one – third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now ?

Let Vani's age be x

Let Vani's father's age be y

Let Vani's grand father's age be z.

$$\frac{x+y+z}{3} = 53 \Longrightarrow x + y + z = 159 \quad \dots(1)$$

$$\frac{x}{4} + \frac{y}{3} + \frac{z}{2} = 65 \Longrightarrow \frac{3x + 4y + 6z}{4} = 65 \Longrightarrow 3x + 4y + 6z = 780 \dots(2)$$

$$(x-4)4 = z - 4 \Longrightarrow 4x - 16 = z - 4$$

$$4x - z = -4 + 16 \Longrightarrow 4x - z = 12 \dots(3)$$

Solve (1) *and* (3)

$$(1) \times 4 \Longrightarrow \cancel{4x} + 4y + 4z = 636$$

$$(3) \Longrightarrow \overset{(-)}{\cancel{4x}} \overset{(-)}{+} 0y \overset{(+)}{-} z \overset{(-)}{=} 12$$

$$4y + 5z = 624 \quad \dots(4)$$

***Solve* (2) *and* (3)**

$$(2) \times 4 \Longrightarrow \cancel{12x} + 16y + 24z = 3120$$

$$(3) \times 3 \Longrightarrow \overset{(-)}{\cancel{12x}} \overset{(-)}{+} 0y \overset{(+)}{-} 3z \overset{(-)}{=} 36$$

$$16y + 27z = 3084 \quad \dots(5)$$

***Solve* (4) *and* (5)**

$$(5) \Longrightarrow \cancel{16y} + 27z = 3084$$

$$(4) \times 4 \Longrightarrow \overset{(-)}{\cancel{16y}} \overset{(-)}{+} 20z \overset{(-)}{=} 2496$$

$$7z = 588$$

$$z = \frac{588}{7} \Longrightarrow z = 84$$

Sub $z = 84$ in (3) $4x - 84 = 12$

$$4x = 12 + 84 \Longrightarrow 4x = 96 \Longrightarrow x = \frac{96}{4} \Longrightarrow x = 24$$

Sub $x = 24, z = 84$ in (1) $x + y + z = 159$

$$24 + y + 84 = 159$$

$$y + 108 = 159 \Longrightarrow y = 159 - 108 \Longrightarrow y = 51$$

$\therefore$ *Vani's age* $= 24$ *years, Vani's father's age* $= 51$ *years*

Vani's grand father's age $= 84$ *years*

4. The sum of the digits of a three − digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number ?

Let the number be $100x + 10y + z$

Reversed number be $100z + 10y + x$

$$x + y + z = 11 \ \ldots(1)$$

$$100z + 10y + x = 5(100x + 10y + z) + 46$$

$$-46 = 500x + 50y + 5z - 100z - 10y - x$$

$$-46 = 499x + 40y - 95z \Longrightarrow 499x + 40y - 95z = -46 \ \ldots(2)$$

$$x + 2y = z$$

$$x + 2y - z = 0 \ \ldots(3)$$

Solve (1) and (3)

$$(1) \quad \Longrightarrow \quad x + y + \cancel{z} = 11$$

$$(3) \quad \Longrightarrow \quad x + 2y \cancel{- z} = 0$$

$$2x + 3y = 11 \quad \ldots(4)$$

Solve (2) and (3)

$$(2) \quad \Longrightarrow \quad 499x + 40y - \cancel{95z} = -46$$

$$(3) \times 95 \Longrightarrow \overset{(-)}{95x} \overset{(-)}{+ 190y} \overset{(+)}{\cancel{- 95z}} \overset{(-)}{=} 0$$

$$404x - 150y = -46 \quad \ldots(5)$$

Solve (4) and (5)

$$(4) \times 50 \Longrightarrow 100x + \cancel{150y} = 550$$

$$(5) \quad \Longrightarrow 404x - \cancel{150y} = -46$$

$$\cancel{504}x = \cancel{504} \Longrightarrow x = 1$$

Sub $x = 1$ in (4)

$$2(1) + 3y = 11 \Longrightarrow 2 + 3y = 11$$

$$3y = 11 - 2 \Longrightarrow 3y = 9 \Longrightarrow y = \frac{9}{3} \Longrightarrow y = 3$$

Sub $x = 1, y = 3$ in (1)

$$1 + 3 + z = 11$$

$$4 + z = 11 \Longrightarrow z = 11 - 4 \Longrightarrow z = 7$$

$\therefore$ The number is $x = 1, y = 3, z = 7$

5. *There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of currencies in each sort.*

$$x + y + z = 12 \ \ldots(1) \qquad 5x + 10y + 20z = 105 \ \ldots(2)$$

$$10x + 5y + 20z = 125 \ \ldots(3)$$

Solve (1) and (3)

$$\begin{array}{lll} (1)\times 5 & \Rightarrow & 5x + \cancel{5y} + 5z = 60 \\ & & (-) \quad (-) \quad (-) \quad (-) \\ (3) & \Rightarrow & 10x + \cancel{5y} + 20z = 125 \\ \hline & & -5x - 15z = -45 \ \ldots(4) \end{array}$$

Solve (2) and (3)

$$\begin{array}{lll} (2) & \Rightarrow & \cancel{5x} + 10y + 20z = 105 \\ & & (-) \quad (-) \quad (-) \quad (-) \\ (3)\times 2 & \Rightarrow & \cancel{10x} + 10y + 40z = 250 \\ \hline & & 15y + 20z = 85 \ \ldots(5) \end{array}$$

Solve (4) and (5)

$$\begin{array}{lll} (4)\times 3 & \Rightarrow & -\cancel{15y} - 45z = -135 \\ (5) & \Rightarrow & \cancel{15y} + 20z = 85 \\ \hline & & -\cancel{25}z = \cancel{50} \Rightarrow z = 2 \end{array}$$

Sub $z = 2$ in (5)

$$15y + 20(2) = 85 \Rightarrow 15y + 40 = 85$$

$$15y = 85 - 40 \Rightarrow 15y = 45 \Rightarrow y = \frac{45}{15} \Rightarrow y = 3$$

Sub $y = 3, z = 2$ in (1)

$$x + 3 + 2 = 12$$

$$x + 5 = 12 \Rightarrow x = 12 - 5 \Rightarrow x = 7$$

∴ The solutions are the number of ₹5 are 7
the number of ₹10 are 3
the number of ₹20 are 2

Exercise 3.2

Example 3.10 Find the GCD of the polynomials x^3+x^2-x+2 and $2x^3-5x^2+5x-3$

Let $f(x) = x^3+x^2-x+2$ *and* $g(x) = 2x^3-5x^2+5x-3$

$$\begin{array}{r|l} & 2 \\ \cline{2-2} x^3+x^2-x+2 & \cancel{2x^3}-5x^2+5x-3 \\ & (\cancel{-})\ (-)\quad (+)\quad (-) \\ & \cancel{2x^3}+2x^2-2x+4 \\ \cline{2-2} & \quad -7x^2+7x-7 \end{array}$$

$-7(x^2-x+1) \longrightarrow$ *Remainder*

$$\begin{array}{r|l} & x+2 \\ \cline{2-2} x^2-x+1 & \cancel{x^3}+x^2-x+2 \\ & (\cancel{-})\ (+)\quad (-) \\ & \cancel{x^3}-x^2+x \\ \cline{2-2} & \quad \cancel{2x^2-2x+2} \\ & \quad (-)\quad (+)\quad (-) \\ & \quad \cancel{2x^2-2x+2} \\ \cline{2-2} & \qquad 0 \end{array}$$

$0 \longrightarrow$ *Remainder*

$G.C.D$ *of* $f(x)$ *and* $g(x) = x^2-x+1$

Example 3.11 Find the GCD of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$

Let $f(x) = 6x^3-30x^2+60x-48$

$= 6(x^3-5x^2+10x-8) = 2\times 3(x^3-5x^2+10x-8)$

$g(x) = 3x^3-12x^2+21x-18$

$g(x) = 3(x^3-4x^2+7x-6)$

$G.C.D = 3$

$$\begin{array}{r|l} & 1 \\ \cline{2-2} x^3-5x^2+10x-8 & \cancel{x^3}-4x^2+7x-6 \\ & (\cancel{-})\ (+)\quad (-)\quad (+) \\ & \cancel{x^3}-5x^2+10x-8 \\ \cline{2-2} & \end{array}$$

$x^2-3x+2 \longrightarrow$ *Remainder*

$$\begin{array}{r|l} & x-2 \\ \cline{2-2} x^2-3x+2 & \cancel{x^3}-5x^2+10x-8 \\ & (\cancel{-})\ (+)\quad (-) \\ & \cancel{x^3}-3x^2+2x \\ \cline{2-2} & \quad \cancel{-2x^2}+8x-8 \\ & \quad (\cancel{+})\quad (-)\quad (+) \\ & \quad \cancel{-2x^2}+6x-4 \\ \cline{2-2} & \end{array}$$

$2x-4 = 2(x-2) \longrightarrow$ *Remainder*

$$\begin{array}{r|l} & x-1 \\ \hline x-2 & x^2-3x+2 \\ & (-)\ (+) \\ & \cancel{x^2}-2x \\ \hline & \quad -x+2 \\ & \quad (+)\ (-) \\ & \quad \cancel{-x}+2 \\ \hline & \qquad 0 \longrightarrow Remainder \end{array}$$

$G.C.D\ of\ f(x), g(x) = 3\ (x-2)$

Example 3.12 Find the LCM of the following

(i) $\mathbf{8x^4y^2, 48x^2y^4}$

$8x^4y^2 = 2\times2\times2\times x^4\times y^2$

$48x^2y^4 = 2\times2\times2\times6\times x^2\times y^4$

$L.C.M = 2\times2\times\ 2\times x^4\times y^4\times6$

$L.C.M = 48\times x^4\times y^4$

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline & 6 \end{array}$$

(ii) $\mathbf{(5x-10), (5x^2-20)}$

$5x-10 = 5(x-2) = 5\ \times(x-2)$

$5x^2-20 = 5(x^2-4)= 5(x^2-2^2) = 5\times(x-2)\times(x+2)$

$L.C.M = 5\times(x-2)\times(x+2)$

$L.C.M = 5(x-2)(x+2)$

(iii) $\mathbf{(x^4-1), (x^2-2x+1)}$

$x^4-1 = (x^2)^2-1^2 = (x^2+1)(x^2-1)= (x^2+1)(x+1)(x-1)$

$x^2-2x+1\ = (x-1)(x-1)= (x-1)^2$

$L.C.M = (x-1)^2\times(x^2+1)\times(x+1)$

$L.C.M = (x^2+1)(x+1)(x-1)^2$

$$\begin{array}{cc} + & \times \\ -2 & 1 \\ -1 & -1 \end{array}$$

(iv) $\mathbf{x^3-27, (x-3)^2, x^2-9}$

$x^3-27 = x^3-3^3 =(x-3)(x^2+3x+9)$

$x^2-9 = x^2-3^2 = (x+3)(x-3)$

$\boxed{a^3-b^3 = (a-b)(a^2+ab+b^2)}$

$(x-3)^2 = (x-3)^2$

$L.C.M = (x-3)^2\times(x^2+3x+9)\times(x+3)$

$L.C.M = (x-3)^2(x+3)(x^2+3x+9)$

1. *Find the GCD of the given polynomials*

(i) $\mathbf{x^4+3x^3-x-3, x^3+x^2-5x+3}$

Let $f(x) =x^4+3x^3-x-3$ *and* $g(x) =x^3+x^2-5x+3$

$$\begin{array}{r|l} & x+2 \\ \cline{2-2} x^3+x^2-5x+3 & \cancel{x^4}+3x^3+0x^2-x-3 \\ & (-)\ (-)\ \ (+)\ \ \ (-) \\ & \cancel{x^4}+x^3-5x^2+3x \\ \cline{2-2} & \quad \cancel{2x^3}+5x^2-4x-3 \\ & \quad (-)\ (-)\ \ (+)\ \ (-) \\ & \quad \cancel{2x^3}+2x^2-10x+6 \\ \cline{2-2} & \qquad 3x^2+6x-9 \longrightarrow \mathit{Remainder} \end{array}$$

$$\begin{array}{r|l} & x-1 \\ \cline{2-2} x^2+2x-3 & \cancel{x^3}+x^2-5x+3 \\ & (-)\ (-)\ \ (+) \\ & \cancel{x^3}+2x^2-3x \\ \cline{2-2} & \quad \cancel{-x^2-2x+3} \\ & \quad (+)\ (+)\ \ (-) \\ & \quad \cancel{-x^2-2x+3} \\ \cline{2-2} & \qquad 0 \longrightarrow \mathit{Remainder} \end{array}$$

$G.C.D\ of\ f(x)\ and\ g(x) = x^2+2x-3$

$(ii)\ \boldsymbol{x^4-1, x^3-11x^2+x-11}$

$Let\ f(x) = x^4-1\ and\ g(x) = x^3-11x^2+x-11$

$$\begin{array}{r|l} & x+11 \\ \cline{2-2} x^3-11x^2+x-11 & \cancel{x^4}+0x^3+0x^2+0x-1 \\ & (-)\ \ (+)\ \ \ (-)\ \ (+) \\ & \cancel{x^4}-11x^3+x^2-11x \\ \cline{2-2} & \quad \cancel{11x^3}-x^2+\cancel{11x}-1 \\ & \quad (-)\ \ (+)\ \ \ (-)\ \ (+) \\ & \quad \cancel{11x^3}-121x^2+\cancel{11x}-121 \\ \cline{2-2} & \quad 120x^2+120 = 120(x^2+1) \neq 0 \longrightarrow \mathit{Remainder} \end{array}$$

$$\begin{array}{r|l} & x-11 \\ \cline{2-2} x^2+1 & \cancel{x^3}-11x^2\cancel{+x}-11 \\ & (-)\ (-) \\ & \cancel{x^3}+\cancel{x} \\ \cline{2-2} & \quad -11x^2-11 \\ & \quad (+)\quad\ (+) \\ & \quad -11x^2-11 \\ \cline{2-2} & \qquad 0 \longrightarrow \mathit{Remainder} \end{array}$$

$G.C.D = x^2+1$

$(iii)\ \boldsymbol{3x^4+6x^3-12x^2-24x,\ \ 4x^4+14x^3+8x^2-8x}$

Let $f(x) = 3x^4+6x^3-12x^2-24x = 3(x^4+2x^3-4x^2-8x)$

$g(x) = 4x^4+14x^3+8x^2-8x = 2(2x^4+7x^3+4x^2-4x)$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^4+2x^3-4x^2-8x & \cancel{2x^4}+7x^3+4x^2-4x \\ & (-)\ (-)\ \ (+)\ \ (+) \\ & \cancel{2x^4}+4x^3-8x^2-16x \\ \hline & \quad 3x^3+12x^2+12x = 3(x^3+4x^2+4x) \end{array}$$

$$\downarrow \ \textit{Remainder}$$

$$\begin{array}{r|l} & \quad x-2 \\ \hline x^3+4x^2+4x & \cancel{x^4}+2x^3-4x^2-8x \\ & (-)\ (-)\ \ (-) \\ & \cancel{x^4}+4x^3+4x^2 \\ \hline & \quad -\cancel{2x^3}-\cancel{8x^2}-\cancel{8x} \\ & \quad (+)\ \ (+)\ \ (+) \\ & \quad -\cancel{2x^3}-\cancel{8x^2}-\cancel{8x} \\ \hline & \qquad 0 \longrightarrow \textit{Remainder} \end{array}$$

$$G.C.D = x(x^3+4x^2+4x)$$

(iv) $3x^3+3x^2+3x+3,\ 6x^3+12x^2+6x+12$

Let $f(x) = 3x^3+3x^2+3x+3 = \enclose{circle}{3}(x^3+x^2+x+1)$

$g(x) = 6x^3+12x^2+6x+12 = 6(x^3+2x^2+x+2)$

$= 2 \times \enclose{circle}{3}(x^3+2x^2+x+2)$

$G.C.D = 3$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^3+x^2+x+1 & \cancel{x^3}+2x^2+x+2 \\ & (-)\ (-)\ \ (-)\ (-) \\ & \cancel{x^3}+\ x^2+x+1 \\ \hline & \qquad x^2+1 \neq 0 \end{array}$$

$$\begin{array}{r|l} & \quad x+1 \\ \hline x^2+1 & \cancel{x^3}+x^2+\cancel{x}+1 \\ & (-)\ (-) \\ & \cancel{x^3}+\cancel{x} \\ \hline & \quad x^2+1 \\ & (-)\ \ (-) \\ & \quad x^2+1 \\ \hline & \qquad 0 \longrightarrow \textit{Remainder} \end{array}$$

$$G.C.D = 3(x^2+1)$$

2. *Find the LCM of the given expressions.*

(i) $4x^2y, 8x^3y^2$

$4x^2y = \boxed{2} \times \boxed{2} \times \enclose{circle}{x^2} \times \enclose{circle}{y}$

$8x^3y^4 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \enclose{circle}{x^3} \times \enclose{circle}{y^2}$

$L.C.M = 2 \times 2 \times x^3 \times y^2 \times 2$

$L.C.M = 8 \times x^3 \times y^2$

(ii) $-9a^3b^2, 12a^2b^2c$

$-9a^3b^2 = -3 \times 3 \times a^3 \times b^2$

$12a^2b^2c = 2 \times 3 \times 2 \times a^2 \times b^2 \times c$

$L.C.M = 3 \times a^3 \times b^2 \times -3 \times 2 \times 2 \times c$

$L.C.M = -36 \times a^3 \times b^2 \times c$

(iii) $16m, -12m^2n^2, 8n^2$

$16m = 2 \times 2 \times 2 \times 2 \times m$

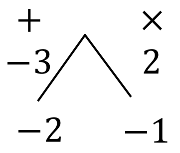
$-12m^2n^2 = -2 \times 2 \times 3 \times m^2 \times n^2$

$8n^2 = 2 \times 2 \times 2 \times n^2$

$L.C.M = 2 \times 2 \times 2 \times m^2 \times n^2 \times -2 \times 3$

$L.C.M = -48 \times m^2 \times n^2$

(iv) $p^2 - 3p + 2, p^2 - 4$

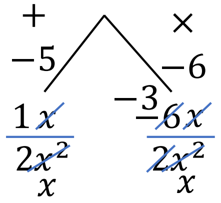


$p^2 - 3p + 2 = (p-2)(p-1)$

$p^2 - 4 = p^2 - 2^2 = (p-2)(p+2)$

$L.C.M = (p-2)(p-1)(p+2)$

(v) $2x^2 - 5x - 3, 4x^2 - 36$



$2x^2 - 5x - 3 = (x-3)(2x+1)$

$4x^2 - 36 = 4(x^2 - 9) = 4(x^2 - 3^2) = 4(x-3)(x+3)$

$L.C.M = (x-3) \times (2x+1) \times (x+3) \times 4$

$L.C.M = 4(x+3)(x-3)(2x+1)$

(vi) $\left(2x^2 - 3xy\right)^2, (4x - 6y)^3, 8x^3 - 27y^3$

$(2x^2 - 3xy)^2 = x^2\,(2x-3y)^2$

$\boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$

$(4x - 6y)^3 = 2^3(2x-3y)^3$

$8x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x-3y)[(2x)^2 + (2x)(3y) + (3y)^2]$

$= (2x-3y)(4x^2 + 6xy + 9y^2)$

$L.C.M = (2x-3y)^3 \times x^2 \times 2^3 \times (4x^2 + 6xy + 9y^2)$

$L.C.M = 8x^2(2x-3y)^3(4x^2 + 6xy + 9y^2)$

$Exercise\ 3.3$

1. $\boldsymbol{Find\ the\ L.C.M\ and\ G.C.D\ for\ the\ following\ and\ verify\ that}$
$\boldsymbol{f(x) \times g(x) = L.C.M \times G.C.D.}$

$\boldsymbol{(i)\ 21x^2y, 35xy^2}$

$$Let\ f(x) = 21x^2y = 3 \times 7 \times x^2 \times y$$

$$g(x) = 35xy^2 = 7 \times 5 \times x \times y^2$$

$$G.C.D = 7xy$$

$$\text{L.C.M} = 7 \times x^2 \times y^2 \times 3 \times 5 = 105x^2y^2$$

$$L.C.M \times G.C.D = f(x) \times g(x)$$

$$105x^2y^2 \times 7xy = 21x^2y \times 35xy^2$$

$$735x^3y^3 = 735x^3y^3$$

$Hence\ verified$

$\boldsymbol{(ii)\ (x^3 - 1)(x + 1), (x^3 + 1)}$

$$\boxed{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

$$Let\ f(x) = (x^3 - \overset{3}{1})(x + 1) = (x - 1)(x^2 + x + 1)(x + 1)$$

$$g(x) = (x^3 + \overset{3}{1}) = (x + 1)(x^2 - x + 1)$$

$$\boxed{G.C.D = x + 1}$$

$$\boxed{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$

$$L.C.M = (x + 1)(x - 1)(x^2 + x + 1)(x^2 - x + 1)$$

$$L.C.M \times G.C.D = f(x) \times g(x)$$

$$(x + 1)(x + 1)(x - 1)(x^2 + x + 1)(x^2 - x + 1) = (x^3 - 1)(x + 1) \times (x^3 + 1)$$

$$(x + 1)(x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1) = (x^3 - 1)(x + 1) \times (x^3 + 1)$$

$$(x + 1)(x^3 + 1)(x^3 - 1) = (x + 1)(x^3 - 1)(x^3 + 1)$$

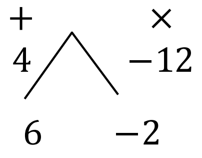
$Hence\ verified$

2. $\boldsymbol{Find\ the\ LCM\ of\ each\ pair\ of\ the\ following\ polynomials}$

$\boldsymbol{(i)\ a^2 + 4a - 12, a^2 - 5a + 6\ whose\ GCD\ is\ a - 2}$

$$Let\ f(x) = a^2 + 4a - 12 = (a + 6)(a - 2)$$

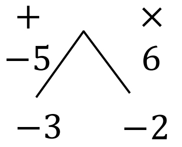
$$g(x) = a^2 - 5a + 6 = (a - 3)(a - 2)$$



$$G.C.D = a - 2$$

$$L.C.M \times G.C.D = f(x) \times g(x)$$

$$L.C.M = \frac{f(x) \times g(x)}{G.C.D} = \frac{(a + 6)(a - 2) \times (a - 3)\cancel{(a - 2)}}{\cancel{(a - 2)}}$$



$$= (a + 6)(a - 2)(a - 3)$$

$$L.C.M = (a - 2)(a - 3)(a + 6)$$

$\boldsymbol{(ii)\ x^4 - 27a^3x, (x-3a)^2\ whose\ GCD\ is\ (x-3a)}$

$$Let\ f(x) = x^4 - 27a^3x = x(x^3 - 27a^3) = x[x^3 - (3a)^3]$$

$$= x(x-3a)[x^2 + (x)(3a) + (3a)^2]$$

$$= x(x-3a)(x^2 + 3ax + 9a^2)$$

$$g(x) = (x-3a)^2 \qquad \boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

$$G.C.D = (x-3a)$$

$$L.C.M \times G.C.D = f(x) \times g(x) \implies L.C.M = \frac{f(x) \times g(x)}{G.C.D}$$

$$L.C.M = \frac{x\cancel{(x-3a)}(x^2 + 3ax + 9a^2) \times (x-3a)^2}{\cancel{(x-3a)}}$$

$$= x(x-3a)^2(x^2 + 3ax + 9a^2)$$

$\boldsymbol{3. Find\ the\ GCD\ of\ each\ pair\ of\ the\ following\ polynomials}$

$\boldsymbol{(i)\ 12(x^4 - x^3), 8(x^4 - 3x^3 + 2x^2)\ whose\ LCM\ is\ 24x^3(x-1)(x-2)}$

$$Let\ f(x) = 12(x^4 - x^3)$$

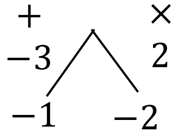
$$g(x) = 8(x^4 - 3x^3 + 2x^2)$$

$$\text{L. C.M} = 24x^3(x-1)(x-2)$$

$$G.C.D = \frac{f(x) \times g(x)}{L.C.M} = \frac{\cancel{12}(x^4 - x^3) \times \overset{4}{\cancel{8}}(x^4 - 3x^3 + 2x^2)}{{}_{\cancel{2}}\cancel{24}x^3(x-1)(x-2)}$$

$$= \frac{4 \times \cancel{x^3}\cancel{(x-1)} \times x^2(x^2 - 3x + 2)}{\cancel{x^3}\cancel{(x-1)}(x-2)}$$

$$= \frac{4 \times x^2\cancel{(x-2)}(x-1)}{\cancel{(x-2)}} = 4x^2(x-1)$$



$\boldsymbol{(ii)\ (x^3 + y^3), (x^4 + x^2y^2 + y^4)\ whose\ LCM\ is\ (x^3 + y^3)(x^2 + xy + y^2)}$

$$Let\ f(x) = (x^3 + y^3)\ and\ g(x) = (x^4 + x^2y^2 + y^4)$$

$$L.C.M = (x^3 + y^3)(x^2 + xy + y^2)$$

$$G.C.D = \frac{f(x) \times g(x)}{L.C.M}$$

$$= \frac{\cancel{(x^3 + y^3)} \times (x^4 + x^2y^2 + y^4)}{\cancel{(x^3 + y^3)}(x^2 + xy + y^2)} = \frac{(x^2 - xy + y^2)\cancel{(x^2 + xy + y^2)}}{\cancel{(x^2 + xy + y^2)}}$$

$$G.C.D = x^2 - xy + y^2$$

4. *Given the LCM and GCD of the two polynomials $p(x)$ and $q(x)$ find the unknown polynomial in the following table*

$S.No.$	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3-10a^2+11a+70$	$a-7$	$a^2-12a+35$	
(ii)	(x^2+y^2) $(x^4+x^2y^2+y^4)$	(x^2-y^2)		(x^4-y^4) (x^2+y^2-xy)

(i) $L.C.M = a^3-10a^2+11a+70$

$G.C.D = a-7$

$p(x) = a^2-12a+35$

$$p(x)\times q(x) = L.C.M\times G.C.D \Longrightarrow q(x) = \frac{L.C.M\times G.C.D}{p(x)}$$

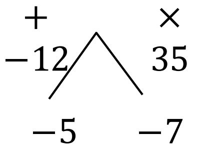
$$= \frac{(a^3-10a^2+11a+70)\times(a-7)}{a^2-12a+35}$$

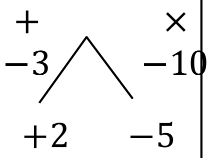
$$\begin{array}{c|cccc} 7 & 1 & -10 & 11 & 70 \\ & 0 & 7 & -21 & -70 \\ \hline & 1 & -3 & -10 & \boxed{0} \end{array}$$

$$= \frac{(a-7)(a^2-3a-10)\times(a-7)}{a^2-12a+35}$$

$$= \frac{(a-7)(a^2-3a-10)\times(a-7)}{a^2-12a+35} = \frac{(a^2-3a-10)(a-7)\cancel{(a-7)}}{(a-5)\cancel{(a-7)}}$$

$$= \frac{(a+2)\cancel{(a-5)}(a-7)}{\cancel{(a-5)}}$$





$\therefore q(x) = (a+2)(a-7)$

(ii)

$L.C.M = (x^2+y^2)(x^4+x^2y^2+y^4)$ and $G.C.D = (x^2-y^2)$

$q(x) = (x^4-y^4)(x^2+y^2-xy)$

$$p(x)\times q(x) = L.C.M\times G.C.D \Longrightarrow p(x) = \frac{L.C.M\times G.C.D}{q(x)}$$

$$p(x) = \frac{(x^2+y^2)(x^4+x^2y^2+y^4)\times(x^2-y^2)}{(x^4-y^4)(x^2+y^2-xy)}$$

$$= \frac{\cancel{(x^2+y^2)}\cancel{(x^2-xy+y^2)}(x^2+xy+y^2)\cancel{(x^2-y^2)}}{\cancel{(x^2+y^2)}\cancel{(x^2-y^2)}\cancel{(x^2+y^2-xy)}} = (x^2+xy+y^2)$$

Exercise 3.4

Example 3.13 Reduce the rational expressions to its lowest form

$(i)\ \dfrac{x-3}{x^2-9}\quad (ii)\ \dfrac{x^2-16}{x^2+8x+16}$

$$\boxed{\begin{aligned} a^2-b^2 &= (a+b)(a-b)\\ x^2-9 &= x^2-3^2\\ &= (x+3)(x-3)\end{aligned}}$$

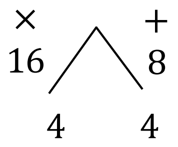
$(i)\ \dfrac{x-3}{x^2-9}$

$$\frac{x-3}{x^2-9} = \frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}} = \frac{1}{x+3}$$

$(ii)\ \dfrac{x^2-16}{x^2+8x+16}$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$x^2-16 = x^2-4^2$$
$$= (x+4)(x-4)$$
$$x^2+8x+16 = (x+4)(x+4)$$



$$\frac{x^2-16}{x^2+8x+16} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+4)}$$
$$= \frac{x-4}{x+4}$$

Example 3.14 Find the excluded values of the following expressions (if any). $(i)\ \dfrac{x+10}{8x}\quad (ii)\ \dfrac{7p+2}{8p^2+13p+5}\quad (iii)\ \dfrac{x}{x^2+1}$

$(i)\ \dfrac{x+10}{8x}$

The expresion $\dfrac{x+10}{8x}$ *is undefined when* $8x=0$ *or* $x=0.$

Hence the excluded value is 0.

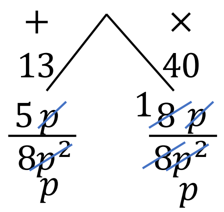
$(ii)\ \dfrac{7p+2}{8p^2+13p+5}$

The expresion $\dfrac{7p+2}{8p^2+13p+5}$ *is undefined when* $8p^2+13p+5=0$

i.e $(8p+5)(p+1)=0 \Longrightarrow 8p+5=0,\ p+1=0$

$8p=-5,\ p=-1$

$p=\dfrac{-5}{8}$ *. Hence the excluded value is* $\dfrac{-5}{8}$ *and* $-1.$



$(iii)\ \dfrac{x}{x^2+1}$

Here $x^2 \geq 0$ *for all* x*. Hence,* $x^2+1 \neq 0$ *for any* x*.*

Therefore, there can be no real excluded values for the given rational expression $\dfrac{x}{x^2+1}$

1. *Reduce each of the following rational expressions to its lowest form.*

$$(i)\ \frac{x^2-1}{x^2+x}$$

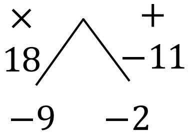
$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$x^2-1 = x^2-1^2 = (x+1)(x-1)$$

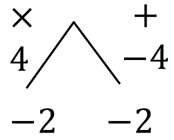
$$\frac{x^2-1}{x^2+x} = \frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}} = \frac{x-1}{x}$$

$$(ii)\ \frac{x^2-11x+18}{x^2-4x+4}$$

$$\boxed{x^2-11x+18=(x-9)(x-2)}$$



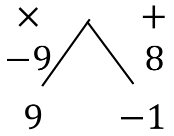
$$\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}$$



$$= \frac{x-9}{x-2}$$

$$\boxed{x^2-4x+4=(x-2)(x-2)}$$

$$(iii)\ \frac{9x^2+81x}{x^3+8x^2-9x}$$



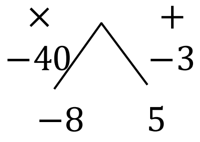
$$9x^2+81x = 9x(x+9)$$

$$x^3+8x^2-9x = x(x^2+8x-9)$$

$$= x(x+9)(x-1)$$

$$\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9\cancel{x}\cancel{(x+9)}}{\cancel{x}\cancel{(x+9)}(x-1)} = \frac{9}{x-1}$$

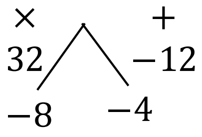
$$(iv)\ \frac{p^2-3p-40}{2p^3-24p^2+64p}$$



$$p^2-3p-40 = (p-8)(p+5)$$

$$2p^3-24p^2+64p = 2p(p^2-12p+32)$$

$$= 2p\,(p-8)(p-4)$$



$$\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{\cancel{(p-8)}(p+5)}{2p\cancel{(p-8)}(p+4)}$$

$$= \frac{p+5}{2p(p-4)}$$

2. *Find the excluded values, if any of the following expressions.*

$$(i)\ \frac{y}{y^2-25}$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

The expression is undefined when $y^2-25=0$

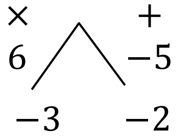
$$y^2-25=0 \Rightarrow\ y^2-5^2=0 \Rightarrow (y+5)(y-5)=0$$

$$y+5=0,\ y-5=0$$

$$y=-5,\ \ y=5$$

$\therefore$ *The excluded values are* $5,-5$

$(\boldsymbol{ii})\ \dfrac{\boldsymbol{t}}{\boldsymbol{t^2-5t+6}}$



The expression is undefined when $t^2-5t+6=0$

$$(t-3)(t-2)=0 \Longrightarrow\ t-3=0,\ t-2=0$$

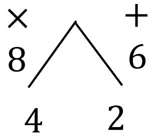
The expression is undefined if $t=3, t=2$

$\therefore$ *The excluded values are* $3,2$

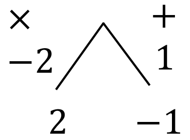
$(\boldsymbol{iii})\ \dfrac{\boldsymbol{x^2+6x+8}}{\boldsymbol{x^2+x-2}}$

$$x^2+6x+8\ =(x+4)(x+2)$$

$$x^2+x-2\ =(x+2)(x-1)$$



$$\frac{x^2+6x+8}{x^2+x-2}=\frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} \Longrightarrow \frac{x^2+6x+8}{x^2+x-2}=\frac{x+4}{x-1}$$



The expression is not defined when $x-1=0$

$$x-1=0 \Longrightarrow x=1$$

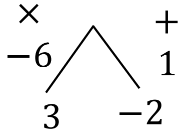
$\therefore$ *The excluded values is* 1

$(\boldsymbol{iv})\ \dfrac{\boldsymbol{x^3-27}}{\boldsymbol{x^3+x^2-6x}}$

$$\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

$$x^3-27=\underset{a^3-b^3}{x^3-3^3}$$

$$=\underset{(a-b)}{(x-3)}\underset{(a^2+ab+b^2)}{(x^2+3x+9)}$$



$$x^3+x^2-6x\ =x(x^2+x-6)$$

$$=x(x+3)(x-2)$$

$$\frac{x^3-27}{x^3+x^2-6x}=\frac{(x-3)(x^2+3x+9)}{x(x+3)(x-2)}$$

This expresion is not defined when $x(x+3)(x-2)=0$

$$x=0, x+3=0, x-2=0$$

$$x=0, x=-3, x=2$$

$\therefore$ *The excluded values are* $0,-3,2$

$Exercise\ 3.5$

$Example\ 3.15$: (i) $\boldsymbol{Multiply: \frac{x^3}{9y^2}\ by\ \frac{27y}{x^5}}$ (ii) $\boldsymbol{Multiply: \frac{x^4b^2}{x-1}\ by\ \frac{x^2-1}{a^4b^3}}$

(i) $\boldsymbol{Multiply: \frac{x^3}{9y^2}\ by\ \frac{27y}{x^5}}$

$$\frac{\cancel{x^3}}{\underset{y}{\cancel{9}\cancel{y^2}}}\times\frac{\overset{3}{\cancel{27}}\cancel{y}}{\underset{x^2}{\cancel{x^5}}}=\frac{3}{x^2y}$$

(ii) $\boldsymbol{Multiply: \frac{x^4b^2}{x-1}\ by\ \frac{x^2-1}{a^4b^3}}$

$$\frac{x^4b^2}{x-1}\times\frac{x^2-1}{a^4b^3}=\frac{x^4\times\cancel{b^2}}{\cancel{x-1}}\times\frac{(x+1)\cancel{(x-1)}}{a^4\times\underset{b}{\cancel{b^3}}}=\frac{x^4(x+1)}{a^4b}$$

$Example\ 3.16\ Find$

(i) $\boldsymbol{\frac{14x^4}{y}\div\frac{7x}{3y^4}}$ (ii) $\boldsymbol{\frac{x^2-16}{x+4}\div\frac{x-4}{x+4}}$ (iii) $\boldsymbol{\frac{16x^2-2x-3}{3x^2-2x-1}\div\frac{8x^2+11x+3}{3x^2-11x-4}}$

(i) $\boldsymbol{\frac{14x^4}{y}\div\frac{7x}{3y^4}}$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

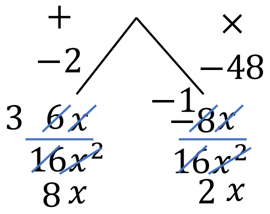
$$\frac{\overset{2}{\cancel{14}}\overset{x^3}{\cancel{x^4}}}{\cancel{y}}\times\frac{3\overset{y^3}{\cancel{y^4}}}{\cancel{7}\cancel{x}}=6x^3y^3$$

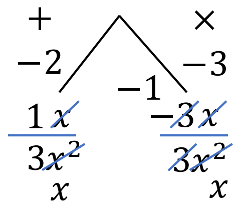
(ii) $\boldsymbol{\frac{x^2-16}{x+4}\div\frac{x-4}{x+4}}$

$$x^2-16=x^2-4^2=(x+4)(x-4)$$

$$\frac{x^2-16}{x+4}\times\frac{x+4}{x-4}=\frac{(x+4)\cancel{(x-4)}}{\cancel{x+4}}\times\frac{\cancel{x+4}}{\cancel{x-4}}=x+4$$

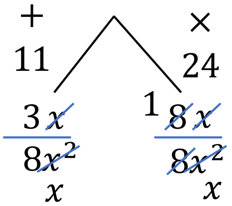
(iii) $\boldsymbol{\frac{16x^2-2x-3}{3x^2-2x-1}\div\frac{8x^2+11x+3}{3x^2-11x-4}}$

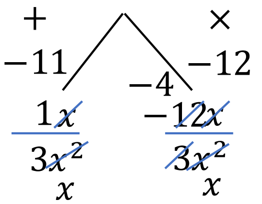




$$16x^2-2x-3=(8x+3)(2x-1)$$

$$3x^2-2x-1=(3x+1)(x-1)$$





$$8x^2+11x+3=(8x+3)(x+1)$$

$$3x^2-11x-4=(3x+1)(x-4)$$

$$\frac{16x^2-2x-3}{3x^2-2x-1}\div\frac{8x^2+11x+3}{3x^2-11x-4}=\frac{16x^2-2x-3}{3x^2-2x-1}\times\frac{3x^2-11x-4}{8x^2+11x+3}$$

$$=\frac{\cancel{(8x+3)}(2x-1)}{\cancel{(3x+1)}(x-1)}\times\frac{\cancel{(3x+1)}(x-4)}{\cancel{(8x+3)}(x+1)}$$

$$=\frac{(2x-1)(x-4)}{(x-1)(x+1)}=\frac{2x^2-8x-x+4}{x^2-1}=\frac{2x^2-9x+4}{x^2-1}$$

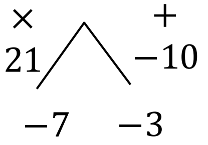
1. *Simplify*

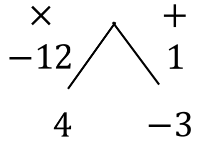
$$(i)\ \frac{4x^2y}{2z^2}\times\frac{6xz^3}{20y^4}\quad (ii)\ \frac{p^2-10p+21}{p-7}\times\frac{p^2+p-12}{(p-3)^2}\quad (iii)\ \frac{5t^3}{4t-8}\times\frac{6t-12}{10t}$$

$$(i)\ \frac{4x^2y}{2z^2}\times\frac{6xz^3}{20y^4}$$

$$\frac{{}^{2}\cancel{4}\cancel{x^2}\cancel{y}}{\cancel{2}\cancel{z^2}}\times\frac{{}^{3}\cancel{6}xz\cancel{{}^3}\,z}{\underset{\cancel{10}\ 5}{\cancel{20}}\underset{y^3}{\cancel{y^4}}}=\frac{3x^3z}{5y^3}$$

$$(ii)\ \frac{p^2-10p+21}{p-7}\times\frac{p^2+p-12}{(p-3)^2}$$





$$p^2-10p+21=(p-7)(p-3)$$

$$p^2+p-12\ =(p+4)(p-3)$$

$$\frac{p^2-10p+21}{p-7}\times\frac{p^2+p-12}{(p-3)^2}=\frac{\cancel{(p-7)}\cancel{(p-3)}}{\cancel{p-7}}\times\frac{(p+4)\cancel{(p-3)}}{\underset{\cancel{(p-3)}}{\cancel{(p-3)^2}}}=p+4$$

$$(iii)\ \frac{5t^3}{4t-8}\times\frac{6t-12}{10t}$$

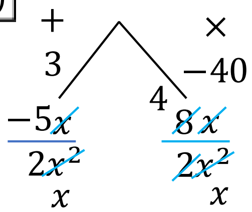
$$\frac{5t^3}{4t-8}\times\frac{6t-12}{10t}=\frac{\overset{t^2}{\cancel{5}\cancel{t^3}}}{4\cancel{(t-2)}}\times\frac{{}^{3}\cancel{6}\cancel{(t-2)}}{\underset{\cancel{2}}{\cancel{10}}\cancel{t}}=\frac{3t^2}{4}$$

2. *Simplify*

$$(i)\ \frac{x+4}{3x+4y}\times\frac{9x^2-16y^2}{2x^2+3x-20}\quad (ii)\ \frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}$$

$$(i)\ \frac{x+4}{3x+4y}\times\frac{9x^2-16y^2}{2x^2+3x-20}$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$



$$9x^2-16y^2=(3x)^2-(4y)^2=(3x+4y)(3x-4y)$$

$$2x^2+3x-20\ =(2x-5)(x+4)$$

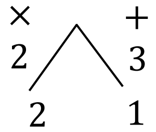
$$\frac{x+4}{3x+4y}\times\frac{9x^2-16y^2}{2x^2+3x-20}=\frac{\cancel{x+4}}{\cancel{3x+4y}}\times\frac{\cancel{(3x+4y)}(3x-4y)}{(2x-5)\cancel{(x+4)}}=\frac{3x-4y}{2x-5}$$

$$(ii)\ \frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}$$

$$\underset{a^3-b^3}{x^3-y^3} = x^3-y^3$$

$$\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

$$= \underset{(a-b)}{(x-y)}\underset{(a^2+ab+b^2)}{(x^2+xy+y^2)}$$



$$3x^2+9xy+6y^2 = 3(x^2+3xy+2y^2) = 3\,(x+2y)(x+y)$$

$$x^2+2xy+y^2 = (x+y)(x+y)$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$x^2-y^2=(x+y)(x-y)$$

$$\frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2} = \frac{\cancel{(x-y)}(x^2+xy+y^2)}{3(x+2y)\cancel{(x+y)}}\times\frac{\cancel{(x+y)}\cancel{(x+y)}}{\cancel{(x+y)}\cancel{(x-y)}}$$

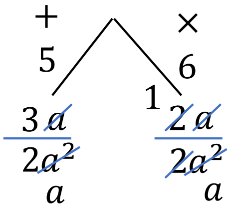
$$=\frac{x^2+xy+y^2}{3(x+2y)}$$

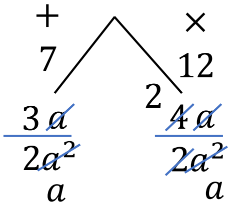
3. $\boldsymbol{Simplify}$ (i) $\mathbf{\frac{2a^2+5a+3}{2a^2+7a+6}\div\frac{a^2+6a+5}{-5a^2-35a-50}}$

(ii) $\mathbf{\frac{b^2+3b-28}{b^2+4b+4}\div\frac{b^2-49}{b^2-5b-14}}$ (iii) $\mathbf{\frac{x+2}{4y}\div\frac{x^2-x-6}{12y^2}}$

(iv) $\mathbf{\frac{12t^2-22t+8}{3t}\div\frac{3t^2+2t-8}{2t^2+4t}}$

(i) $\frac{2a^2+5a+3}{3a^2+7a+6}\div\frac{a^2+6a+5}{-5a^2-35a-50}$





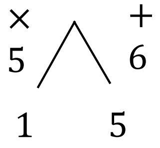
$$2a^2+5a+3=(2a+3)(a+1)$$

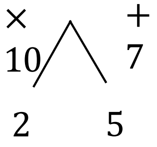
$$2a^2+7a+6=(2a+3)(a+2)$$

$$a^2+6a+5=(a+1)(a+5)$$

$$-5a^2-35a-50=-5(a^2+7a+10)$$

$$=-5(a+2)(a+5)$$

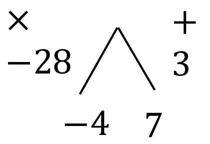


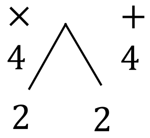


$$\frac{2a^2+5a+3}{2a^2+7a+6}\div\frac{a^2+6a+5}{-5a^2-35a-50}=\frac{2a^2+5a+3}{2a^2+7a+6}\times\frac{-5a^2-35a-50}{a^2+6a+5}$$

$$=\frac{\cancel{(2a+3)}\cancel{(a+1)}}{\cancel{(2a+3)}\cancel{(a+2)}}\times\frac{-5\cancel{(a+2)}\cancel{(a+5)}}{\cancel{(a+1)}\cancel{(a+5)}}=-5$$

(ii) $\mathbf{\frac{b^2+3b-28}{b^2+4b+4}\div\frac{b^2-49}{b^2-5b-14}}$





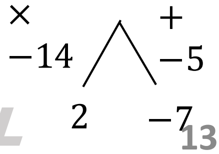
$$b^2+3b-28=(b-4)(b+7)$$

$$b^2+4b+4=(b+2)(b+2)$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$b^2-49=b^2-7^2=(b+7)(b-7)$$

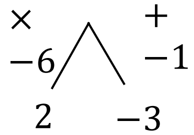
$$b^2-5b-14=(b+2)(b-7)$$



$$\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14} = \frac{b^2+3b-28}{b^2+4b+4} \times \frac{b^2-5b-14}{b^2-49}$$

$$= \frac{(b-4)\cancel{(b+7)}}{\cancel{(b+2)}(b+2)} \times \frac{\cancel{(b+2)}\cancel{(b-7)}}{\cancel{(b+7)}\cancel{(b-7)}} = \frac{b-4}{b+2}$$

(iii) $\boldsymbol{\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}}$



$x^2-x-6 = (x+2)(x-3)$

$$\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2} = \frac{x+2}{4y} \times \frac{12y^2}{x^2-x-6}$$

$$= \frac{\cancel{x+2}}{\cancel{4y}} \times \frac{\overset{3y}{\cancel{12y^2}}}{\cancel{(x+2)}(x-3)} = \frac{3y}{x-3}$$

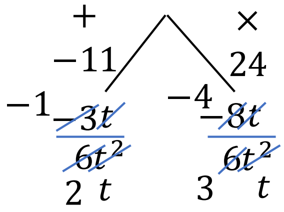
(iv) $\boldsymbol{\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}}$

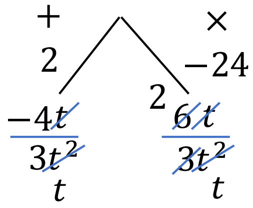
$12t^2-22t+8 = 2(6t^2-11t+4)$

$= 2(2t-1)(3t-4)$

$3t^2+2t-8 = (3t-4)(t+2)$

$2t^2+4t = 2t(t+2)$



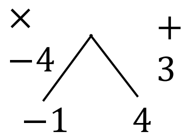


$$\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t} = \frac{12t^2-22t+8}{3t} \times \frac{2t^2+4t}{3t^2+2t-8}$$

$$= \frac{2(2t-1)\cancel{(3t-4)}}{3\cancel{t}} \times \frac{\cancel{2t}\cancel{(t+2)}}{\cancel{(3t-4)}\cancel{(t+2)}} = \frac{4(2t-1)}{3}$$

4. If $\boldsymbol{x = \frac{a^2+3a-4}{3a^2-3}}$ **and** $\boldsymbol{y = \frac{a^2+2a-8}{2a^2-2a-4}}$ **find the value of** $\boldsymbol{x^2y^{-2}}$**.**

$$x = \frac{a^2+3a-4}{3a^2-3}$$

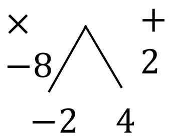


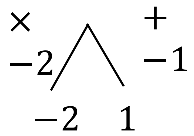
$a^2+3a-4 = (a-1)(a+4)$

$3a^2-3 = 3(a^2-1) = 3(a^2-1^2) = 3(a+1)(a-1)$

$$x = \frac{\cancel{(a-1)}(a+4)}{3(a+1)\cancel{(a-1)}} \Longrightarrow x = \frac{a+4}{3(a+1)}$$

$$y = \frac{a^2+2a-8}{2a^2-2a-4}$$





$a^2+2a-8 = (a-2)(a+4)$

$2a^2-2a-4 = 2(a^2-a-2) = 2(a-2)(a+1)$

$$y=\frac{\cancel{(a-2)}(a+4)}{2\cancel{(a-2)}(a+1)} \Longrightarrow y=\frac{a+4}{2(a+1)}$$

$$x^2y^{-2}=\frac{x^2}{y^2}=\frac{\frac{(a+4)^2}{9(a+1)^2}}{\frac{(a+4)^2}{4(a+1)^2}}=\frac{\frac{\cancel{(a+4)^2}}{9\cancel{(a+1)^2}}}{\frac{\cancel{(a+4)^2}}{4\cancel{(a+1)^2}}}=\frac{\frac{1}{9}}{\frac{1}{4}}=\frac{1}{9}\times\frac{4}{1}$$

$$\boxed{x^2y^{-2}=\frac{4}{9}}$$

5. *If a polynomial* $p(x)=x^2-5x-14$ *is divided by another polynomial* $q(x)$ *we get* $\frac{x-7}{x+2}$*, find* $q(x)$.

$$\frac{p(x)}{q(x)}=\frac{x-7}{x+2} \Longrightarrow \frac{x^2-5x-14}{q(x)}=\frac{x-7}{x+2}$$

$$\begin{array}{ccc} \times & & + \\ -14 & \wedge & -5 \\ 2 & & -7 \end{array}$$

$$\frac{(x+2)\cancel{(x-7)}}{q(x)}=\frac{\cancel{x-7}}{x+2} \Longrightarrow q(x)=(x+2)^2$$

$$\boxed{(a+b)^2=a^2+2ab+b^2}$$

$q(x)=x^2+2(2)x+2^2$

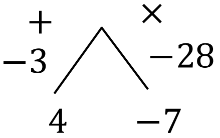
$q(x)=x^2+4x+4$ *is another polynomial*

$Exercise\ 3.6$

$Example\ 3.17$: $Find\ \dfrac{x^2+20x+36}{x^2-3x-28}-\dfrac{x^2+12x+4}{x^2-3x-28}$

$$\frac{x^2+20x+36}{x^2-3x-28}-\frac{x^2+12x+4}{x^2-3x-28}=\frac{x^2+20x+36-(x^2+12x+4)}{x^2-3x-28}$$

$$=\frac{\cancel{x^2}+20x+36-\cancel{x^2}-12x-4}{x^2-3x-28}$$

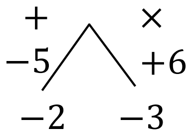


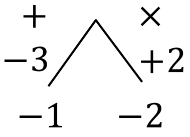
$$=\frac{8x+32}{(x+4)(x-7)}$$

$$=\frac{8\cancel{(x+4)}}{\cancel{(x+4)}(x-7)}=\frac{8}{x-7}$$

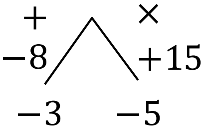
$Example\ 3.18$: $Simplify\ \dfrac{1}{x^2-5x+6}+\dfrac{1}{x^2-3x+2}-\dfrac{1}{x^2-8x+15}$

$$=\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15}$$



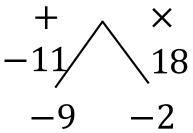


$$=\frac{1}{(x-2)(x-3)}+\frac{1}{(x-1)(x-2)}-\frac{1}{(x-3)(x-5)}$$



$$=\frac{(x-1)(x-5)+(x-3)(x-5)\ -(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$=\frac{x^2-5x-x+5\ +\ x^2-5x-3x+15\ -(x^2-2x-x+2)}{(x-1)(x-2)(x-3)(x-5)}$$



$$=\frac{\cancel{x^2}-5x-\cancel{x}+5+x^2-5x-3x+15-\cancel{x^2}+2x+\cancel{x}-2}{(x-1)(x-2)(x-3)(x-5)}$$

$$=\frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}=\frac{(x-9)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}(x-3)(x-5)}$$

$$=\frac{x-9}{(x-1)(x-3)(x-5)}$$

1. $Simplify\ (i)\dfrac{x(x+1)}{x-2}+\dfrac{x(1-x)}{x-2}\ (ii)\dfrac{x+2}{x+3}+\dfrac{x-1}{x-2}\ (iii)\dfrac{x^3}{x-y}+\dfrac{y^3}{y-x}$

$(i)\dfrac{x(x+1)}{x-2}+\dfrac{x(1-x)}{x-2}$

$$\frac{x(x+1)+x(1-x)}{x-2}=\frac{\cancel{x^2}+x+x-\cancel{x^2}}{x-2}=\frac{2x}{x-2}$$

$(ii)\dfrac{x+2}{x+3}+\dfrac{x-1}{x-2}$

$$=\frac{(x+2)(x-2)+(x-1)(x+3)}{(x+3)(x-2)} \qquad \boxed{a^2-b^2=(a+b)(a-b)}$$

$$=\frac{(x^2-2^2)+x^2+3x-x-3}{(x+3)(x-2)}=\frac{x^2-4+x^2+2x-3}{(x+3)(x-2)}=\frac{2x^2+2x-7}{(x+3)(x-2)}$$

$$(\boldsymbol{iii})\ \frac{\boldsymbol{x^3}}{\boldsymbol{x-y}}+\frac{\boldsymbol{y^3}}{\boldsymbol{y-x}}$$

$$=\frac{x^3}{x-y}+\frac{y^3}{-(x-y)}=\frac{x^3}{x-y}-\frac{y^3}{x-y}=\frac{x^3-y^3}{x-y}$$

$$=\frac{(\cancel{x-y})(x^2+xy+y^2)}{\cancel{x-y}}=x^2+xy+y^2 \quad \boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

$$\mathbf{2.}\ \boldsymbol{Simplify}\ (\boldsymbol{i})\frac{\boldsymbol{(2x+1)(x-2)}}{\boldsymbol{x-4}}-\frac{\boldsymbol{(2x^2-5x+2)}}{\boldsymbol{x-4}}\ (\boldsymbol{ii})\ \frac{\boldsymbol{4x}}{\boldsymbol{x^2-1}}-\frac{\boldsymbol{x+1}}{\boldsymbol{x-1}}$$

$$(\boldsymbol{i})\frac{\boldsymbol{(2x+1)(x-2)}}{\boldsymbol{x-4}}-\frac{\boldsymbol{(2x^2-5x+2)}}{\boldsymbol{x-4}}$$

$$\frac{(2x+1)(x-2)}{x-4}-\frac{(2x^2-5x+2)}{x-4}=\frac{2x^2-4x+x-2-(2x^2-5x+2)}{x-4}$$

$$=\frac{\cancel{2x^2}-3x-2\cancel{-2x^2}+5x-2}{x-4}$$

$$=\frac{2x-4}{x-4}=\frac{2(x-2)}{x-4}$$

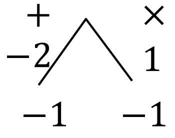
$$(\boldsymbol{ii})\ \frac{\boldsymbol{4x}}{\boldsymbol{x^2-1}}-\frac{\boldsymbol{x+1}}{\boldsymbol{x-1}}$$

$$\frac{4x}{x^2-1^2}-\frac{x+1}{x-1}=\frac{4x}{(x+1)(x-1)}-\frac{x+1}{x-1} \qquad \boxed{a^2-b^2=(a+b)(a-b)} \quad \boxed{(a+b)^2=a^2+2ab+b^2}$$

$$=\frac{4x-(x+1)(x+1)}{(x+1)(x-1)}$$

$$=\frac{4x-(x+1)^2}{(x+1)(x-1)}=\frac{4x-(x^2+2x+1^2)}{(x+1)(x-1)}$$

$$=\frac{4x-x^2-2x-1}{(x+1)(x-1)}=\frac{-x^2+2x-1}{(x+1)(x-1)}$$



$$=\frac{-(x^2-2x+1)}{(x+1)(x-1)}=\frac{-(x-1)(\cancel{x-1})}{(x+1)(\cancel{x-1})}=\frac{-(x-1)}{(x+1)}=\frac{1-x}{1+x}$$

$$\mathbf{3.}\ \boldsymbol{Subtract}\ \frac{\mathbf{1}}{\boldsymbol{x^2+2}}\ \boldsymbol{from}\ \frac{\boldsymbol{2x^3+x^2+3}}{\boldsymbol{(x^2+2)^2}}$$

$$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}=\frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2}$$

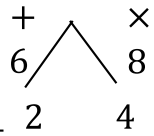
$$=\frac{2x^3\cancel{+x^2}+3\cancel{-x^2}-2}{(x^2+2)^2}=\frac{2x^3+1}{(x^2+2)^2}$$

$\mathbf{4.}$ ***Which rational expression should be subtracted from***

$$\frac{\boldsymbol{x^2+6x+8}}{\boldsymbol{x^3+8}}\ \textbf{\textit{to get}}\ \frac{\mathbf{3}}{\boldsymbol{x^2-2x+4}}$$

$$\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$$

$$\frac{x^2+6x+8}{x^3+8}-\frac{3}{x^2-2x+4}=\frac{(x+2)(x+4)}{x^3+2^3}-\frac{3}{x^2-2x+4}$$



$$=\frac{(x+2)(x+4)}{(x+2)(x^2-2x+2^2)}-\frac{3}{x^2-2x+4}$$

$$=\frac{\cancel{(x+2)}(x+4)}{\cancel{(x+2)}(x^2-2x+4)}-\frac{3}{x^2-2x+4}$$

$$=\frac{x+4}{x^2-2x+4}-\frac{3}{x^2-2x+4}=\frac{x+4-3}{x^2-2x+4}$$

$$=\frac{x+1}{x^2-2x+4}$$

$\mathbf{5.}$ $\boldsymbol{If\ A=\frac{2x+1}{2x-1}, B=\frac{2x-1}{2x+1}\ find\ \frac{1}{A-B}-\frac{2B}{A^2-B^2}}$

$$\frac{1}{A-B}-\frac{2B}{A^2-B^2}=\frac{1}{A-B}-\frac{2B}{(A+B)(A-B)}$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$=\frac{A+B-2B}{(A+B)(A-B)}=\frac{\cancel{A-B}}{(A+B)\cancel{(A-B)}}=\frac{1}{A+B}$$

$$\frac{1}{A+B}=\frac{1}{\frac{2x+1}{2x-1}+\frac{2x-1}{2x+1}}=\frac{1}{\frac{(2x+1)(2x+1)+(2x-1)(2x-1)}{(2x-1)(2x+1)}}$$

$$=\frac{1}{\frac{(2x+1)^2+(2x-1)^2}{(2x)^2-1^2}}=\frac{1}{\frac{2[(2x)^2+1^2]}{4x^2-1}}=\frac{1}{\frac{2[4x^2+1]}{4x^2-1}}$$

$$\boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$$

$$=\frac{4x^2-1}{2[4x^2+1]}$$

$\mathbf{6.}$ $\boldsymbol{If\ A=\frac{x}{x+1}, B=\frac{1}{x+1}\ prove\ that\ \frac{(A+B)^2+(A-B)^2}{A\div B}=\frac{2(x^2+1)}{x(x+1)^2}}$

$$L.H.S=\frac{(A+B)^2+(A-B)^2}{A\div B}$$

$$\boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$$

$$=\frac{2(A^2+B^2)}{\frac{A}{B}}=2(A^2+B^2)\times\frac{B}{A}\quad where\ A=\frac{x}{x+1},B=\frac{1}{x+1}$$

$$=2\left[\left(\frac{x}{x+1}\right)^2+\left(\frac{1}{x+1}\right)^2\right]\times\frac{\frac{1}{\cancel{x+1}}}{\frac{x}{\cancel{x+1}}}=2\left[\frac{x^2}{(x+1)^2}+\frac{1^2}{(x+1)^2}\right]\times\frac{1}{x}$$

$$=2\left[\frac{x^2+1}{(x+1)^2}\right]\times\frac{1}{x}=\frac{2(x^2+1)}{x(x+1)^2}=R.H.S$$

7. *Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?*

Time taken by Pari to complete a work = 4 hrs.

$$5\overline{)\,12\,}\,(2$$
$$\frac{10}{2}$$

$$\therefore Pari's\ 1\ hour\ work=\frac{1}{4}$$

Time taken by Yuvan to complete the same work = 6 hrs.

$$\boxed{\frac{2}{5}\ hrs=\frac{2}{\cancel{5}}\times\overset{12}{\cancel{60}}}$$

$$\therefore Yuvan's\ 1\ hour\ work=\frac{1}{6}$$

$$\text{(Pari + Yuvan)'s 1 day's work}=\left(\frac{1}{4}+\frac{1}{6}\right)=\frac{3+2}{12}=\frac{5}{12}$$

$$\text{Both Pari and Yuvan will complete the work in}=2\frac{2}{5}\ hrs$$

$$=2\ hrs\ 24\ min.$$

8. *Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy?*

Let x be the weight of an apple and y be the weight of a banana

$$x+y=50\ \ldots(1)$$

Iniya bought apples = ₹ 1800 ⟹ weight × price per kg of apple = 1800

$$x\times price\ per\ kg\ of\ apple=1800\Longrightarrow Price\ per\ kg\ of\ apple=\frac{1800}{x}$$

Iniya bought bananas = ₹ 600 ⟹ weight × price per kg of bananas = 600

$$y\times price\ per\ kg\ of bananas=600\Longrightarrow Price\ per\ kg\ of\ bananas=\frac{600}{y}$$

Given : Price per kg for the apple = twice the price per kg of bananas

$$\frac{\overset{3}{\cancel{1800}}}{x}=2\times\frac{\cancel{600}}{y}\Longrightarrow\frac{3}{x}=\frac{2}{y}\Longrightarrow y=\frac{2x}{3}\ \ldots(2)$$

$$sub\ y = \frac{2x}{3}\ in\ (1)\ x + y = 50$$

$$x + \frac{2x}{3} = 50 \Longrightarrow \frac{3x + 2x}{3} = 50 \Longrightarrow \frac{5x}{3} = 50 \Longrightarrow 5x = 150$$

$$x = \frac{\cancel{150}^{30}}{\cancel{5}} \Longrightarrow \boxed{x = 30}$$

$$sub\ x = 30\ in\ (2)\ y = \frac{2x}{3}$$

$$y = \frac{2 \times 30}{3} \Longrightarrow y = 2 \times 10 \Longrightarrow \boxed{y = 20}$$

$\therefore$ *She bought* 30 *kg of apple* & 20 *kg of banana*.

$Exercise\ 3.7$

$Example\ 3.19\ Find\ the\ square\ root\ of\ the\ following\ expressions$

$(i)\ 256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}\ (ii)\ \dfrac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

$\boldsymbol{(i)\ 256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$

$$= \sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$$

$$= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times (x-a)^8 \times (x-b)^4 \times (x-c)^{16} \times (x-d)^{20}}$$

$$= 2 \times 2 \times 2 \times 2 \times (x-a)^4 \times (x-b)^2 \times (x-c)^8 \times (x-d)^{10}$$

$$= |16(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}|$$

$$= 16|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}|$$

$$\boxed{256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$\boldsymbol{(ii)\ \dfrac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}}$

$$= \sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \sqrt{\frac{12 \times 12 \times a^8 \times b^{12} \times c^{16}}{9 \times 9 \times f^{12} \times g^4 \times h^{14}}}$$

$$= \left|\frac{12a^4b^6c^8}{9f^6g^2h^7}\right| = \frac{\overset{4}{\cancel{12}}}{\underset{3}{\cancel{9}}}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right| = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

Example 3.20 Find the square root of the following expressions

$\boldsymbol{(i)\ 16x^2 + 9y^2 - 24xy + 24x - 18y + 9}$

$\boldsymbol{(ii)\left(6x^2 + x - 1\right)\left(3x^2 + 2x - 1\right)\left(2x^2 + 3x + 1\right)}$

$\boldsymbol{(iii)\ \left[\sqrt{15}x^2 + \left(\sqrt{3} + \sqrt{10}\right)x + \sqrt{2}\right]\left[\sqrt{5}x^2 + \left(2\sqrt{5} + 1\right)x + 2\right]}$
$\boldsymbol{\left[\sqrt{3}x^2 + \left(\sqrt{2} + 2\sqrt{3}\right)x + 2\sqrt{2}\right]}$

$\boldsymbol{(i) 16x^2 + 9y^2 - 24xy + 24x - 18y + 9}$

$$\sqrt{16x^2 + 9y^2 - 24xy + 24x - 18y + 9}$$

$$= \sqrt{(4x)^2 + (-3y)^2 + (3)^2 + 2(4x)(-3y) + 2(-3y)(3) + 2(4x)(3)}$$

$$a^2 \quad + (-b)^2 \quad + c^2 \quad + 2a(-b) \quad + 2(-b)(c) \quad + 2(c)a$$

$$= \sqrt{(4x - 3y + 3)^2} = |4x - 3y + 3|$$

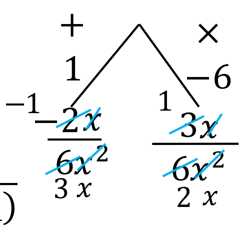
$$(a - b + c)^2$$

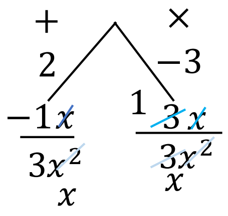
$\boldsymbol{(ii)\left(6x^2 + x - 1\right)\left(3x^2 + 2x - 1\right)\left(2x^2 + 3x + 1\right)}$

$$6x^2 + x - 1 = (3x - 1)(2x + 1)$$

$$3x^2 + 2x - 1 = (3x - 1)(x + 1)$$

$$2x^2 + 3x + 1 = (2x + 1)(x + 1)$$

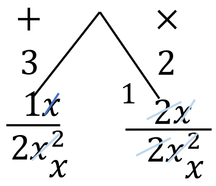




$$\sqrt{(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)}$$

$$= \sqrt{(3x - 1)(2x + 1)(3x - 1)(x + 1)(2x + 1)(x + 1)}$$

$$= |(3x - 1)(2x + 1)(x + 1)|$$



$$(iii)\left[\sqrt{15}x^2+\left(\sqrt{3}+\sqrt{10}\right)x+\sqrt{2}\right]\left[\sqrt{5}x^2+\left(2\sqrt{5}+1\right)x+2\right]$$
$$\left[\sqrt{3}x^2+\left(\sqrt{2}+2\sqrt{3}\right)x+2\sqrt{2}\right]$$

let us factorize the polynomials

$$\sqrt{15}x^2+\left(\sqrt{3}+\sqrt{10}\right)x+\sqrt{2}=\sqrt{15}x^2+\sqrt{3}x+\sqrt{10}x+\sqrt{2}$$
$$=\underbrace{\sqrt{5}\sqrt{3}x^2+\sqrt{3}x}+\underbrace{\sqrt{5}\sqrt{2}x+\sqrt{2}}$$
$$=\sqrt{3}x\left(\sqrt{5}x+1\right)+\sqrt{2}\left(\sqrt{5}x+1\right)$$
$$=\left(\sqrt{5}x+1\right)\left(\sqrt{3}x+\sqrt{2}\right)$$

$$\sqrt{5}x^2+\left(2\sqrt{5}+1\right)x+2=\underbrace{\sqrt{5}x^2+2\sqrt{5}x}+\underbrace{x+2}$$
$$=\sqrt{5}x(x+2)+1(x+2)$$
$$=\left(\sqrt{5}x+1\right)(x+2)$$

$$\sqrt{3}x^2+\left(\sqrt{2}+2\sqrt{3}\right)x+2\sqrt{2}=\underbrace{\sqrt{3}x^2+\sqrt{2}x}+\underbrace{2\sqrt{3}x+2\sqrt{2}}$$
$$=x\left(\sqrt{3}x+\sqrt{2}\right)+2\left(\sqrt{3}x+\sqrt{2}\right)$$
$$=\left(\sqrt{3}x+\sqrt{2}\right)(x+2)$$

$$=\sqrt{\begin{matrix}\left[\sqrt{15}x^2+\left(\sqrt{3}+\sqrt{10}\right)x+\sqrt{2}\right]\left[\sqrt{5}x^2+\left(2\sqrt{5}+1\right)x+2\right]\\ \left[\sqrt{3}x^2+\left(\sqrt{2}+2\sqrt{3}\right)x+2\sqrt{2}\right]\end{matrix}}$$

$$=\sqrt{\left(\sqrt{5}x+1\right)\left(\sqrt{3}x+\sqrt{2}\right)\left(\sqrt{5}x+1\right)(x+2)\left(\sqrt{3}x+\sqrt{2}\right)(x+2)}$$

$$=\left|\left(\sqrt{5}x+1\right)\left(\sqrt{3}x+\sqrt{2}\right)(x+2)\right|$$

1. *Find the square root of the following rational expressions.*

$$(i)\ \frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}\quad (ii)\ \frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}\quad (iii)\ \sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^2}}$$

$$(i)\ \frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$$

$$=\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}}=\sqrt{\frac{20\times20\times x^4\times y^{12}\times z^{16}}{10\times10\times x^8\times y^4\times z^4}}$$

$$=\left|\frac{20x^2y^6z^8}{10x^4y^2z^2}\right|=\frac{\cancel{20}}{\cancel{10}}\left|\frac{x^2y^6z^8}{x^4y^2z^2}\right|=2\left|\frac{x^2y^6z^8}{x^4y^2z^2}\right|$$

$$(ii)\ \frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$$

$$7x^2+2\sqrt{14}x+2=\left(\sqrt{7}x\right)^2+2\times\sqrt{7}x\times\sqrt{2}+\left(\sqrt{2}\right)^2$$
$$a^2\quad+\quad 2ab\quad\quad+b^2$$
$$=\left(\sqrt{7}x+\sqrt{2}\right)^2$$

$$= \sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}}$$

$$x^2-\frac{1}{2}x+\frac{1}{16} = \underset{a^2}{(x)^2} \underset{-\ 2ab}{-\ 2\times x\times\frac{1}{4}} \underset{+}{+} \underset{b^2}{\left(\frac{1}{4}\right)^2}$$

$$= \left(x-\frac{1}{4}\right)^2$$

$$= \sqrt{\frac{\left(\sqrt{7}x+\sqrt{2}\right)^2}{\left(x-\frac{1}{4}\right)^2}} = \left|\frac{\sqrt{7}x+\sqrt{2}}{x-\frac{1}{4}}\right|$$

$$= \left|\frac{\sqrt{7}x+\sqrt{2}}{\frac{4x-1}{4}}\right| = \left|\frac{4}{1}\times\frac{\sqrt{7}x+\sqrt{2}}{4x-1}\right| = 4\left|\frac{\sqrt{7}x+\sqrt{2}}{4x-1}\right|$$

$$(\boldsymbol{iii})\ \sqrt{\frac{\mathbf{121}(\boldsymbol{a+b})^{\mathbf{8}}(\boldsymbol{x+y})^{\mathbf{8}}(\boldsymbol{b-c})^{\mathbf{8}}}{\mathbf{81}(\boldsymbol{b-c})^{\mathbf{4}}(\boldsymbol{a-b})^{\mathbf{12}}(\boldsymbol{b-c})^{\mathbf{4}}}}$$

$$= \sqrt{\frac{11\times 11\times(a+b)^8\times(x+y)^8\times(b-c)^8}{9\times 9\times(b-c)^4\times(a-b)^{12}\times(b-c)^4}}$$

$$= \left|\frac{11(a+b)^4(x+y)^4\overset{(b-c)^2}{\cancel{(b-c)^4}}}{9(b-c)^2(a-b)^6\cancel{(b-c)^2}}\right| = \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4(b-c)^2}{(b-c)^2(a-b)^6}\right|$$

2. *Find the square root of the following*

$$(\boldsymbol{i})\ \mathbf{4}\boldsymbol{x}^{\mathbf{2}}+\mathbf{20}\boldsymbol{x}+\mathbf{25}\quad(\boldsymbol{ii})\ \mathbf{9}\boldsymbol{x}^{\mathbf{2}}-\mathbf{24}\boldsymbol{xy}+\mathbf{30}\boldsymbol{xz}-\mathbf{40}\boldsymbol{yz}+\mathbf{25}\boldsymbol{z}^{\mathbf{2}}+\mathbf{16}\boldsymbol{y}^{\mathbf{2}}$$

$$(\boldsymbol{iii})\ \mathbf{1}+\frac{\mathbf{1}}{\boldsymbol{x}^{\mathbf{6}}}+\frac{\mathbf{2}}{\boldsymbol{x}^{\mathbf{3}}}\ (\boldsymbol{iv})\left(\mathbf{4}\boldsymbol{x}^{\mathbf{2}}-\mathbf{9}\boldsymbol{x}+\mathbf{2}\right)\left(\mathbf{7}\boldsymbol{x}^{\mathbf{2}}-\mathbf{13}\boldsymbol{x}-\mathbf{2}\right)\left(\mathbf{28}\boldsymbol{x}^{\mathbf{2}}-\mathbf{3}\boldsymbol{x}-\mathbf{1}\right)$$

$$(\boldsymbol{v})\left(\mathbf{2}\boldsymbol{x}^{\mathbf{2}}+\frac{\mathbf{17}}{\mathbf{6}}\boldsymbol{x}+\mathbf{1}\right)\left(\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{x}^{\mathbf{2}}+\mathbf{4}\boldsymbol{x}+\mathbf{2}\right)\left(\frac{\mathbf{4}}{\mathbf{3}}\boldsymbol{x}^{\mathbf{2}}+\frac{\mathbf{11}}{\mathbf{3}}\boldsymbol{x}+\mathbf{2}\right)$$

$$(\boldsymbol{i})\ \mathbf{4}\boldsymbol{x}^{\mathbf{2}}+\mathbf{20}\boldsymbol{x}+\mathbf{25}$$

$$= \sqrt{4x^2+20x+25} = \sqrt{\underset{a^2}{(2x)^2} \underset{+}{+} \underset{2\ a\ \ b}{2(2x)(5)} \underset{+}{+} \underset{b^2}{(5)^2}} = \sqrt{\underset{(a+b)^2}{(2x+5)^2}}$$

$$= |2x+5|$$

$$(\boldsymbol{ii})\ \mathbf{9}\boldsymbol{x}^{\mathbf{2}}-\mathbf{24}\boldsymbol{xy}+\mathbf{30}\boldsymbol{xz}-\mathbf{40}\boldsymbol{yz}+\mathbf{25}\boldsymbol{z}^{\mathbf{2}}+\mathbf{16}\boldsymbol{y}^{\mathbf{2}}$$

$$\sqrt{9x^2-24xy+30xz-40yz+25z^2+16y^2}$$

$$= \sqrt{\underset{a^2}{(3x)^2}+\underset{+(-b)^2}{(-4y)^2}+\underset{+c^2}{(5z)^2}+\underset{+2a(-b)}{2(3x)(-4y)}+\underset{+2(-b)(c)}{2(-4y)(5z)}+\underset{+2(c)a}{2(5z)(3x)}}$$

$$= \sqrt{\underset{(a-b+c)^2}{(3x-4y+5z)^2}} = |3x-4y+5z|$$

$$(\boldsymbol{iii})\ \mathbf{1}+\frac{\mathbf{1}}{\boldsymbol{x}^{\mathbf{6}}}+\frac{\mathbf{2}}{\boldsymbol{x}^{\mathbf{3}}}$$

$$= \sqrt{1+\frac{1}{x^6}+\frac{2}{x^3}} = \sqrt{\underset{a^2\ +\ b^2\ \ +2ab}{1^2+\left(\frac{1}{x^3}\right)^2+\frac{2}{x^3}}} = \sqrt{\underset{(a+b)^2}{\left(1+\frac{1}{x^3}\right)^2}}$$

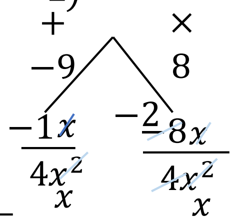
$$= \left|1+\frac{1}{x^3}\right|$$

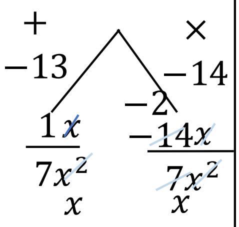
$(iv)\left(4x^2-9x+2\right)\left(7x^2-13x-2\right)\left(28x^2-3x-1\right)$

$4x^2-9x+2 = (4x-1)(x-2)$

$7x^2-13x-2 = (7x+1)(x-2)$

$28x^2-3x-1 = (7x+1)(4x-1)$

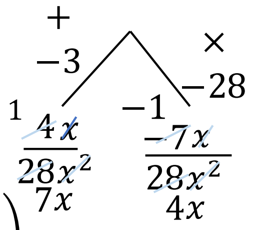




$$\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)}$$

$$= \sqrt{(4x-1)(x-2)(7x+1)(x-2)(7x+1)(4x-1)}$$

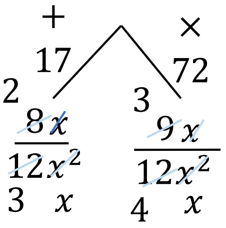
$$= |((4x-1)(x-2)(7x+1)|$$



$(v)\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

$$\sqrt{\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)}$$

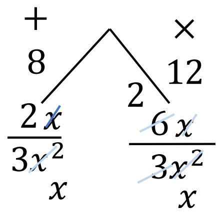
$$= \sqrt{\left(\frac{12x^2+17x+6}{6}\right)\left(\frac{3x^2+8x+4}{2}\right)\left(\frac{4x^2+11x+6}{3}\right)}$$

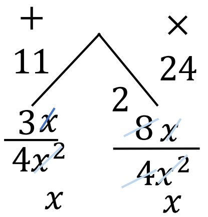


$12x^2+17x+6 = (3x+2)(4x+3)$

$3x^2+8x+4 = (3x+2)(x+2)$

$4x^2+11x+6 = (4x+3)(x+2)$





$$= \sqrt{\frac{(3x+2)(4x+3)}{6} \times \frac{(3x+2)(x+2)}{2} \times \frac{(4x+3)(x+2)}{3}}$$

$$= \sqrt{\frac{(3x+2)^2(4x+3)^2(x+2)^2}{36}} = \left|\frac{(3x+2)(4x+3)(x+2)}{6}\right|$$

$$= \frac{1}{6}\,|(3x+2)(4x+3)(x+2)|$$

Exercise 3.8

Example 3.21: Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

$$
\begin{array}{r|l}
 & 8x^2 - x + 1 \\
\hline
8x^2 & \cancel{64x^4} - 16x^3 + 17x^2 - 2x + 1 \\
 & (-) \\
 & \cancel{64x^4} \\
\hline
16x^2 - x & \cancel{-16x^3} + 17x^2 \\
 & \quad (-) \\
 & \cancel{-(+)16x^3} + x^2 \\
\hline
16x^2 - 2x + 1 & \cancel{16x^2 - 2x + 1} \\
 & (-) \quad (+) \\
 & \cancel{(-)16x^2 - 2x + 1} \\
\hline
 & 0
\end{array}
$$

$$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$$

Example 3.22: Find the square root of the expression

$$\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$$

$$
\begin{array}{r|l}
 & \frac{2x}{y} + 5 \\
\hline
\frac{2x}{y} & \cancel{\frac{4x^2}{y^2}} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2} \\
 & (-) \\
 & \cancel{\frac{4x^2}{y^2}} \\
\hline
\frac{4x}{y} + 5 & \cancel{\frac{20x}{y}} + 13 \\
 & (-) \quad (-) \\
 & \cancel{\frac{20x}{y}} + 25 \\
\hline
 & -12 - \frac{30y}{x} + \frac{9y^2}{x^2}
\end{array}
$$

$$
\begin{array}{r|l}
 & \frac{2x}{y} + 5 - \frac{3y}{x} \\
\hline
\frac{4x}{y} + 10 - \frac{3y}{x} & \cancel{-12 - \frac{30y}{x} + \frac{9y^2}{x^2}} \\
 & (+) \quad (+) \quad (-) \\
 & \cancel{-12 - \frac{30y}{x} + \frac{9y^2}{x^2}} \\
\hline
 & 0
\end{array}
$$

$$\sqrt{\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}} = \left|\frac{2x}{y}+5-\frac{3y}{x}\right|$$

Example 3.23: If $9x^4+12x^3+28x^2+ax+b$ is a perfect square, find the values of a and b.

$$\begin{array}{r|l}
 & 3x^2+2x+4 \\ \hline
3x^2 & \cancel{9x^4}+12x^3+28x^2+ax+b \\
 & (-) \\
 & \cancel{9x^4} \\ \hline
6x^2+2x & \cancel{12x^3}+28x^2 \\
 & (-)\quad(-) \\
 & \cancel{12x^3}+4x^2 \\ \hline
6x^2+4x+4 & \cancel{24x^2}+ax+b \\
 & (-)\quad(-)\quad(-) \\
 & \cancel{24x^2}+16x+16 \\ \hline
 & 0
\end{array}$$

$$\boxed{\begin{array}{l} a-16=0,\ b-16=0 \\ a=16,\ b=16 \end{array}}$$

Since the given polynomials is a perfect square.

$$\therefore\ a = 16 \text{ and } b = 16$$

1. *Find the square root of the following polynomials by division method.* (*i*) $x^4-12x^3+42x^2-36x+9$

(*ii*) $37x^2-28x^3+4x^4+42x+9$

(*iii*) $16x^4+8x^2+1$ (*iv*) $121x^4-198x^3-183x^2+216x+144$

(*i*) $x^4-12x^3+42x^2-36x+9$

$$\begin{array}{r|l}
 & x^2-6x+3 \\ \hline
x^2 & \cancel{x^4}-12x^3+42x^2-36x+9 \\
 & (-) \\
 & \cancel{x^4} \\ \hline
2x^2-6x & \cancel{-12x^3}+42x^2 \\
 & (+)\quad(-) \\
 & \cancel{-12x^3}+36x^2 \\ \hline
2x^2-12x+3 & \cancel{6x^2-36x+9} \\
 & (-)\quad(+)\quad(-) \\
 & \cancel{6x^2-36x+9} \\ \hline
 & 0
\end{array}$$

$$\sqrt{x^4-12x^3+42x^2-36x+9} = |x^2-6x+3|$$

(*ii*) $37x^2-28x^3+4x^4+42x+9$

$$\begin{array}{r|l}
 & 2x^2 - 7x - 3 \\ \hline
2x^2 & \cancel{4x^4} - 28x^3 + 37x^2 + 42x + 9 \\
 & (-) \\
 & \cancel{4x^4} \\ \hline
4x^2 - 7x & \quad \cancel{-28x^3} + 37x^2 \\
 & \quad (+) \qquad (-) \\
 & \quad \cancel{-28x^3} + 49x^2 \\ \hline
4x^2 - 14x - 3 & \qquad \cancel{-12x^2 + 42x + 9} \\
 & \qquad (+) \quad (-) \quad (-) \\
 & \qquad \cancel{-12x^2 + 42x + 9} \\ \hline
 & \qquad 0
\end{array}$$

$$\sqrt{37x^2 - 28x^3 + 4x^4 + 42x + 9} = |2x^2 - 7x - 3|$$

$(\boldsymbol{iii})$ $\mathbf{16x^4 + 8x^2 + 1}$

$$\begin{array}{r|l}
 & 4x^2 + 1 \\ \hline
4x^2 & \cancel{16x^4} + 0x^3 + 8x^2 + 0x + 1 \\
 & (\\
 & \cancel{16x^4} \\ \hline
 &) \\
8x^2 + 1 & \quad \cancel{8x^2 + 1} \\
 & \quad (-) \quad (-) \\
 & \quad \cancel{8x^2 + 1} \\ \hline
 & \quad 0
\end{array}$$

$$\sqrt{16x^4 + 8x^2 + 1} = |4x^2 + 1|$$

$(\boldsymbol{iv})$ $\mathbf{121x^4 - 198x^3 - 183x^2 + 216x + 144}$

$$\begin{array}{r|l}
 & 11x^2 - 9x - 12 \\ \hline
11x^2 & \cancel{121x^4} - 198x^3 - 183x^2 + 216x + 144 \\
 & (-) \\
 & \cancel{121x^4} \\ \hline
22x^2 - 9x & \quad \cancel{-198x^3} - 183x^2 \\
 & \quad (+) \qquad (-) \\
 & \quad \cancel{-198x^3} + 81x^2 \\ \hline
22x^2 - 18x - 12 & \qquad \cancel{-264x^2 + 216x + 144} \\
 & \qquad (+) \quad (-) \quad (-) \\
 & \qquad \cancel{-264x^2 + 216x + 144} \\ \hline
 & \qquad 0
\end{array}$$

$$\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = |11x^2 - 9x - 12|$$

2. *Find the square root of the expression* $\boldsymbol{\frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}}$

$$\begin{array}{r|l}
 & \frac{x}{y}-5+\frac{y}{x} \\
\hline
\frac{x}{y} & \cancel{\frac{x^2}{y^2}}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2} \\
 & (-) \\
 & \cancel{\frac{x^2}{y^2}} \\
\hline
\frac{2x}{y}-5 & \cancel{-\frac{10x}{y}}+27 \\
 & (+) \qquad (-) \\
 & \cancel{-\frac{10x}{y}}+25 \\
\hline
\frac{2x}{y}-10+\frac{y}{x} & \cancel{2-\frac{10y}{x}+\frac{y^2}{x^2}} \\
 & (-) \quad (+) \quad (-) \\
 & \cancel{2-\frac{10y}{x}+\frac{y^2}{x^2}} \\
\hline
 & 0
\end{array}$$

$$\sqrt{\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}}=\left|\frac{x}{y}-5+\frac{y}{x}\right|$$

3. *Find the values of a and b if the following polynomials are perfect squares*

(*i*) $4x^4-12x^3+37x^2+bx+a$ (*ii*) $ax^4+bx^3+361x^2+220x+100$

(*i*) $4x^4-12x^3+37x^2+bx+a$

$$\begin{array}{r|l}
 & 2x^2-3x+7 \\
\hline
2x^2 & \cancel{4x^4}-12x^3+37x^2+bx+a \\
 & (-) \\
 & \cancel{4x^4} \\
\hline
4x^2-3x & \cancel{-12x^3}+37x^2 \\
 & (+) \qquad (-) \\
 & \cancel{-12x^3}+9x^2 \\
\hline
4x^2-6x+7 & \cancel{28x^2}+bx+a \\
 & (-) \quad (+) \quad (-) \\
 & \cancel{28x^2}-42x+49 \\
\hline
 & 0
\end{array}$$

$a-49=0, b+42=0$

$a=49, b=-42$

Since the given polynomials is a perfect square.

$\therefore\ a=49$ *and* $b=-42$

$(ii)\ ax^4 + bx^3 + 361x^2 + 220x + 100$

$$\begin{array}{r|l} & 10 + 11x + 12x^2 \\ \hline 10 & \cancel{100} + 220x + 361x^2 + bx^3 + ax^4 \\ & (-) \\ & \cancel{100} \\ \hline 20 + 11x & \cancel{220x} + 361x^2 \\ & (-) \quad (-) \\ & \cancel{220x} + 121x^2 \\ \hline 20 + 22x + 12x^2 & \cancel{240x^2} + bx^3 + ax^4 \\ & (-) \quad (-) \quad (-) \\ & \cancel{240x^2} + 264x^3 + 144 \\ \hline & 0 \end{array}$$

$a - 144 = 0,$
$b - 264 = 0$
$a = 144, b = 264$

Since the given polynomials is a perfect square.

Thus, $a = 144$ *and* $b = 264$

4. *Find the values of m and n if the following expressions are perfect sqaures*

$(i)\ \frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n \quad (ii)\ x^4 - 8x^3 + mx^2 + nx + 16$

$(i)\ \frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n$

$$\begin{array}{r|l} & \frac{x}{y} - \frac{3}{x} + 2 \\ \hline \frac{1}{x^2} & \cancel{\frac{1}{x^4}} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n \\ & (-) \\ & \cancel{\frac{1}{x^4}} \\ \hline \frac{2}{x^2} - \frac{3}{x} & -\cancel{\frac{6}{x^3}} + \frac{13}{x^2} \\ & (+) \quad (-) \\ & -\cancel{\frac{6}{x^3}} + \frac{9}{x^2} \\ \hline \frac{2}{x^2} - \frac{6}{x} + 2 & \frac{4}{x^2} + \cancel{\frac{m}{x}} + n \\ & (-) \quad (+) \quad (-) \\ & \cancel{\frac{4}{x^2}} - \frac{12}{x} + 4 \\ \hline & 0 \end{array}$$

$m + 12 = 0, n - 4 = 0 \Longrightarrow m = -12, n = -32$

Since the given polynomials is a perfect square.

Thus, $m = -24$ *and* $n = 4$

(ii) $x^4 - 8x^3 + mx^2 + nx + 16$

$$
\begin{array}{r|l}
 & x^2-4x+4 \\ \hline
x^2 & \cancel{x^4}-8x^3+mx^2+nx+16 \\
 & (-) \\
 & \cancel{x^4} \\ \hline
2x^2-4x & \cancel{-8x^3}+mx^2 \\
 & (+)\qquad(-) \\
 & \cancel{-8x^3}+16x^2 \\ \hline
2x^2-8x+4 & (m-16)x^2+nx+\cancel{16} \\
 & \qquad(-)\quad(+)\quad(-) \\
 & \qquad 8x^2-32x+\cancel{16} \\ \hline
 & \qquad\qquad 0
\end{array}
$$

Since the given polynomials is a perfect square.

$$m - 16 - 8 = 0,\ n + 32 = 0$$

$$m - 24 = 0,\quad n = -32$$

$$m = 24$$

Thus, $m = 24$ and $n = -32$

Exercise 3.9

Example 3.24: Find the zeroes of the quadratic expression:

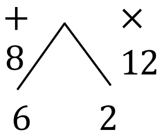
$$\boldsymbol{x^2+8x+12}$$

Let $p(x)=x^2+8x+12=(x+2)(x+6)$

so the values of $x^2+8x+12$ is zero

when $x+2=0$ and $x+6=0$ i.e $x=-2$ and $x=-6$

$\therefore$ The zeros of $x^2+8x+12$ are -2 and -6



Example 3.25: Write down the quadratic equation in general form for in general form for which sum and product of the roots are given below:

$i)\ 9, 14\quad ii)\ -\frac{7}{2}, \frac{5}{2}\quad iii)\ -\frac{3}{5}, -\frac{1}{2}$

(i) Given: Sum of the roots $=9$, Product of the roots $=14$

General form of the quadratic equation :

$x^2-(\text{sum of the roots})x+\text{product of the roots}=0$

$$x^2-9x+14=0$$

(ii) Given: Sum of the roots $=-\frac{7}{2}$, Product of the roots $=\frac{5}{2}$

$$x^2-\left(-\frac{7}{2}\right)x+\frac{5}{2}=0 \Rightarrow 2x^2+7x+5=0$$

multiplying both side by 2

(iii) Given: Sum of the roots $=-\frac{3}{5}$, Product of the roots $=-\frac{1}{2}$

$$x^2-\left(-\frac{3}{5}\right)x+\left(-\frac{1}{2}\right)=0 \Rightarrow 10x^2-\overset{2}{\cancel{10}}\left(-\frac{3}{\cancel{5}}\right)x+\overset{5}{\cancel{10}}\left(-\frac{1}{\cancel{2}}\right)=0$$

multiplying both side by 10

$\boxed{L.C.M\ of\ 5\ and\ 2=10}$

$$10x^2+6x-5=0$$

Example 3.26: Find the sum and product of the roots for each of the following quadratic equations:

$(i)\ x^2+8x-65=0\quad (ii)\ 2x^2+5x+7=0\quad (iii)\ kx^2-k^2x-2k^3$

Let α and β be the roots of the given quadratic equation

$(i)\ x^2+8x-65=0$

$a=1,\ b=8,\ c=-65$

$$\alpha+\beta=-\frac{b}{a} \Rightarrow \alpha+\beta=-\frac{8}{1} \Rightarrow \alpha+\beta=-8$$

$$\alpha\beta=\frac{c}{a} \Rightarrow \alpha\beta=\frac{-65}{1} \Rightarrow \alpha\beta=-65$$

Sum of the roots: $\alpha+\beta=-8$ and Product of the roots: $\alpha\beta=-65$

$(ii)\ 2x^2+5x+7=0$

$a=2,\ b=5,\ c=7$

$$\alpha+\beta=-\frac{b}{a}\Rightarrow\alpha+\beta=-\frac{5}{2}$$

$$\alpha\beta=\frac{c}{a}\ \Rightarrow\ \alpha\beta=\frac{7}{2}$$

$Sum\ of\ the\ roots:\ \alpha+\beta=-\frac{5}{2}\ \ and\ Product\ \ of\ the\ roots: \alpha\beta=\frac{7}{2}$

$\boldsymbol{(iii)\ kx^2-k^2x-2k^3}$

$a=k,\ b=-k^2, c=-2k^3$

$$\alpha+\beta=-\frac{b}{a}\Rightarrow\alpha+\beta=-\frac{(-k^2)}{k}\Rightarrow\alpha+\beta=-\frac{\overset{k}{(-\cancel{k^2})}}{\cancel{k}}$$

$$\alpha+\beta=k$$

$$\alpha\beta=\frac{c}{a}\ \Rightarrow\ \alpha\beta=\frac{-2\overset{k^2}{\cancel{k^3}}}{\cancel{k}}\Rightarrow\alpha\beta=-2k^2$$

$Sum\ of\ the\ roots:\ \alpha+\beta=k\ \ and\ Product\ \ of\ the\ roots: \alpha\beta=-2k^2$

1. *Determine the quadratic equations, whose sum and product of roots are*

$\boldsymbol{(i)-9,20\qquad(ii)\frac{5}{3},4\quad(ii)-\frac{3}{2},-1\ (iv)-(-2-a)^2,(a+5)^2}$

$(i)\ Given: Sum\ of\ the\ roots=-9,\quad Product\ \ of\ the\ roots=20$

$$\boxed{Quadratic\ equation: x^2-(sum\ of\ the\ roots)x+product\ of\ the\ roots=0}$$

$$x^2-(-9)x+20=0\ \Rightarrow x^2+9x+20=0$$

$(ii)\ Given: Sum\ of\ the\ roots=\frac{5}{3},\ Product\ \ of\ the\ roots=4$

$$x^2-\left(\frac{5}{3}\right)x+4=0\quad\Rightarrow\quad 3x^2-5x+12=0$$

$multiplying\ both\ side\ \ by\ 3$

$(iii)\ Given: Sum\ of\ the\ roots=-\frac{3}{2},\ Product\ \ of\ the\ roots=-1$

$$x^2-\left(-\frac{3}{2}\right)x-1=0\quad\Rightarrow\quad 2x^2+3x-2=0$$

$multiplying\ both\ side\ \ by\ 2$

$(iv)\ Given: Sum\ of\ the\ roots=-(2-a)^2,\ Product\ \ of\ the\ roots=(a+5)^2$

$$\boxed{Quadratic\ equation: x^2-(sum\ of\ the\ roots)x+product\ of\ the\ roots=0}$$

$$x^2-[-(2-a)^2]x+(a+5)^2=0\Rightarrow x^2+(2-a)^2x+(a+5)^2=0$$

$\therefore The\ required\ equation\ is\ x^2+(2-a)^2x+(a+5)^2=0$

2. *Find the sum and product of the roots for each of the following quadratic equations.*

$\boldsymbol{(i)x^2+3x-28=0\ (ii)x^2+3x=0\ (iii)\ 3+\frac{1}{a}=\frac{10}{a^2}\ (iv)\ 3y^2-y-4=0}$

$(i)\ x^2 + 3x - 28 = 0$

$Given\ equation\ is : x^2 + 3x - 28 = 0$

$a = 1,\ \ b = 3,\ c = -28$

$$\therefore Sum\ of\ the\ roots = \alpha + \beta = -\frac{b}{a} = -\frac{3}{1}$$

$$\boxed{Sum\ of\ the\ roots = -3}$$

$$Product\ of\ the\ roots = \alpha\beta = \frac{c}{a} = -\frac{28}{1}$$

$$\boxed{Product\ of\ the\ roots = -28}$$

$ii)\ x^2 + 3x = 0$

$a = 1,\ \ b = 3,\ c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{3}{1} = -3$$

$$\alpha\beta = \frac{c}{a} = \frac{0}{1} = 0$$

$Sum\ of\ the\ roots = -3\ and\ Product\ of\ the\ roots = 0$

$iii)\ 3 + \frac{1}{a} = \frac{10}{a^2}$

$$\frac{3a+1}{\cancel{a}} = \frac{10}{\cancel{a^2}\,a} \Rightarrow 3a + 1 = \frac{10}{a} \Rightarrow 3a^2 + a - 10 = 0$$

$a = 3,\ b = 1,\ c = -1$

$$\alpha + \beta = -\frac{b}{a} = -\frac{1}{3}$$

$$\alpha\beta = \frac{c}{a} = -\frac{4}{3}$$

$$Sum\ of\ the\ roots = -\frac{1}{3}\ and\ Product\ of\ the\ roots = -\frac{4}{3}$$

$iv)\ 3y^2 - y - 4 = 0$

$a = 3,\ b = -1,\ c = -4$

$$\therefore \alpha + \beta = -\frac{b}{a} = \frac{1}{3}$$

$$\alpha\beta = \frac{c}{a} = -\frac{4}{3}$$

$$Sum\ of\ the\ roots = \frac{1}{3} and\ Product\ of\ the\ roots = -\frac{4}{3}$$

$Example\ 3.27:\ Solve: 2x^2 - 2\sqrt{6}x + 3 = 0$

$$2x^2 - 2\sqrt{6x} + 3 = 0\ \ (By\ splitting\ the\ middle\ term)$$

$$2x^2 - \sqrt{6}x - \sqrt{6}x + 3 = 0$$

$$\underbrace{\sqrt{2}x \times \sqrt{2}x - \sqrt{2} \times \sqrt{3} \times x}_{} - \underbrace{\sqrt{2} \times \sqrt{3} \times x + \sqrt{3} \times \sqrt{3}}_{} = 0$$

$$\sqrt{2}x\left(\sqrt{2}x - \sqrt{3}\right) - \sqrt{3}\left(\sqrt{2}x - \sqrt{3}\right) = 0$$

$$\left(\sqrt{2}x - \sqrt{3}\right)\left(\sqrt{2}x - \sqrt{3}\right) = 0$$

$$\sqrt{2}x - \sqrt{3} = 0 \ \ or \ \sqrt{2}x - \sqrt{3} = 0$$

$$\sqrt{2}x = \sqrt{3} \ \ or \ \sqrt{2}x = \sqrt{3}$$

$$Therefore\ the\ Solution\ is\ x = \frac{\sqrt{3}}{\sqrt{2}}$$

$Example\ 3.28$: $Solve$: $\mathbf{2m^2 + 19m + 30 = 0}$

$$2m^2 + 19m + 30 = 0$$

$$2m^2 + 4m + 15m + 30 = 0$$

$$2m(m+2) + 15(m+2) = 0$$

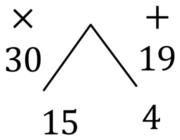
$$(m+2)(2m+15) = 0$$

$$m + 2 = 0\ ,\ 2m + 15 = 0$$

$$m = -2\ ,\ 2m = -15$$

$$m = -\frac{15}{2}$$

$$Therefore\ the\ roots\ are\ -2, -\frac{15}{2}$$



$Example\ 3.29$: $Solve\ \mathbf{x^4 - 13x^2 + 42 = 0}$

$$x^4 - 13x^2 + 42 = 0$$

$$(x^2)^2 - 13x^2 + 42 = 0$$

$$Let\ x^2 = a$$

$$a^2 - 13a + 42 = 0$$

$$(a-7)(a-6) = 0$$

$$(a-7) = 0\ ,(a-6) = 0$$

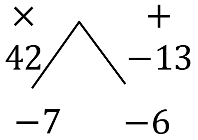
$$a = 7, a = 6$$

$$Since\ a = x^2$$

$$x^2 = 7 \Longrightarrow x = \pm\sqrt{7}$$

$$x^2 = 6 \Longrightarrow x = \pm\sqrt{6}$$

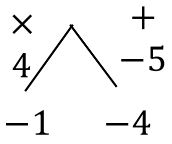
$$Therefore\ the\ roots\ are\ x = \pm\sqrt{7}, \pm\sqrt{6}$$



$Example\ 3.30$: $Solve$: $\mathbf{\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}}$

$$Let\ y = \frac{x}{x-1}\ then \frac{1}{y} = \frac{x-1}{x}$$

$$\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2} \Rightarrow y+\frac{1}{y}=\frac{5}{2} \Rightarrow \frac{y^2+1}{y}=\frac{5}{2}$$



$$2y^2+2=5y \Rightarrow 2y^2-5y+2=0$$

$$2y^2-1y-4y+2=0 \Rightarrow y(2y-1)-2(2y-1)=0$$

$$(2y-1)(y-2)=0 \Rightarrow 2y-1=0\,, y-2=0$$

$$2y=1\,, y=2$$

$$\boxed{y=\frac{1}{2}}$$

$$put\ y=\frac{1}{2}\ in\ y=\frac{x}{x-1}$$

$$\frac{1}{2}=\frac{x}{x-1} \Rightarrow \frac{x}{x-1}=\frac{1}{2} \Rightarrow 2x=x-1 \Rightarrow 2x-x=-1$$

$$\boxed{x=-1}$$

$$put\ y=2\ in\ y=\frac{x}{x-1}$$

$$2=\frac{x}{x-1} \Rightarrow \frac{x}{x-1}=2 \Rightarrow x=2(x-1)$$

$$x=2x-2 \Rightarrow 2x-x=2$$

$$x=2$$

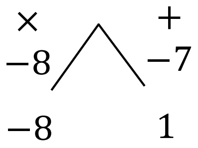
$Therefore\ the\ roots\ are\ x=-1, 2$

1. *Solve the following quadratic equations by factorization method*

$\boldsymbol{i)\ 4x^2-7x-2=0\quad ii) 3(p^2-6)=p(p+5)\quad iii)\sqrt{a(a-7)}=3\sqrt{2}}$

$\boldsymbol{iv)\ \sqrt{2}x^2+7x+5\sqrt{2}=0\quad v)\ 2x^2-x+\frac{1}{8}=0}$

$(i)\ 4x^2-7x-2=0$



$$4x^2-8x+x-2=0 \Rightarrow 4x(x-2)+1(x-2)=0$$

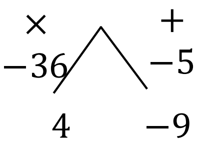
$$(4x+1)(x-2)=0 \Rightarrow 4x+1=0\,, x-2=0$$

$$4x=-1\,, x=2$$

$$x=-\frac{1}{4}$$

$$Roots\ are\ \left\{-\frac{1}{4}, 2\right\}$$

$\boldsymbol{ii)\ 3(p^2-6)=p(p+5)}$



$$3p^2-18=p^2+5p$$

$$3p^2-18-p^2-5p=0 \Rightarrow 2p^2-5p-18=0$$

$$2p^2+4p-9p-18=0 \Rightarrow 2p(p+2)-9(p+2)=0$$

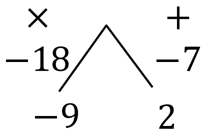
$$(2p-9)(p+2)=0 \Rightarrow 2p-9=0\,, p+2=0$$

$$2p=9, p=-2$$

$$p = \frac{9}{2}$$

$$Roots\ are\ \left\{\frac{9}{2}, -2\right\}$$

$$\boldsymbol{iii)}\ \sqrt{\boldsymbol{a(a-7)}} = \boldsymbol{3\sqrt{2}}$$



$$\sqrt{a(a-7)} = 3\sqrt{2}$$

Squaring on both sides

$$\left(\sqrt{a(a-7)}\right)^2 = \left(3\sqrt{2}\right)^2 \implies a(a-7) = 9 \times 2$$

$$a^2 - 7a = 18 \implies a^2 - 7a - 18 = 0$$

$$(a-9)(a+2) = 0 \implies a - 9 = 0\,, a + 2 = 0$$

$$a = 9\,, a = -2$$

$$Roots\ are\ 9, -2$$

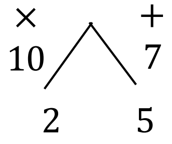
$$\boldsymbol{iv)}\ \boldsymbol{\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0}$$

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0 \implies \underbrace{\sqrt{2}x^2 + \sqrt{2} \times \sqrt{2}x} + \underbrace{5x + 5\sqrt{2}} = 0$$

$$\sqrt{2}x\left(x + \sqrt{2}\right) + 5\left(x + \sqrt{2}\right) = 0 \implies \left(x + \sqrt{2}\right)\left(\sqrt{2}x + 5\right) = 0$$

$$\sqrt{2}x + 5 = 0\,, x + \sqrt{2} = 0$$

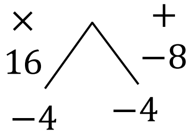


$$\sqrt{2}x = -5\,, x = -\sqrt{2}$$

$$x = -\frac{5}{\sqrt{2}}$$

$$\therefore Roots\ are - \frac{5}{\sqrt{2}}\ , -\sqrt{2}$$

$$\boldsymbol{v)}\ \boldsymbol{2x^2 - x + \frac{1}{8} = 0}$$



$$2x^2 - x + \frac{1}{8} = 0 \qquad \implies 16x^2 - 8x + 1 = 0$$

(multiplying both side by 8*)*

$$16x^2 - 4x - 4x + 8 = 0 \implies 4x(4x-1) - 1(4x-1) = 0$$

$$(4x-1)(4x-1) = 0 \implies 4x - 1 = 0, 4x - 1 = 0$$

$$4x = 1\,, 4x = 1$$

$$x = \frac{1}{4}$$

$$\therefore The\ roots\ is \frac{1}{4}$$

2. *The number of volley ball that must be scheduled in a league by* $\boldsymbol{G(n) = \dfrac{n^2 - n}{2}}$ ***where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?***

$$G(n) = \frac{n^2 - n}{2} = 15$$

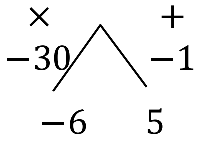
$$n^2 - n = 30 \Longrightarrow \; n^2 - n - 30 = 0$$

$$(n - 6)(n + 5) = 0$$

$$n - 6 = 0, n + 5 = 0$$

$$n = 6, n = -5$$

$\therefore$ *The number of terms in the league* $= 6$



EXERCISE 3. 11

Example 3.31: Solve $x^2 - 3x - 2 = 0$

$$x^2 - 3x - 2 = 0 \Rightarrow x^2 - 3x = 2$$

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 2 + \left(\frac{3}{2}\right)^2 \Rightarrow \left(x - \frac{3}{2}\right)^2 = 2 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{8+9}{4} \Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$$

$$x - \frac{3}{2} = \sqrt{\frac{17}{4}} \Rightarrow x - \frac{3}{2} = \pm\frac{\sqrt{17}}{2}$$

$$x = \frac{3}{2} + \frac{\sqrt{17}}{2} \quad or\ x = \frac{3}{2} - \frac{\sqrt{17}}{2}$$

$$\therefore x = \frac{3+\sqrt{17}}{2}, \frac{3-\sqrt{17}}{2}$$

Example 3.32: Solve $2x^2 - x - 1 = 0$

$$\boxed{\frac{\frac{1}{2}}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}}$$

$$\underset{\div 2}{2x^2 - x - 1 = 0} \Rightarrow x^2 - \frac{x}{2} - \frac{1}{2} = 0$$

$$x^2 - \frac{x}{2} = \frac{1}{2} \Rightarrow x^2 - \frac{x}{2} + \left(\frac{1}{4}\right)^2 = \frac{1}{2} + \left(\frac{1}{4}\right)^2$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{1}{2} + \frac{1}{16} \Rightarrow \left(x - \frac{1}{4}\right)^2 = \frac{8+1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{9}{16} \Rightarrow x - \frac{1}{4} = \sqrt{\frac{9}{16}} \Rightarrow x - \frac{1}{4} = \pm\frac{3}{4}$$

$$x = \frac{1}{4} \pm \frac{3}{4} \Rightarrow x = \frac{1 \pm 3}{4} \Rightarrow x = \frac{1+3}{4}, x = \frac{1-3}{4}$$

$$x = \frac{4}{4}, x = \frac{-2}{4} \Rightarrow x = 1, -\frac{1}{2}$$

Example 3.33: Solve $x^2 + 2x - 2 = 0$ by formula method

$$x^2 + 2x - 2 = 0$$

Compare: $ax^2 + bx + c = 0$

$$a = 1,\ b = 2,\ c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+8}}{2}\;\Rightarrow\;x=\frac{-2\pm\sqrt{12}}{2}\;\Rightarrow\;x=\frac{-2\pm\sqrt{4\times3}}{2}$$

$$x=\frac{-2\pm2\sqrt{3}}{2}\;\Rightarrow\;x=\frac{\cancel{2}(-1\pm\sqrt{3})}{\cancel{2}}\;\Rightarrow\;x=-1\pm\sqrt{3}$$

$$\therefore x=-1+\sqrt{3},-1-\sqrt{3}$$

Example 3.34: Solve $2x^2-3x-3=0$ by formula method

$$2x^2-3x-3=0$$

$Compare: ax^2+bx+c=0$

$$a=2,\;b=-3,c=-3$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\;\Rightarrow x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$$

$$x=\frac{3\pm\sqrt{9+24}}{2}\;\;\Rightarrow\;x=\frac{3\pm\sqrt{33}}{2}$$

$$Therefore,\;x=\frac{3+\sqrt{33}}{4},\frac{3-\sqrt{33}}{4}$$

Example 3.35: Solve $3p^2+2\sqrt{5}-5=0$ by formula method

$$3p^2+2\sqrt{5}-5=0$$

$Compare: ax^2+bx+c=0$

$$a=3,b=2\sqrt{5},\;c=-5$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\;\;\Rightarrow x=\frac{-2\sqrt{5}\pm\sqrt{\left(2\sqrt{5}\right)^2-4(3)(-5)}}{2(3)}$$

$$x=\frac{-2\sqrt{5}\pm\sqrt{(4\times5)+60}}{6}\;\;\Rightarrow x=\frac{-2\sqrt{5}\pm\sqrt{20+60}}{6}$$

$$x=\frac{-2\sqrt{5}\pm\sqrt{80}}{6}\;\;\Rightarrow\;x=\frac{-2\sqrt{5}\pm\sqrt{16\times5}}{6}\;\;\Rightarrow x=\frac{-\cancel{2}\sqrt{5}\pm\overset{2}{\cancel{4}}\sqrt{5}}{\cancel{6}\;3}$$

$$x=\frac{-\sqrt{5}\pm2\sqrt{5}}{3}\Rightarrow x=\frac{-\sqrt{5}+2\sqrt{5}}{3},\frac{-\sqrt{5}-2\sqrt{5}}{3}\;\Rightarrow x=\frac{\sqrt{5}}{3},\frac{-\cancel{3}\sqrt{5}}{\cancel{3}}$$

$$Therefore,\;x=\frac{\sqrt{5}}{3},-\sqrt{5}$$

Example 3.36: Solve $pqx^2-\;(p+q)^2x+(p+q)^2=0$ by formula method

$$pqx^2-\;(p+q)^2x+(p+q)^2=0$$

$Compare: ax^2+bx+c=0$

$$a=pq,b=-(p+q)^2,c=(p+q)^2$$

$$x=-\frac{b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-[-(p+q)^2]\pm\sqrt{[-(p+q)^2]^2-4(pq)(p+q)^2}}{2(pq)}$$

$$x=\frac{(p+q)^2\pm\sqrt{(p+q)^4-4(pq)(p+q)^2}}{2(pq)}$$

$$=\frac{(p+q)^2\pm\sqrt{(p+q)^2\ [(p+q)^2-4pq]}}{2pq}$$

$$x=\frac{(p+q)^2\pm\sqrt{(p+q)^2[p^2+q^2+2pq-4pq]}}{2pq}$$

$$=\frac{(p+q)^2\pm\sqrt{(p+q)^2[p^2+q^2-2pq]}}{2pq}$$

$$x=\frac{(p+q)^2\pm\sqrt{(p+q)^2(p-q)^2}}{2pq}=\frac{(p+q)^2\pm(p+q)(p-q)}{2pq}$$

$$x=\frac{(p+q)^2\pm(p+q)(p-q)}{2pq}\Longrightarrow x=\frac{(p+q)[p+q\pm(p-q)]}{2(pq)}$$

$$x=\frac{(p+q)[p+\cancel{q}+p-\cancel{q}]}{2pq},\frac{(p+q)[\cancel{p}+q-\cancel{p}+q]}{2pq}$$

$$\therefore x=\frac{p+q}{\cancel{2p}q}\times\cancel{2p},\frac{p+q}{\cancel{2}p\cancel{q}}\times\cancel{2q}\Longrightarrow x=\frac{p+q}{q},\frac{p+q}{p}$$

1. *Solve the a following quadratic equations by completing the square method*

i) $9x^2-12x+4=0$

$$\boxed{\frac{\frac{4}{3}}{2}=\frac{4}{3}\times\frac{1}{2}=\frac{2}{3}}$$

The given equation is $9x^2-12x+4=0$

$$9x^2-12x=-4$$

$$\div\ by\ 9$$

$$x^2-\frac{\overset{4}{\cancel{12}}}{{}_{3}\cancel{9}}x=-\frac{4}{9}\Longrightarrow x^2-\frac{4}{3}x=-\frac{4}{9}$$

$$x^2-\frac{4}{3}x+\left(\frac{2}{3}\right)^2=-\frac{4}{9}+\left(\frac{2}{3}\right)^2\Longrightarrow\left(x-\frac{2}{3}\right)^2=-\frac{4}{9}+\frac{4}{9}$$

$$\left(x-\frac{2}{3}\right)^2=0\Longrightarrow\left(x-\frac{2}{3}\right)\left(x-\frac{2}{3}\right)=0\Longrightarrow x=\frac{2}{3},\frac{2}{3}$$

$$\therefore\ Solution\ set\ x=\{{}^{2}/_{3},{}^{2}/_{3}\}$$

$\boldsymbol{ii)\ \dfrac{5x+7}{x-1} = 3x+2}$

$ii)\ Given\ Equation\ is\ \dfrac{5x+7}{x-1} = 3x+2$

$$5x+7 = (3x+2)(x-1) \Longrightarrow\ 5x+7 = 3x^2 - 3x + 2x - 2$$

$$5x+7 = 3x^2 - x - 2 \ \Longrightarrow\ \ 0 = 3x^2 - 5x - 7 - x - 2$$

$$3x^2 - 6x - 9 = 0$$

$$\div 3 \qquad \Longrightarrow\ x^2 - 2x - 3 = 0$$

$$x^2 - 2x = 3$$

$$x^2 - 2x + 1 = 3 + 1$$

$$(x-1)^2 = 4 \ \Longrightarrow x - 1 = \sqrt{4} \ \Longrightarrow x - 1 = \pm 2$$

$$x - 1 = 2,\ x - 1 = -2 \ \Longrightarrow x = 2+1,\ x = -2+1$$

$$x = 3, x = -1$$

$$Solution\ set = \{3, -1\}$$

$\boldsymbol{2.\ Solve\ the\ following\ quadratic\ equations\ by\ formula\ method}$

$\boldsymbol{i)\ 2x^2 - 5x + 2 = 0}$

$i)\ Given\ equation\ is\ 2x^2 - 5x + 2 = 0$

$$Compare: ax^2 + bx + c = 0$$

$$a = 2,\ b = -5, c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ \Longrightarrow\ x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 16}}{4} \ \Longrightarrow\ x = \frac{5 \pm \sqrt{9}}{4}$$

$$x = \frac{5 \pm 3}{4} \ \Longrightarrow\ x = \frac{5+3}{4}, \frac{5-3}{4} \ \Longrightarrow x = \frac{\overset{2}{\cancel{8}}}{\cancel{4}}, \frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}}$$

$$x = 2, \frac{1}{2}$$

$\boldsymbol{ii)\ \sqrt{2}f^2 - 6f + 3\sqrt{2} = 0}$

$Given\ equation\ is\ \sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

$$Compare: ax^2 + bx + c = 0$$

$$a = \sqrt{2}, b = -6, c = 3\sqrt{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ \Longrightarrow\ x = \frac{6 \pm \sqrt{(-6)^2 - 4(\sqrt{2})(3\sqrt{2})}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 - 12(2)}}{6} \ \Longrightarrow\ x = \frac{6 \pm \sqrt{36 - 24}}{2\sqrt{2}} \ \Longrightarrow\ x = \frac{6 \pm \sqrt{12}}{2\sqrt{2}}$$

$$x = \frac{6 \pm \sqrt{4 \times 3}}{2\sqrt{2}} \Rightarrow x = \frac{\overset{3}{\cancel{6}} \pm \cancel{2}\sqrt{3}}{\cancel{2}\sqrt{2}} \Rightarrow x = \frac{3 \pm \sqrt{3}}{\sqrt{2}}$$

$$x = \frac{3 + \sqrt{3}}{\sqrt{2}}, \frac{3 - \sqrt{3}}{\sqrt{2}}$$

$iii)\ 3y^2 - 20y - 23 = 0$

Given equation is $3y^2 - 20y - 23 = 0$

Compare: $ax^2 + bx + c = 0$

$a = 3, b = -20, c = -23$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow y = \frac{20 \pm \sqrt{(-20)^2 - 4(3)(-23)}}{2(3)}$$

$$y = \frac{20 \pm \sqrt{400 - 4(3)(-23)}}{6} = \frac{20 \pm \sqrt{400 + 276}}{6} = \frac{20 \pm \sqrt{676}}{6}$$

$$\begin{array}{r|l} 2 & 676 \\ \hline 2 & 338 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$y = \frac{20 \pm \sqrt{2 \times 2 \times 13 \times 13}}{6} \Rightarrow y = \frac{20 \pm 26}{6}$$

$$y = \frac{20 + 26}{6}, \frac{20 - 26}{6} \Rightarrow y = \frac{46}{6}, -\frac{6}{6}$$

$$y = \frac{23}{3}, -1$$

$iv)\ 36y^2 - 12ay + (a^2 - b^2) = 0$

Given equation is $36y^2 - 12ay + (a^2 - b^2) = 0$

Compare: $Ax^2 + Bx + C = 0$

$A = 36, B = -12a, C = a^2 - b^2$

$$y = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \Rightarrow y = \frac{12a \pm \sqrt{(-12a)^2 - 4(36)(a^2 - b^2)}}{2(36)}$$

$$y = \frac{12a \pm \sqrt{144a^2 - 144(a^2 - b^2)}}{72} = \frac{12a \pm \sqrt{\cancel{144a^2} - \cancel{144a^2} + 144b^2)}}{72}$$

$$y = \frac{12a \pm \sqrt{144b^2}}{72} = \frac{12a \pm 12b}{72} \Rightarrow y = \frac{\cancel{12}(a \pm b)}{\cancel{72}\ 6}$$

$$y = \frac{a \pm b}{6} \Rightarrow y = \frac{a + b}{6}, \frac{a - b}{6}$$

3. A ball roll a down a slope and travels a distance $dt = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet

Let t be the time in seconds when the distance traveled by the ball is 11.25 feet.

$$\text{Distance travelled by the ball} = 11.25 \text{ feet}.$$
$$d = 11.25 \text{ feet}$$

Given : $dt = t^2 - 0.75t$ where $d = 11.25$ feet

$$t^2 - 0.75t = 11.25 \implies t^2 - 0.75t - 11.25 = 0$$

Multiply each side by 100.

$$100t^2 - 75t = 1125 \implies 4t^2 - 3t - 45 = 0$$

Divide both side by 25.

$$\text{Compare}: ax^2 + bx + c = 0$$
$$a = 4,\ b = -3,\ c = -45$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies t = \frac{3 \pm \sqrt{(-3)^2 - 4(4)(-45)}}{2(4)}$$

$$t = \frac{3 \pm \sqrt{9 + (16)(45)}}{8} \implies t = \frac{3 \pm \sqrt{729}}{8} \implies t = \frac{3 \pm 27}{8}$$

$$t = \frac{3+27}{8}, \frac{3-27}{8} \implies t = \frac{30}{8}, \frac{-24}{8} \implies t = \frac{15}{4}, -3$$

$$\therefore t = 3.75, t = -3 \implies \text{But } t \neq -3$$
$$\therefore t = 3.75 \text{ sec}$$

EXERCISE 3.12

Example 3.37: The product of Kumaran's age (in years) two years ago his age four years from now is one more than twice his present age. What is his present age?

Let the present age of Kumaran's be x years

Two years ago, his age $= (x-2)$ years

Four years from now, his age $= (x+4)$ years

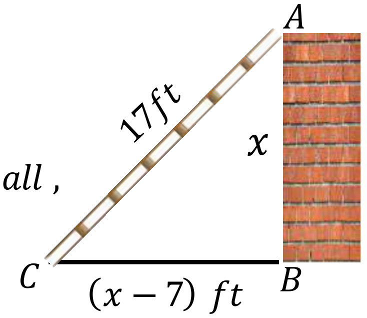
Given: $(x-2)(x+4) = 1 + 2x$

$$x^2 + 4x - 2x - 8 = 1 + 2x \implies x^2 + 2x - 8 = 1 + 2x$$
$$x^2 + \cancel{2x} - 8 - 1 - \cancel{2x} = 0 \implies x^2 - 8 - 1 = 0$$
$$x^2 - 9 = 0 \implies x^2 - 3^2 = 0 \implies (x+3)(x-3) = 0$$
$$x + 3 = 0, x - 3 = 0 \implies x = -3, x = 3$$

$\therefore x = 3$ ($x = -3$ is not possible, since age cannot be negative)

Kumaran's age is 3 years

Example 3.38: A ladder 17 feet long is leaning against a wall. If a ladder, vertical wall and the floor from the bottom of the wall to the ladder from a right angle, find the height of the wall where the top of the ladder meets if the distance between bottom of the ladder is 7 feet less than the height of the wall



Let the height of the wall $AB = x$ *feet*

Lenght of the ladder, $AC = 17ft$

Distance between bottom of the ladder and the wall ,

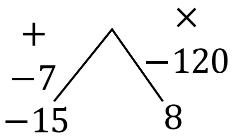
$BC = (x-7)ft$

By pythagoras theoram, $AC^2 = AB^2 + BC^2$

$$(17)^2 = x^2 + (x-7)^2$$

$$289 = x^2 + x^2 - 14x + 49 \Longrightarrow 289 - 49 = 2x^2 - 14x$$

$$\underset{\div 2}{240 = 2x^2 - 14x} \Longrightarrow x^2 - 7x = 120 \Longrightarrow x^2 - 7x - 120 = 0$$



$$hence\ (x-15)(x+8) = 0 \Longrightarrow x = 15, x = -8$$

Therefore Height of the wll $AB = 15\ ft$

(Rejecting -8 *as height cannot be negative value*

Example 3.39: A flock of swans contained x^2 members. As clouds gatehered, 10x went to a lake and one eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total

Given that there are x^2 *swans*

As per the given data: $x^2 - 10x - \frac{1}{8}x^2 = 6$

multiply by 8 on both side

$$8x^2 - 80x - x^2 = 48 \Longrightarrow 7x^2 - 80x - 48 = 0$$

$$a = 7, b = -80, c = -48$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{80 \pm \sqrt{(-80)^2 - 4(7)(-48)}}{14}$$

$$= \frac{80 \pm \sqrt{6400 - 4(7)(-48)}}{14} = \frac{80 \pm \sqrt{6400 + 1344}}{14} = \frac{80 \pm \sqrt{7744}}{14}$$

$$= \frac{80 \pm 88}{14} = \frac{80 + 88}{14}, \frac{80 - 88}{14}$$

$$x = -\frac{8}{14}, \frac{168}{14}$$

Therefor $x = 12, -{}^{4}/_{7}$

Here $x = -{}^{4}/_{7}$ *is not possible as the number of swans cannot be negative*

Hence $x = 12$. *Therefore total number of swans is* $x^2 = 144$

Example 3.40: A passanger train takes 1 hr more than an express train to travel a distance of 240km from chennai to virudhachalam. The speed of train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

Let the average speed of passenger train be x km/hr

Then the average speed of express train will be $(x+20)\,km/hr$

Time Taken by the passanger to cover distance of 240 km : $T_1 = \dfrac{240}{x}\ hr$

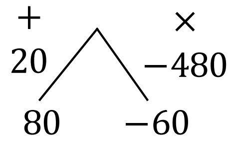
Time taken by express train to cover distance of 240 km : $T_2 = \dfrac{240}{x+20}\ hr$

$$Given:\ T_1 - T_2 = 1$$

$$\frac{240}{x} - \frac{240}{x+20} = 1 \Rightarrow 240\left[\frac{1}{x+20} - \frac{1}{x}\right] = 1 \Rightarrow 240\left[\frac{x+20-x}{x(x+20)}\right] = 1$$

$$240\left[\frac{20}{x^2+20x}\right] = 1 \Rightarrow \frac{480}{x^2+20x} = 1 \Rightarrow 480 = x^2+20x$$

$$x^2+20x = 480 \Rightarrow x^2+20x-480 = 0$$



$$(x+80)(x-60) = 0 \Rightarrow x+80 = 0, x-60 = 0$$

$$x = -80,\ x = 60$$

Not possible

Average speed of the passenger train is $x = 60\ km/hr$

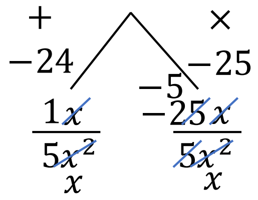
$$\begin{aligned} Average\ speed\ of\ the\ express\ train &= x+20\ km/hr \\ &= 60+20\ km/hr \\ &= 80\ km/hr \end{aligned}$$

1. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number

Let x be the number and its reciprocal is $\dfrac{1}{x}$

Given difference between a number and its reciprocal $= \dfrac{24}{5}$

$$x - \frac{1}{x} = \frac{24}{5} \Rightarrow \frac{x^2-1}{x} = \frac{24}{5} \Rightarrow 5x^2-5 = 24x$$



$$5x^2-24x-5 = 0 \Rightarrow (5x+1)(x-5) = 0$$

$$5x+1 = 0, x-5 = 0 \Rightarrow 5x = -1, x = 5$$

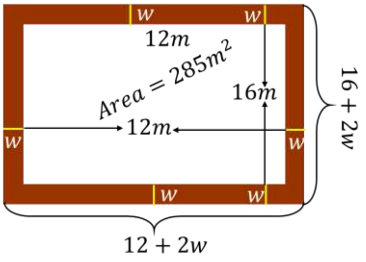
$$x = -\frac{1}{5}, 5$$

$\therefore$ *The required numbers are* $5, -\dfrac{1}{5}$

2. The garden measuring 12m by 16m is to have a pedestrian that is w meters wide installed all the way around so that it increases the total area to $285m^2$, what is the width of the pathway?

Given the dimension of the garden $= 16m \times 12m$

Let w be the equal width of the pedestrian pathway



$$\text{Total Area} = 285m^2$$

$$(16+2w)(12+2w) = 285$$

$$192 + 32w + 24w + 4w^2 = 285$$

$$192 + 56w + 4w^2 - 285 = 0$$

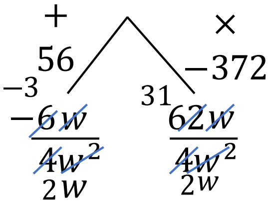
$$4w^2 + 56w - 93 = 0 \Longrightarrow (2w-3)(2w+31) = 0$$

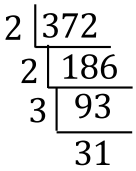
$$2w - 3 = 0,\ 2w + 31 = 0$$

$$2w = 3,\ 2w = -31$$

$$w = \frac{3}{2}, -\frac{31}{2}\ (\textit{w cannot be negative})$$

$$\therefore w = {}^{3}/_{2} = 1.5\ m$$





$\therefore$ *Width of the path away* $= 1.5m$

3. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

Let x km/hr be the original speed of the bus

Distance covered by a Bus $= 90\ km$

"Let T_1 be the time taken to cover the distance of 90km in x speed"

$$T_1 = \frac{90}{x}\ \ldots(1)$$

"Let T_2 be the time taken to cover the distance of 90km in x + 15 speed"

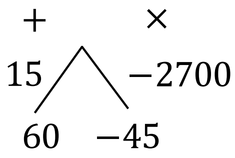
$$T_2 = \frac{90}{x+15}\ \ldots(2)$$

$$T_2 - T_1 = 30mins \Longrightarrow T_1 - T_2 = \frac{1}{2}hr$$

$$\left(\therefore 30\ min = \frac{1}{2}\ hr\right)$$

$$\frac{90}{x+15} - \frac{90}{x} = \frac{1}{2} \Longrightarrow 90\left(\frac{1}{x+15} - \frac{1}{x}\right) = \frac{1}{2}$$

$$90\left(\frac{1}{x} - \frac{1}{x+15}\right) = \frac{1}{2} \Longrightarrow 90\left[\frac{\cancel{x} + 15 - \cancel{x}}{x(x+15)}\right] = \frac{1}{2}$$



$$90\left[\frac{15}{x(x+15)}\right] = \frac{1}{2} \Longrightarrow \frac{1350}{x^2+15x} = \frac{1}{2} \Longrightarrow 1350 \times 2 = x^2 + 15x$$

$x^2 + 15x = 2700 \implies x^2 + 15x - 2700 = 0$

$(x + 60)(x - 45) = 0 \implies x + 60 = 0\,,\ x - 45 = 0$

$x = -60\,,\ x = 45$

Speed cannot be negative

$\therefore$ *The original speed of the car is* 75 *km/hr*.

2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

4. *A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present age*

Let x be the present age of a girl and y be the present age of her sister

Now girl age = twice as old as her sister

$x = 2y \ldots (1)$

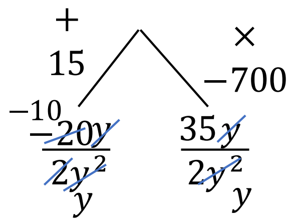
After Five year

Age of a girl $= x + 5$ *Age of her sister* $= y + 5$

The product of their ages $= 375$

$(x + 5)(y + 5) = 375 \ldots (2)$

sub $x = 2y$ *in* $(2) \implies (2y + 5)(y + 5) = 375$



$2y^2 + 10y + 5y + 25 = 375 \implies 2y^2 + 15y + 25 - 375 = 0$

$2y^2 + 15y - 350 = 0 \implies (y - 10)(2y + 35) = 0$

$y - 10 = 0\,,\ 2y + 35 = 0 \implies y = 10\,,\ 2y = -35$

$y = -\frac{35}{2}$ $(y\ can't\ be\ -ve)$

sub $y = 10$ *in* $(1)\ x = 2y$

$x = 2(10) \implies x = 20$

$\therefore$ *Their present ages are* 20, 10 *years old*.

$\therefore$ *Their present age of a girl* : $x = 20$

The present age of her sister : $y = 10$

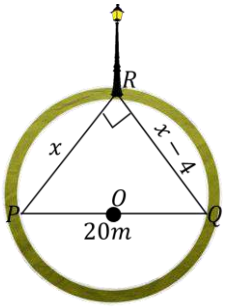
5. *A Pole has to be errected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4m. is it possible to do so? If answer is yes at what distance from the two gates should the pole errected*

Diameter of the circle $PQ = 20\ m$

A Pole has to be errected at a point R

Let $PR = x$

The difference of its distances from two diametrically opposite fixed gates P and Q on the boundary = $4m$



$PR - RQ = 4m \implies PR - RQ = 4m$

$x - RQ = 4 \implies x - 4 = RQ$

$RQ = x - 4$

$PQ^2 = PR^2 + RQ^2 \implies 20^2 = x^2 + (x-4)^2$

$400 = x^2 + x^2 - 2(x)(4) + 4^2 \implies 2x^2 - 8x + 16 = 400$

$2x^2 - 8x + 16 - 400 = 0 \implies 2x^2 - 8x + 384 = 0$

$2x^2 - 8x + 384 = 0$
$\div 2$ $\implies x^2 - 4x - 192 = 0$

$$\begin{array}{cc} + & \times \\ -4 & -192 \\ -16 & 12 \end{array}$$

$(x-16)(x+12) = 0 \implies x - 16 = 0\,,\, x + 12 = 0$

$x = 16,\ x = -12 \quad (x\ can't\ be\ -ve)$

$\therefore x = 16\ m \implies PR = 16m$

$RQ = x - 4 \implies RQ = 16 - 4$

$RQ = 12m$

$\therefore$ *Pole should be erected at a distance of* $16m, 12m$ *from the two gates*

6. *From a group of* $\boldsymbol{2x^2}$ ***black bees, square root of half of the group went to a tree. Again eight − ninth of the bees went to the same tree. the remaining two caught up in a fragrant lotus. How many bees were there in the total?***

Group of black bees $= 2x^2$

Square root of half of the group $= \sqrt{\frac{1}{2}(2x^2)} = \sqrt{x^2} = x$

Eight − ninth of the bees in the group $= \frac{8}{9}(2x^2) = \frac{16x^2}{9}$

Remaining number of bees $= 2$

$2x^2 = x + \frac{16x^2}{9} + 2 \implies 18x^2 = 9x + 16x^2 + 18$

$\times$ *by* 9

$18x^2 - 9x - 16x^2 - 18 = 0 \implies 2x^2 - 9x - 18 = 0$

$$\begin{array}{cc} + & \times \\ -9 & -36 \\ \dfrac{\overset{-6}{-\cancel{12}\cancel{x}}}{\underset{x}{\cancel{2}\cancel{x^2}}} & \dfrac{3\cancel{x}}{\underset{x}{2\cancel{x^2}}} \end{array}$$

$(x-6)(2x+3) = 0 \implies x - 6 = 0\,, 2x + 3 = 0 \implies x = 6\,, 2x = -3$

$x = -\frac{3}{2}\ (x\ can't\ be\ -ve)$

$\therefore$ *The total number of bees* $= 2x^2$ *, where* $x = 6$

$= 2(6^2) = 2(36)$

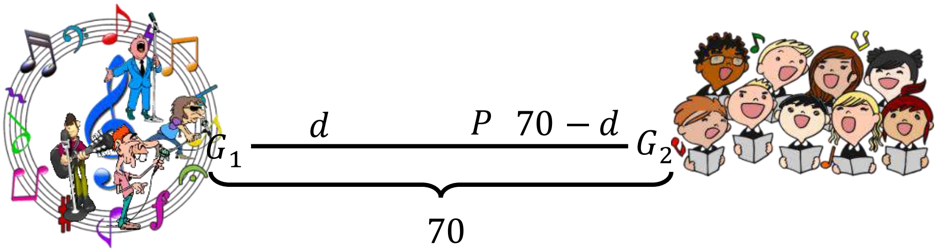
$= 72$

7. Music is been played in two opposite galleries with certain group of pepole. In the first gallery a group of 4 singers were singing in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70m. Where should a person stand for hearing the same intensity of the singers voice(hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).

let "d" is the distance from gallery of 4 singers

The distance from gallery of 9 singers $= 70 - d$

$\therefore\ G_1G_2 = 70\text{m}, G_1P = dm, G_2P = (70-d)m$



The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances

$$4:9 = d^2:(70-d)^2 \implies \frac{4}{9} = \frac{d^2}{(70-d)^2} \implies 4(70-d)^2 = 9d^2$$

$$19600 - 560d + 4d^2 = 9d^2 \implies 0 = 9d^2 - 4d^2 + 560d - 19600$$

$$5d^2 + 560d - 19600 = 0 \implies d^2 + 112d - 3920 = 0$$

$\div 5$

$+$ $\times$

112 -3920

140 28

$$d^2 + 112d - 3920 = 0 \implies (d+140)(d-28) = 0$$

2	3920
2	1960
2	980
2	490
5	245
7	49
7	7
	1

$d + 140 = 0\ ,\ d - 28 = 0$

$d = -140,\quad d = 28$

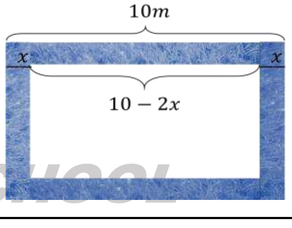
$\therefore d = 28\ m$

The person should stand 28m from Gallery 1

The distance of a person from gallery $2 = 70 - 28$

$= 42$

9. There is square field whose side is 10m. A square flower bed is prepared in its centre leaving a gravel path all around the flower bed. The total cost of laying the flower bed and gravelling the path a ₹3 and ₹4 per square metre respectively is ₹364. Find the width of the gravel path



Let x be the width of the gravel

Side of a square field $= 10m$

Area of square field $= 10^2 = 100m^2$

$Side\ of\ a\ flower\ bed = 10 - 2x$

$Area\ of\ flower\ bed = (10 - 2x)2$

$= 10^2 - 2(10)(2x) + (2x)^2$

$Area\ of\ flower\ bed = 100 - 40x + 4x^2$

$Area\ of\ gravel\ path = Area\ of\ a\ square\ field\ - Area\ of\ flower\ bed$

$Area\ of\ gravel\ path$

$= 100 - (100 - 40x + 4x^2) = 1\cancel{0}0 - 1\cancel{0}0 - 4x^2 + 40x$

$Area\ of\ gravel\ path = 40x - 4x^2$

$The\ cost\ of\ laying\ the\ flower\ bed\ ₹3/sq.m = 3(100 - 40x + 4x^2)$

$The\ cost\ of\ laying\ the\ flower\ bed = 300 - 120x + 12x^2$

$The\ cost\ of\ laying\ the\ gravel\ path\ ₹4/sq.m = 4(40x - 4x^2)$

$The\ cost\ of\ laying\ the\ gravel = 160x - 16x^2$

$The\ total\ cost\ of\ laying\ the\ flower\ bed\ and\ gravelling\ the\ path = ₹364$

$$300 - 120x + 12x^2 + 160x - 16x^2 = 364$$

$$300 - 4x^2 + 40x - 16x^2 = 364$$

$$300 - 4x^2 + 40x - 364 = 0 \Longrightarrow -4x^2 + 40x - 64 = 0$$

$\div -4$

$$x^2 - 10x + 16 = 0 \Longrightarrow (x - 8)(x - 2) = 0$$

$+ \qquad \times$

$-10 \qquad 16$

$-8 \qquad -2$

$$x - 8 = 0\ ,\ x - 2 = 0$$

$$x = 8,\ \ x = 2$$

$\therefore width\ of\ the\ path = 2m$

9. *Two women together took 100 eggs to a marker, one had more than the other. Both sold them for the same sum of the money. The first then said to the second: " If I had your eggs, I would have earned ₹15, to which the second replied: If I had your eggs, i would have earned ₹$6\frac{2}{3}$ How many eggs did each had in the beginning?*

$Let\ x\ be\ the\ number\ of\ eggs\ of\ 1^{st}\ women\ and$

$Let\ y\ be\ the\ number\ of\ eggs\ of\ 2^{nd}\ women$

$$x + y = 100 \Longrightarrow y = 100 - x$$

$Let\ p\ and\ q\ be\ the\ cost\ price\ of\ each\ egg\ for\ the\ 1^{st}\ and\ 2^{nd}\ women.$

Given : $Both\ sold\ them\ for\ the\ same\ sum\ of\ money$

$$px = qy \implies px = q(100 - x)$$

$$\frac{p}{q} = \frac{100 - x}{x} \quad \dots (1)$$

$The\ first\ then\ said\ to\ the\ second:$ If I had your eggs, I would have earned ₹15"

$$py = ₹15 \implies p(100 - x) = 15 \dots (2)$$

Second replied: If I had your eggs, i would have earned ₹$6\frac{2}{3}$

$$qx = 6\frac{2}{3} \Rightarrow qx = \frac{20}{3} \quad \ldots (3)$$

$$Divide\ (2)\ \&\ (3)$$

$$\frac{p(100-x)}{qx} = \frac{15}{\frac{20}{3}} \Rightarrow \frac{p(100-x)}{qx} = \overset{3}{\cancel{15}} \times \frac{3}{\underset{4}{\cancel{20}}}$$

$$\frac{p(100-x)}{qx} = \frac{9}{4} \Rightarrow \frac{p}{q} = \frac{9x}{4(100-x)} \quad where\ \frac{p}{q} = \frac{100-x}{x}$$

$$\frac{100-x}{x} = \frac{9x}{4(100-x)} \Rightarrow 4(100-x)^2 = 9x^2$$

$$4(x^2 - 2 \times 100x + 100^2) = 9x^2 \Rightarrow 4(x^2 - 200x + 10000) = 9x^2$$

$$4x^2 - 800x + 40000 = 9x^2 \Rightarrow 4x^2 - 800x + 40000 - 9x^2 = 0$$

$$-5x^2 - 800x + 40000 = 0 \Rightarrow 5x^2 + 800x - 40000 = 0$$

$$\div 5$$

$$x^2 + 160x - 8000 = 0$$

$$+ \quad \times$$

$$160 \quad -8000$$

$$-200 \quad 40$$

$$(x+200)(x-40) = 0 \Rightarrow x + 200 = 0, x - 40 = 0$$

$$x = -200, \quad x = 40$$

$\therefore 1^{st}$ *Women had number of eggs* $x = 40$ *and*

2^{nd} *women had number of eggs* $: y = 100 - x$

$$= 100 - 40 = 60$$

10. *The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm. Find the length of the smallest side.*

Length of hypotenuse side $= 25\ cm$

Perimeter of the triangle $= 56cm$

$$AB + BC + AC = 56 \Rightarrow 25 + x + AC = 56$$

let us take $BC = x$

$$AC = 56 - 25 - x \Rightarrow AC = 31 - x$$

$\therefore$ *In* $\Delta ABC,\ AC^2 + BC^2 = 25^2$

$$(31-x)^2 + x^2 = 625$$

$$31^2 - 2(31)(x) + x^2 + x^2 = 625$$

$$961 - 62x + 2x^2 = 625 \Rightarrow 961 - 62x + 2x^2 - 625 = 0$$

$$2x^2 - 62x + 336 = 0$$

$$\div 2$$

$$\Rightarrow x^2 - 31x + 168 = 0$$

$$+ \quad \times$$

$$-31 \quad 168$$

$$-24 \quad -7$$

$$(x-24)(x-7) = 0 \Rightarrow x - 24 = 0, \quad x - 7 = 0$$

$$x = 24, \quad x = 7$$

$BC = x = 7cm$

$AC = 31 - x$
$= 31 - 7 = 24$

$\therefore The\ sides\ of\ the\ \Delta\ are\ 7cm, 24cm, 25cm$

$Length\ of\ the\ smallest\ side = 7cm$

Exercise 3.13

Example 3.41*: *Determine the nature of roots for the following quadratic equations

$\boldsymbol{i)\ x^2 - x - 20 = 0 \quad ii)\ 9x^2 - 24x + 16 = 0 \quad iii)\ 2x^2 - 2x + 9 = 0}$

$\boldsymbol{i)\ x^2 - x - 20 = 0}$

$Here\ a = 1, b = -1, c = -20$

$Now\ \Delta = b^2 - 4ac$

$\Delta = (-1)^2 - 4(1)(-20) = 1 + 80 = 81$

$Here\ \Delta = 81 > 0.\ So, the\ equation\ will\ have\ real\ and\ unequal\ roots$

$\boldsymbol{ii)\ 9x^2 - 24x + 16 = 0}$

$Here\ a = 9, b = -24, c = 16$

$Now\ \Delta = b^2 - 4ac$

$\Delta = (-24)^2 - 4(9)(16) = 576 - 576 = 0$

$Here\ \Delta = 0. So, the\ equation\ will\ have\ real\ and\ equal\ roots$

$\boldsymbol{iii)\ 2x^2 - 2x + 9 = 0}$

$Here\ a = 2, b = -2, c = 9$

$\Delta = b^2 - 4ac$

$\Delta = (-2)^2 - 4(2)(9) = 4 - 72 = -68$

$Here\ \Delta = -68 < 0.\ So, the\ equation\ will\ have\ no\ real\ roots$

Example 3.41: **(*i*)*Find the values of* k *for which the quadratic equation* $\mathbf{kx^2 - (8k + 4) + 81 = 0}$ *has real and equal roots***
(*ii*)*Find the values of* ksuch *that the quadratic equation* $\mathbf{(k + 9)x^2 + (k + 1)x + 1 = 0}$ *has no real roots*

$kx^2 - (8k + 4) + 81 = 0$

$Since\ the\ equation\ has\ real\ and\ equal\ roots\ \Delta = 0.$

$b^2 - 4ac = 0$

$Here\ a = k, b = -(8k + 4), c = 81$

$(8k + 4)^2 - 4(k)(81) = 0 \implies (8k)^2 + 2(8k)(4) + 4^2 - 324k = 0$

$64k^2 + 64k + 16 - 324k = 0$

$64k^2 - 260k + 16 = 0$

$\div 4 \qquad \implies 16k^2 - 65k + 4 = 0$

$(16k - 1)(k - 4) = 0 \implies 16k - 1 = 0, k - 4 = 0$

$+ \qquad \times$

$-65 \qquad 64$

-4

$\frac{-\cancel{64}\cancel{k}}{\cancel{16}\cancel{k^2}} \qquad \frac{-1\cancel{k}}{16\cancel{k^2}}$

$k \qquad k$

$$16k = 1, k = 4 \Rightarrow k = \frac{1}{16} \text{ or } k = 4$$

$\boldsymbol{ii)\ (k+9)x^2 + (k+1)x + 1 = 0}$

$Since\ the\ equation\ has\ no\ real\ roots, \Delta < 0$

$$b^2 - 4ac < 0$$

$Here\ a = k + 9, b = k + 1, c = 1$

$$(k+1)^2 - 4(k+9)(1) < 0$$

$$k^2 + 2k + 1 - 4k - 36 < 0$$

$$k^2 - 2k - 35 < 0$$

$$(k+5)(k-7) < 0$$

$\times \quad +$

$-35 \quad -2$

$-7 \quad 5$

$Therefore\ -5 < k < 7$

$\{If\ \alpha < \beta\ and\ (x-\alpha)(x-\beta) < 0\ then, \alpha < x < \beta\}$

$\boldsymbol{Example\ 3.43: Prove\ that\ the\ equation}$
$\boldsymbol{x^2(p^2+q^2) + 2x(pr+qs) + r^2 + s^2 = 0\ has\ no\ real\ roots. If\ ps = qr,}$
$\boldsymbol{then\ show\ that\ the\ roots\ are\ real\ and\ equal.}$

$The\ Given\ quadratic\ equation\ is\ x^2(p^2+q^2) + 2x(pr+qs) + r^2 + s^2 = 0$

$Here\ a = p^2 + q^2, b = 2(pr+qs), c = r^2 + s^2$

$$\Delta = b^2 - 4ac$$

$$= [2(pr+qs)]^2 - 4(p^2+q^2)(r^2+s^2)$$

$$= 4(pr+qs)^2 - 4(p^2+q^2)(r^2+s^2)$$

$$= 4[(pr+qs)^2 - (p^2+q^2)(r^2+s^2)]$$

$$= 4[(pr+qs)^2 - (p^2r^2 + p^2s^2 + q^2r^2 + q^2s^2)]$$

$$= 4[\cancel{p^2r^2} + 2pqrs + \cancel{q^2s^2} - \cancel{p^2r^2} - p^2s^2 - q^2r^2 - \cancel{q^2s^2}]$$

$$= 4[-p^2s^2 - q^2r^2 + 2pqrs] = -4[p^2s^2 + q^2r^2 - 2pqrs]$$

$$= -4[(ps-qr)]^2 < 0 \ldots (1)$$

$Since\ \Delta = b^2 - 4ac < 0, the\ roots\ are\ not\ equal$

$If\ ps = qr\ then\ \Delta = -4[(ps-qr)]^2$

$$\Delta = -4[qr - qr]^2 = 0$$

$Thus\ \Delta = 0\ if\ ps = qr\ and\ so\ the\ roots\ will\ be\ real\ and\ equal$

$\boldsymbol{1)\ Determine\ the\ nature\ of\ the\ roots\ for\ the\ following\ quadratic}$
$\boldsymbol{equations}$

i) $\mathbf{15x^2 + 11x + 2 = 0}$ **ii)** $\mathbf{x^2 - x + 1 = 0}$ **iii)** $\mathbf{\sqrt{2}t^2 - 3t + 3\sqrt{2}}$

iv) $\mathbf{9y^2 - 6\sqrt{2}y + 2 = 0}$ **v)** $\mathbf{9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0}$

i) $\mathbf{15x^2 + 11x + 2 = 0}$

$$a = 15, b = 11, c = 2$$

$\therefore \Delta = b^2 - 4ac = 121 - 4 \times 15 \times 2 = 121 - 120 = 1 > 0$

$\therefore$ *The equations will have real and unequal roots*

$\mathbf{ii)}\ \mathbf{x^2 - x + 1 = 0}$

$a = 1\ , b = -1\ , c = 1$

$\therefore \Delta = b^2 - 4ac$

$= 1 - 4(1)(1) \quad = 1 - 4 = -3 < 0$

$\therefore$ *The equations will have no real roots*

$\boldsymbol{iii)}\ \boldsymbol{\sqrt{2}t^2 - 3t + 3\sqrt{2}}$

$a = \sqrt{2}\ , b = -3, c = 3\sqrt{2}$

$\therefore \Delta = b^2 - 4ac\ = 3^2 - 4(\sqrt{2})(3\sqrt{2})$

$= 9 - 4(\sqrt{2})(3\sqrt{2}) = 9 - 12(2)$

$= 9 - 24\ = -15 < 0$

$\therefore$ *The equations will have no real roots*

$\boldsymbol{iv)}\ \boldsymbol{9y^2 - 6\sqrt{2}y + 2 = 0}$

$a = 9\ , b = -6\sqrt{2}\ , c = 2$

$\therefore \Delta = b^2 - 4ac$

$= (-6\sqrt{2})^2 - 4(9)(2) = 36 \times 2 - 72 = 0$

$\therefore$ *The roots are real and equal*

$\boldsymbol{v)}\ \boldsymbol{9a^2b^2x^2 - 24abcdx + 16c^2d^2\ = 0, a \neq 0, b \neq\ 0}$

$a\ = 9a^2b^2\ , b\ = 24abcd\ , c\ = 16c^2d^2$

$\therefore \Delta = b^2 - 4ac$

$= (24abcd)^2 - 4(9a^2b^2)(16c^2d^2)$

$= 576a^2b^2c^2d^2 - 4(9a^2b^2)(16c^2d^2)$

$= 576a^2b^2c^2d^2 - 576a^2b^2c^2d^2 = 0$

$\therefore$ *The roots are real and equal*

2) *Find the value(s) of* k*for which the roots of the following equations are real and equal*

$\boldsymbol{i)}\ \boldsymbol{(5k - 6)x^2 + 2kx + 1 = 0}\ \boldsymbol{ii)}\ \boldsymbol{kx^2 + (6k + 2)x + 16 = 0}$

i) Given equations $(5k - 6)x^2 + 2kx + 1\ = 0$ *are real and equal*

$a\ = 5k - 6, b\ = 2k\ , c\ = 1$

$\therefore \Delta = b^2 - 4ac\ = 0$

$(2k)^2 - 4(5k - 6)(1) = 0$

$4k^2 - 4(5k - 6)\ = 0 \Longrightarrow 4k^2 - 20k + 24 = 0$

$k^2 - 5k + 6 = 0\ \Longrightarrow (k - 3)(k - 2)\ = 0$

$\times \quad +$
$6 \quad -5$
$-3 \quad -2$

$k - 3\ = 0,\ k - 2\ = 0$

$k\ = 3,\ k\ = 4$

$\therefore k = 3, 2$

ii) Given the roots of $kx^2+(6k+2)x+16=0$ are real and equal

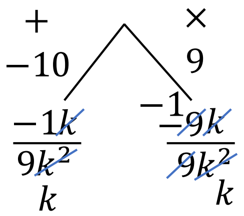
$$a=k\,,b=6k+2\,,c=16$$

$$\therefore \Delta=b^2-4ac=0$$

$$(6k+2)^2-4(k)(16)=0\Longrightarrow 36k^2+2\times 6k\times 2+4-64k=0$$

$$36k^2+24k+4-64k=0\Longrightarrow 36k^2-40k+4=0$$

$$\div 4$$



$$9k^2-10k+1=0\Longrightarrow (9k-1)(k-1)=0$$

$$9k-1=0,k-1=0\Longrightarrow 9k=1,k=1$$

$$k=\frac{1}{9}\ or\ k=1\ \Longrightarrow\ \therefore k=1,{}^{1}/_{9}$$

3) If the roots of $(a-b)x^2+(b-c)x+(c-a)=0$ are real and equal, then prove that b,a,c are in arithmetic progression.

Given equation is $(a-b)x^2+(b-c)x+(c-a)=0$ *are real and equal*

To prove b,a,c *are in A.P*

Here $A=a-b,B=b-c,C=c-a$

$$\Delta=B^2-4AC=0$$

$$(b-c)^2-4(a-b)(c-a)=0$$

$$(b^2+c^2-2bc)-4(ac-bc-a^2+ab)=0$$

$$b^2+c^2-2bc-4ac+4bc+4a^2-4ab=0$$

$$4a^2+b^2+c^2-4ab+2bc-4ac=0$$

$$(2a)^2+b^2+c^2-2\times(2a)\times b+2\times bc-2\times(2a)\times c=0$$

$$(2a-b-c)^2=0\ \Longrightarrow 2a-b-c=0\ \Longrightarrow\ 2a=b+c$$

$$a+a=b+c\ \Longrightarrow\ \underset{t_2-t_1}{a-b}=\underset{t_3-t_2}{c-a}$$

$$\therefore b,a,c \text{ are in } A.P$$

6) If a,b are real then show that the roots of the equation $(a-b)x^2-6(a+b)x-\ 9(a-b)=0$ are real and unequal

To prove the roots of $(a-b)x^2-6(a+b)x-\ 9(a-b)=0$ *are real and unequal*

Here $A=a-b,B=-6(a+b),c=-9(a-b)$

$$\Delta=B^2-4AC \qquad \boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$$

$$=[-6(a+b)]^2-\ 4(a-b)\times-9(a-b)$$

$$=36(a+b)^2+36(a-b)^2=36[(a+b)^2+(a-b)^2]$$

$$=36[2(a^2+b^2)]=72(a^2+b^2)>0$$

$\therefore$ *The roots of the equation are real and unequal.*

5) $\boldsymbol{If\ the\ roots\ of\ the\ equation\ (c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0\ are\ real\ and\ equal\ prove\ that\ either\ a = 0\ (or)\ a^3 + b^3 + c^3 = 3abc}$

$Given\ roots\ of\ the\ equation (c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0\ are\ real\ \&\ equal$

$To\ prove\ that\ either\ a = 0 (or) a^3 + b^3 + c^3 = 3abc$

$Here\ A = c^2 - ab, B = -2(a^2 - bc), C = b^2 - ac$

$Roots\ are\ real\ \&\ equal\ , then\ \Delta = B^2 - 4AC = 0$

$$[-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$4(a^4 - 2a^2bc + b^2c^2) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$$

$$4a^4 - 8a^2bc + 4b^2\cancel{c^2} - \cancel{4b^2}c^2 + 4ab^3 + 4ac^3 - 4a^2bc = 0$$

$$4a^4 - 12a^2bc + 4ab^3 + 4ac^3 = 0$$

$$4a(a^3 - 3abc + b^3 + c^3) = 0$$

$$4a = 0 \quad or \quad (a^3 - 3abc + b^3 + c^3) = 0$$

Hence proved $a = 0 \quad or \quad a^3 + b^3 + c^3 = 3abc$

EXERCISE 3.14

Example 3.44: If the difference between the root of the equation $x^2 - 13x + k$ is 17 find k.

Let α, β be the root of the equation.

Given difference between the root $= 17$

$$\alpha - \beta = 17 \ldots (1)$$

$$x^2 - 13x + k = 0$$

Here, $a = 1, b = -13, c = k$

Sum of the roots: $\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13$

$$\alpha + \beta = 13 \ldots (2)$$

Solve (1) & (2)

$$\alpha - \cancel{\beta} = 17$$

$$\alpha + \cancel{\beta} = 13$$

$$2\alpha = 30 \implies \alpha = \frac{30}{2}$$

$$\alpha = 15$$

sub $\alpha = 15$ in (2) $\alpha + \beta = 13$

$$15 + \beta = 13 \implies \beta = 13 - 15$$

$$\beta = -2$$

Product of the roots : $\alpha\beta = \frac{c}{a} = \frac{k}{1}$

$$\alpha\beta = k \implies 15 \times (-2) = k$$

$$\boxed{k = -30}$$

Example 3.45: If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of

i) $(\alpha - \beta)$ ii) $\alpha^2 + \beta^2$ iii) $\alpha^3 - \beta^3$ iv) $\alpha^4 + \beta^4$ v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Let α, β be the root of the equation $x^2 + 7x + 10 = 0$.

Here, $a = 1, b = 7, c = 10$

Sum of the roots : $\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7$

$$\boxed{\alpha + \beta = -7}$$

product of the roots : $\alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$

$$\boxed{\alpha\beta = 10}$$

$i)\ \alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$=\sqrt{(-7)^2-4\times10}=\sqrt{49-40}$

$=\sqrt{9}\ =3$

$\alpha-\beta=3$

$$\boxed{\begin{aligned}(\alpha-\beta)^2&=(\alpha+\beta)^2-4\alpha\beta\\ \alpha-\beta\ &=\sqrt{(\alpha+\beta)^2-4\alpha\beta}\end{aligned}}$$

$ii)\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-7)^2-2\times10=49-20$

$\alpha^2+\beta^2\ =29$

$$\boxed{\begin{aligned}(\alpha+\beta)^2&=\alpha^2+\beta^2\ +2\alpha\beta\\ (\alpha+\beta)^2-2\alpha\beta&=\alpha^2+\beta^2\\ \alpha^2+\beta^2\ &=(\alpha+\beta)^2-2\alpha\beta\end{aligned}}$$

$iii)\ \alpha^3-\beta^3\ =(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$

$=(3)^3\ +3(10)(3)=27+90$

$\alpha^3+\beta^3\ =117$

$$\boxed{\begin{aligned}(\alpha-\beta)^3\ &=\alpha^3-\beta^3-3\alpha\beta(\alpha-\beta)\\ (\alpha-\beta)^3\ +3\alpha\beta(\alpha-\beta)&=\alpha^3-\beta^3\\ \alpha^3-\beta^3\ &=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)\end{aligned}}$$

$iv)\ \alpha^4+\beta^4$

$=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2\ =29^2-2\times(10)^2$

$=641\ \ (where\ \alpha^2+\beta^2=29)$

$\alpha^4+\beta^4\ =641$

$v)\ \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$ $\boxed{where\ \alpha+\beta=-7\ and\alpha\beta=10}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}\ =\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{(-7)^2-2(10)}{10}=\frac{49-20}{10}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{29}{10}$$

$$\boxed{\begin{aligned}(\alpha+\beta)^3&=\alpha^3+\beta^3\ +3\alpha\beta(\alpha+\beta)\\ (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)&=\alpha^3+\beta^3\\ \alpha^3+\beta^3\ &=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\end{aligned}}$$

$vi)\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}\ =\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{(-343)-3\big(10\times(-7)\big)}{10}=\frac{-343+210}{10}=\frac{-133}{10}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{-133}{10}$$

Example 3.46: If α,β are the roots of the equation $3x^2+7x-2=0$, find the values of $i)\ \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}\ \ ii)\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

Let α,β be the root of the equation $3x^2+7x-2=0$.

Here, $a=3,\ b=7,\ c=-2$

Sum of the roots : $\alpha+\beta=\dfrac{-b}{a}=\dfrac{-7}{3}$

$$\boxed{\alpha+\beta=\frac{-7}{3}}$$

$$product\ of\ the\ roots: \ \alpha\beta=\frac{c}{a}=\frac{-2}{3}$$

$$\boxed{\alpha\beta=\frac{-2}{3}}$$

$$i)\ \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^2-2\left(-\frac{2}{3}\right)}{-\frac{2}{3}}=\frac{\frac{49}{9}+\frac{4}{3}}{-\frac{2}{3}}=\frac{\frac{49+12}{9}}{-\frac{2}{3}}$$

$$=\frac{\frac{61}{9}}{-\frac{2}{3}}=\frac{61}{\cancel{9}_{3}}\times\frac{\cancel{-3}}{2}=\frac{-61}{6}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-61}{6}$$

$$ii)\ \frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$$

$$=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^3-3\left(-\frac{2}{3}\right)\left(-\frac{7}{3}\right)}{-\frac{7}{3}}$$

$$=-\cancel{\frac{7}{3}}\left[\frac{\left(-\frac{7}{3}\right)^2-3\left(-\frac{2}{3}\right)}{-\cancel{\frac{7}{3}}}\right]=\left[\left(-\frac{7}{3}\right)^2+\cancel{3}\left(\frac{2}{\cancel{3}}\right)\right]=\left[\frac{49}{9}+2\right]=\left[\frac{49+18}{9}\right]=\frac{67}{9}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{67}{9}$$

Example 3.47: *If α,β are the roots of the equation $2x^2-x-1=0$, then form the equation whose roots are*

i) $\boldsymbol{\frac{1}{\alpha},\frac{1}{\beta}}$ ***ii)*** $\boldsymbol{\alpha^2\beta,\beta^2\alpha}$ ***iii)*** $\boldsymbol{2\alpha+\beta,2\beta+\alpha}$

Let α,β *be the root of the equation* $2x^2-x-1=0$

$$Here,\ a=2,\ b=-1, c=-1$$

$$Sum\ of\ the\ roots: \alpha+\beta=\frac{-b}{a}=\frac{-(-1)}{2}=\frac{1}{2}$$

$$\alpha+\beta=\frac{1}{2}$$

$$product\ of\ the\ roots: \ \alpha\beta=\frac{c}{a}=-\frac{1}{2}$$

$$\alpha\beta=-\frac{1}{2}$$

$i)$ *Given roots are* $\frac{1}{\alpha},\frac{1}{\beta}$

$$Sum\ of\ the\ roots = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{1}{2}}{-\frac{1}{2}} = \frac{1}{2}\times-\frac{2}{1} = -1$$

$$\therefore Sum\ of\ the\ roots = -1$$

$$Product\ of\ the\ roots = \frac{1}{\alpha}\times\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-\frac{1}{2}}$$

$$= 1\times\left(-\frac{2}{1}\right) = -2$$

$$Product\ of\ the\ roots = -2$$

The required equation is $x^2-(sum\ of\ the\ roots)x$
$+(product\ of\ the\ roots) = 0$

$$x^2-(-1)x-2 = 0$$

$$x^2+x-2 = 0$$

$\boldsymbol{ii)}$ ***Given roots are*** $\boldsymbol{\alpha^2\beta,\beta^2\alpha}$

$$Sum\ of\ the\ roots = \alpha^2\beta+\beta^2\alpha$$

$$= \alpha\beta(\alpha+\beta) = -\frac{1}{2}\left(\frac{1}{2}\right)$$

$$Sum\ of\ the\ roots = -\frac{1}{4}$$

$$Product\ of\ the\ roots = (\alpha^2\beta)\times(\beta^2\alpha)$$

$$= \alpha^3\beta^3 = (\alpha\beta)^3 = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$$

The required equation is $x^2-(sum\ of\ the\ roots)x$
$+(Product\ of\ the\ roots) = 0$

$$x^2-\left(-\frac{1}{4}\right)x-\frac{1}{8} = 0 \Rightarrow x^2+\frac{1}{4}x-\frac{1}{8} = 0$$

multipying by 8 on both side

$$8x^2+2x-1 = 0$$

$\boldsymbol{iii)}$ $\boldsymbol{2\alpha+\beta, 2\beta+\alpha}$

$$Sum\ of\ the\ roots = 2\alpha+\beta+2\beta+\alpha = 3\alpha+3\beta$$

$$= 3(\alpha+\beta) = 3\left(\frac{1}{2}\right)$$

$$Sum\ of\ the\ roots = \frac{3}{2}$$

$$Product\ of\ the\ roots = (2\alpha+\beta)(2\beta+\alpha)$$

$$= 4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta = 5\alpha\beta+2(\alpha^2+\beta^2)$$

$$= 5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]$$

$$= 5\left(-\frac{1}{2}\right) + 2\left[\left(\frac{1}{2}\right)^2 - \cancel{2} \times \left(-\frac{1}{\cancel{2}}\right)\right] = -\frac{5}{2} + 2\left[\frac{1}{4} + 1\right]$$

$$= -\frac{5}{2} + \cancel{2}\left[\frac{1+4}{{}_2\cancel{4}}\right] = -\frac{5}{2} + \frac{5}{2} = 0$$

The required equation is $x^2 - (Sum\ of\ the\ roots)x$
$+(product\ of\ the\ roots) = 0$

$$x^2 - \frac{3}{2}x + 0$$

$$\times 2 \quad \Longrightarrow \ 2x^2 - 3x = 0$$

1. Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.

$$\textbf{i)}\ \frac{\alpha}{3\beta} + \frac{\beta}{3\alpha} \quad \textbf{ii)}\ \frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha} \quad \textbf{iii)}\ (3\alpha - 1)(2\beta - 1) \quad \textbf{iv)}\ \frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha}$$

$$\textbf{i)}\ \frac{\alpha}{3\beta} + \frac{\beta}{3\alpha}$$

$$\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha} = \frac{\alpha^2 + \beta^2}{3\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{3\alpha\beta}$$

$$\textbf{ii)}\ \frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha} = \frac{\beta + \alpha}{\alpha^2\beta^2} = \frac{\alpha+\beta}{(\alpha\beta)^2}$$

$$\textbf{iii)}\ (3\alpha - 1)(3\beta - 1)$$

$$(3\alpha - 1)(3\beta - 1) = 9\alpha\beta - 3\alpha - 3\beta + 1$$

$$= 9\alpha\beta - 3(\alpha + \beta) + 1$$

$$\textbf{iv)}\ \frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha}$$

$$\frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha} = \frac{\alpha^2 + 3\alpha + \beta^2 + 3\beta}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 3(\alpha + \beta\ )}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta + 3(\alpha+\beta)}{\alpha\beta}$$

2. The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. Without solving for the roots, find

$$\textbf{i)}\frac{1}{\alpha} + \frac{1}{\beta}\ \textbf{ii)}\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\ \textbf{iii)}\ \frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2}$$

Given α, β *are the roots of* $2x^2 - 7x + 5 = 0$

$$a = -2,\ b = -7, c = 5$$

Sum of the roots : $\alpha + \beta = -\frac{b}{a}$

$$\boxed{\alpha + \beta = \frac{7}{2}}$$

$$product\ of\ the\ roots: \quad \alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$\boxed{\alpha\beta = \frac{5}{2}}$$

$$\boldsymbol{i)\ \frac{1}{\alpha} + \frac{1}{\beta}}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{7}{2}}{\frac{5}{2}}$$

$$= \frac{7}{\cancel{2}} \times \frac{\cancel{2}}{5}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7}{5}$$

$$\boldsymbol{ii)\ \frac{\alpha}{\beta} + \frac{\beta}{\alpha}}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\frac{49}{4} - 5}{\frac{5}{2}} = \frac{49 - 20}{\cancel{4}_{2}} \times \frac{\cancel{2}}{5}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{29}{10}$$

$$iii)\ \frac{\alpha + 2}{\beta + 2} + \frac{\beta + 2}{\alpha + 2}$$

$$\frac{\alpha + 2}{\beta + 2} + \frac{\beta + 2}{\alpha + 2} = \frac{(\alpha + 2)^2 + (\beta + 2)^2}{(\alpha + 2)(\beta + 2)}$$

$$= \frac{\alpha^2 + 2 \times \alpha \times 2 + 2^2 + \beta^2 + 2 \times \beta \times 2 + 2^2}{(\alpha + 2)(\beta + 2)}$$

$$= \frac{\alpha^2 + 4\alpha + 4 + \beta^2 + 4\beta + 4}{(\alpha + 2)(\beta + 2)} = \frac{\alpha^2 + \beta^2 + 4\alpha + 4\beta + 8}{(\alpha + 2)(\beta + 2)}$$

$$= \frac{(\alpha^2 + \beta^2) + 4(\alpha + \beta) + 8}{\alpha\beta + 2\alpha + 2\beta + 4} = \frac{(\alpha + \beta)^2 - 2\alpha\beta + 4(\alpha + \beta) + 8}{\alpha\beta + 2(\alpha + \beta) + 4}$$

$$= \frac{\frac{49}{4} - 5 + \overset{2}{\cancel{4}}\left(\frac{7}{\cancel{2}}\right) + 8}{\frac{5}{2} + \cancel{2}\left(\frac{7}{\cancel{2}}\right) + 4} = \frac{\frac{49}{4} + 3 + 14}{\frac{5}{2} + 11} = \frac{\frac{49}{4} + 17}{\frac{5}{2} + 11} = \frac{\frac{49 + 68}{4}}{\frac{5 + 22}{2}}$$

$$= \frac{\overset{13\ \cancel{39}}{\cancel{117}}}{2\ \cancel{4}} \times \frac{\cancel{2}}{\underset{\cancel{9}\ \ 3}{\cancel{27}}} = \frac{13}{6}$$

$$\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2} = \frac{13}{6}$$

3. *The roots of the equation* $x^2+6x-4=0$ *are* α, β. *Find the quadratic equation whose roots are*

i) α^2 *and* β^2 ii) $\frac{2}{\alpha}$ *and* $\frac{2}{\beta}$ iii) $\alpha^2\beta$ *and* $\beta^2\alpha$

Given α, β *are the roots of* $x^2+6x-4=0$

$a = 1, b = 6, c = -4$

Sum of the roots: $\alpha+\beta = \frac{-b}{a} = -\frac{6}{1}$

$\alpha+\beta = -6$

Product of the roots: $\alpha\beta = \frac{c}{a} = -\frac{4}{1}$

$\alpha\beta = -4$

i) *To find the equation whose roots are* α^2, β^2

Sum of the root $= \alpha^2+\beta^2$

$$= (\alpha+\beta)^2 - 2\alpha\beta = (-6)^2 - 2(-4) = 36+8 = 44$$

Product of the root $= \alpha^2\beta^2$

$$= (\alpha\beta)^2 = (-4)^2 = 16$$

∴ *The required equation is*

$$x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$$

$$\boxed{x^2 - 44x + 16 = 0}$$

ii) *To find the equation whose roots are* $\frac{2}{\alpha}, \frac{2}{\beta}$

Sum of the roots $= \frac{2}{\alpha} + \frac{2}{\beta}$

$$= 2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) = \cancel{2}\left(\frac{\overset{3}{\cancel{-6}}}{\underset{\cancel{2}}{\cancel{-4}}}\right)$$

Sum of the roots $= 3$

Product of the roots $= \frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{\cancel{4}}{\cancel{-4}}$

Product of the roots $= -1$

∴ *The required equation is*

$$x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$$

$$x^2 - 3x - 1 = 0$$

$iii)$ *To find the equation whose roots are* $\alpha^2\beta, \beta^2\alpha$

$$Sum\ of\ the\ roots = \alpha^2\beta + \beta^2\alpha$$
$$= \alpha\beta(\alpha+\beta) = -4(-6)$$
$$Sum\ of\ the\ roots = 24$$
$$Product\ of\ the\ roots = \alpha^2\beta \times \alpha\beta^2$$
$$= (\alpha\beta)^3 = (-4)^3$$
$$Product\ of\ the\ roots = -64$$

$\therefore$ *The required equation is*

$$x^2 - (Sum\ of\ the\ roots)x + Product\ of\ the\ roots = 0$$
$$x^2 - 24x - 64 = 0$$

4. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$ Find the values of a.

Given α, β *are the roots of* $7x^2 + ax + 2 = 0$

$$\mathrm{a} = 7, \mathrm{b} = \mathrm{a}, \mathrm{c} = 2$$

$$Given : \beta - \alpha = \frac{-13}{7} \Rightarrow \alpha - \beta = \frac{13}{7}$$

$$Sum\ of\ the\ roots: \alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-a}{7}$$

$$Product\ of\ the\ roots: \alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{2}{7}$$

$$\alpha - \beta = \frac{13}{7} \Rightarrow (\alpha - \beta)^2 = \frac{169}{49}$$
$$(\alpha+\beta)^2 - 4\alpha\beta = \frac{169}{49} \Rightarrow \left(\frac{-a}{7}\right)^2 - 4\left(\frac{2}{7}\right) = \frac{169}{49}$$
$$\frac{a^2}{49} - \frac{8}{7} = \frac{169}{49} \Rightarrow \frac{a^2}{49} - \frac{56}{49} = \frac{169}{49}$$
$$\frac{a^2 - 56}{49} = \frac{169}{49} \Rightarrow a^2 - 56 = 169$$
$$a^2 = 169 + 56 \Rightarrow a^2 = 225 \Rightarrow a = \sqrt{225}$$
$$\therefore \quad a = 15, -15$$

5. If one root of the eqtion $2y^2 - ay + 64 = 0$ is twice the other then find the find the values of a.

Let α, β *be the roots of* $2y^2 - ay + 64 = 0$

$\mathrm{a} = 2, \mathrm{b} = -\mathrm{a}, \mathrm{c} = 64$

$$Sum\ of\ the\ roots: \alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-(-a)}{2}$$
$$\alpha + \beta = \frac{a}{2}$$

Product of the roots: $\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{\overset{32}{\cancel{64}}}{{}_{1}\cancel{2}}$

$$\alpha\beta = 32$$

Given $\alpha = 2\beta$

$\therefore \alpha + \beta = \frac{a}{2} \Rightarrow 3\beta = \frac{a}{2} \Rightarrow \beta = \frac{a}{2} \times \frac{1}{3} \Rightarrow \beta = \frac{a}{6}$

$\alpha\beta = 32 \Rightarrow \cancel{2}\beta \times \beta = \cancel{32}^{16} \Rightarrow \beta^2 = 16 \Rightarrow \beta = \sqrt{16}$

$$\beta = \pm 4$$

sub $\beta = \pm 4$ in $\beta = \frac{a}{6}$

$\therefore \frac{a}{6} = \pm 4 \Rightarrow \boxed{a = \pm 24}$

6. *If the roots of the equation* $3x^2 + kx + 81 = 0$ *(having real roots) is the square of the other then find k.*

Let α, β be the roots of $3x^2 + kx + 81 = 0$

$a = 3, b = k, c = 81$

Sum of the roots : $\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-k}{3} \ldots (1)$

product of the roots : $\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{\overset{27}{\cancel{81}}}{\cancel{3}_{1}} \Rightarrow \alpha\beta = 27 \ldots (2)$

Given : $\alpha = \beta^2$

sub $\alpha = \beta^2$ in (2)

$\beta^2 \times \beta = 27 \Rightarrow \beta^3 = 27 \Rightarrow \beta = \sqrt[3]{27} \Rightarrow \beta = 3$

sub $\beta = 3$ in $\alpha = \beta^2$

$\alpha = 3^2 \Rightarrow \alpha = 9$

$\therefore \beta = 3 , \alpha = 9$

sub $\alpha = 9$ and $\beta = 3$ in (1)

$\alpha + \beta = \frac{-k}{3} \Rightarrow 9 + 3 = \frac{-k}{3}$

$12 = \frac{-k}{3} \Rightarrow 12 = \frac{-k}{3} \Rightarrow -k = 36$

$\boxed{k = -36}$

MATRICES

EXERCISE 3.16

Example 3.53: Consider the following information regarding the number of men and women workers in three factories I, II and III.

Factory	*Men*	*Women*
I	**23**	**18**
II	**47**	**36**
III	**15**	**16**

Represent the above information in the form of a matrix. What does the entry in the second row and first column represent?

The information is represented in the form of a 3×2 *matrix as follows:*

$$A = \begin{pmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{pmatrix}$$

The entry in the second row and first column represent that there are 47 men workers in factory II.

Example 3.54: If a matrix has 16 elements, what are the possible orders it can have?

We know that a matrix of order $m \times n$*, has mn elemetns. Thus, to find all possible orders of a matrix with 16 elements, we will find all ordered pairs of natural numbers whose product is 16.*

Such ordered pairs are $(1,16), (16,1), (4,4), (8,2), (2,8)$

Hence, possible orders are $1 \times 16, 16 \times 1, 4 \times 4, 2 \times 8, 8 \times 2$

Example 3.55: Construct a 3×3 matrix whose elements are $a_{ij} = i^2 j^2$

The general 3×3 *matrix is given by* $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $a_{ij} = i^2 j^2$

$a_{11} = 1^2 \times 1^2 = 1 \times 1 = 1;$ $a_{12} = 1^2 \times 2^2 = 1 \times 4 = 4;$

$a_{21} = 2^2 \times 1^2 = 4 \times 1 = 4;$ $a_{22} = 2^2 \times 2^2 = 4 \times 4 = 16;$

$a_{31} = 3^2 \times 1^2 = 9 \times 1 = 9;$ $a_{32} = 3^2 \times 2^2 = 9 \times 4 = 36;$

$a_{13} = 1^2 \times 3^2 = 1 \times 9 = 9$

$a_{23} = 2^2 \times 3^2 = 4 \times 9 = 36$

$a_{33} = 3^2 \times 3^2 = 9 \times 9 = 81$

Hence, the required matrix is $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$

$\boldsymbol{Example\ 3.56: Find\ the\ value\ of\ a, b, c, d\ from\ the\ equation}$

$$\begin{bmatrix} \boldsymbol{a-b} & \boldsymbol{2a+c} \\ \boldsymbol{2a-b} & \boldsymbol{3c+d} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{0} & \mathbf{2} \end{bmatrix}$$

$The\ given\ matrices\ are\ equal.\ Thus\ all\ corresponding\ elements\ are\ equal.$

$a-b=1\ldots(1) \quad 2a+c=5\ldots(2) \quad 2a-b=0\ldots(3) \quad 3c+d=2\ldots(4)$

$2a=b\ldots(5)$

$Put\ 2a=b\ in\ (1) \quad a-b=1$

$a-2a=1 \Rightarrow -a=1 \Rightarrow a=-1$

$Put\ a=-1\ in\ (5)\ 2a=b$

$2(-1)=b \Rightarrow -2=b \Rightarrow b=-2$

$Put\ a=-1\ in\ (2),\ 2a+c=5$

$2(-1)+c=5 \Rightarrow -2+c=5 \Rightarrow c=5+2$

$c=7$

$Put\ c=7\ in\ (4)\ 3c+d=2$

$3(7)+d=2 \Rightarrow 21+d=2 \Rightarrow d=2-21 \Rightarrow d=-19$

$Therefore, a=-1, b=-2, c=7, d=-19$

$$\mathbf{1.}\boldsymbol{In\ the\ matrix\ A} = \begin{bmatrix} \mathbf{8} & \mathbf{9} & \mathbf{4} & \mathbf{3} \\ \mathbf{-1} & \sqrt{\mathbf{7}} & \frac{\sqrt{\mathbf{3}}}{\mathbf{2}} & \mathbf{5} \\ \mathbf{1} & \mathbf{4} & \mathbf{3} & \mathbf{0} \\ \mathbf{6} & \mathbf{8} & \mathbf{-11} & \mathbf{1} \end{bmatrix}, \boldsymbol{write}$$

$\boldsymbol{(i)\ The\ number\ of\ elements}$

$\boldsymbol{(ii)\ The\ order\ of\ the\ matrix}$

$\boldsymbol{(iii) Write\ the\ element\ corresponding\ to\ a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}}$

$$A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$$

$(i)\ The\ number\ of\ elements = 16$

$(ii)\ The\ order\ of\ the\ matrix = No\ of\ rows \times No\ of\ columns = 4 \times 4$

$(iii) a_{22} = \sqrt{7},\ a_{23} = \frac{\sqrt{3}}{2},\ a_{24} = 5,\ a_{34} = 0,\ a_{43} = -11,\ a =_{44} = 1$

$\mathbf{2.}\boldsymbol{If\ a\ matrix\ has\ 18\ elements, what\ are\ the\ possible\ orders\ it\ can\ have? What\ if\ it\ has\ 6\ elements?}$

$Given\ that\ a\ matrix\ has\ 18\ elements.$

$\therefore The\ possible\ orders\ are\ 1 \times 18,\ 18 \times 1, 9 \times 2, 2 \times 9, 6 \times 3,\ 3 \times 6.$

$If\ a\ matrix\ has\ 6\ elements, then\ the\ possible\ orders\ are$

$1 \times 6,\ 6 \times 1,\ 2 \times 3, 3 \times 2.$

3. $\boldsymbol{Construct\ a\ 3 \times 3\ matrix\ whose\ elements\ are\ given\ by}$

$(\boldsymbol{i})\ \boldsymbol{a_{ij} = |i - 2j|}$

$Since, the\ matrix\ is\ of\ order\ 3 \times 3, there\ will\ be\ 9\ elements.$

$a_{11} = |1 - 2(1)| = |1 - 2| = |-1| = 1$

$a_{12} = |1 - 2(2)| = |1 - 4| = |-3| = 3$

$a_{13} = |1 - 2(3)| = |1 - 6| = |-5| = 5$

$a_{21} = |2 - 2(1)| = |2 - 2| = 0$

$a_{22} = |2 - 2(2)| = |2 - 4| = |-2| = 2$

$a_{23} = |2 - 2(3)| = |1 - 6| = |-4| = 4$

$a_{31} = |3 - 2(1)| = |3 - 2| = 1$

$a_{32} = |3 - 2(2)| = |3 - 4| = |-1| = 1$

$a_{33} = |3 - 2(3)| = |3 - 6| = |-3| = 3$

$$\therefore The\ matrix\ is \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$$

$(\boldsymbol{ii})\ \boldsymbol{a_{ij} = \dfrac{(i+j)^3}{3}}$

$a_{11} = \dfrac{(1+1)^3}{3} = \dfrac{(2)^3}{3} = \dfrac{8}{3}$

$a_{12} = \dfrac{(1+2)^3}{3} = \dfrac{(3)^3}{3} = \dfrac{27}{3} = 9$

$a_{13} = \dfrac{(1+3)^3}{3} = \dfrac{(4)^3}{3} = \dfrac{64}{3}$

$a_{21} = \dfrac{(2+1)^3}{3} = \dfrac{(3)^3}{3} = \dfrac{27}{3} = 9$

$a_{22} = \dfrac{(2+2)^3}{3} = \dfrac{(4)^3}{3} = \dfrac{64}{3}$

$a_{23} = \dfrac{(2+3)^3}{3} = \dfrac{(5)^3}{3} = \dfrac{125}{3}$

$a_{31} = \dfrac{(3+1)^3}{3} = \dfrac{(4)^3}{3} = \dfrac{64}{3}$

$a_{32} = \dfrac{(3+2)^3}{3} = \dfrac{(4)^3}{3} = \dfrac{125}{3}$

$a_{33} = \dfrac{(3+3)^3}{3} = \dfrac{(6)^3}{3} = \dfrac{216}{3} = 72$

$$\therefore The\ matrix\ is \begin{pmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{pmatrix}$$

4. $\boldsymbol{If\ A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}\ then\ find\ the\ transpose\ of\ A.}$

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix} \Longrightarrow A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

5. $\boldsymbol{If\ A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}\ then\ find\ the\ transpose\ of\ -A.}$

$$A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix} \Longrightarrow -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

6. $\boldsymbol{If\ A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}\ then\ verify\ \left(A^T\right)^T = A}$

$$A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} \Longrightarrow A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix} = A$$

$\therefore (A^T)^T = A \quad Hence\ verified.$

7. $\boldsymbol{Find\ the\ values\ of\ x, y\ and\ z\ from\ the\ following\ equations.}$

$$\boldsymbol{(i) \begin{bmatrix} 12 & 3 \\ x & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} y & z \\ 3 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix} \quad (iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}}$$

$$(i) \begin{bmatrix} 12 & 3 \\ x & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} y & z \\ 3 & 5 \end{bmatrix}$$

$Equating\ the\ corresponding\ elements$

$x = 3,\ y = 12,\ z = 3$

$(ii)\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$x+y=6\ldots(1), 5+z=5\ldots(2), xy=8\ldots(3)$

$From\ (2)\ \ 5+z=5$

$z=5-5 \Rightarrow z=0$

$From\ (3)\ \ xy=8 \ \Rightarrow y=\frac{8}{x}$

$sub\ y=\frac{8}{x}\ in\ (1)\ x+y=6$

$x+\frac{8}{x}=6 \ \Rightarrow \ \frac{x^2+8}{x}=6 \ \Rightarrow x^2+8=6x$

+ ×
−6 8
−4 −2

$x^2-6x+8=0 \Rightarrow (x-4)(x-2)=0$

$x-4=0,\ x-2=0$

$x=4, x=2$

$when\ x=4\ in\ y=\frac{8}{x} \ \Rightarrow y=\frac{8}{4} \ \Rightarrow y=2$

$when\ x=2\ in\ \ y=\frac{8}{x} \ \Rightarrow y=\frac{8}{2} \ \Rightarrow \ y=4$

$\therefore x=4, y=2, z=0\ \ or\ \ \therefore x=2, y=4, z=0$

$(iii)\ \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

$x+y+z=9 \ \ \ldots(1),\ x+z=5\ldots(2),\ y+z=7\ldots(3)$

$Solve\ \ (1)\ and\ (2)$

$$\begin{array}{l} \cancel{x}+y+\cancel{z}=9 \\ (-)\ \ (-)\ \ (-)\ \ \ (-) \\ \hline \cancel{x}+0y+\cancel{z}=5 \\ \hline \qquad y=4 \end{array}$$

$Sub\ y=4\ in\ (3)\ \ y+z=7$

$4+z=7 \ \Rightarrow \ z=7-4$

$z=3$

$Sub\ z=3\ in\ (2)\ x+z=5$

$x+3=5 \ \Rightarrow x=5-3$

$x=2$

$\therefore x=2,\ y=4,\ z=3.$

EXERCISE 3.17

Example 3.57: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$, ***find*** $A + B$

$$A + B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

Example 3.58: Two examinations were conducted for three groups of students namely group 1, group 2, group 3 and their data on average of marks for the subjects, Tamil, English, Science and Mathematics are given beow in the form of matrics A and B. Find the total marks of both the examinations for all the three groups.

$$A = \begin{array}{c} \\ Group\ 1 \\ Group\ 2 \\ Group\ 3 \end{array} \left(\begin{array}{cccc} Tamil & English & Science & Mathematics \\ 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{array}\right)$$

$$B = \begin{array}{c} \\ Group\ 1 \\ Group\ 2 \\ Group\ 3 \end{array} \left(\begin{array}{cccc} Tamil & English & Science & Mathematics \\ 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{array}\right)$$

$$A = \begin{array}{c} \\ Group\ 1 \\ Group\ 2 \\ Group\ 3 \end{array} \left(\begin{array}{cccc} Tamil & English & Science & Mathematics \\ 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{array}\right)$$

$$B = \begin{array}{c} \\ Group\ 1 \\ Group\ 2 \\ Group\ 3 \end{array} \left(\begin{array}{cccc} Tamil & English & Science & Mathematics \\ 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{array}\right)$$

The total marks in both the examinations for all the three groups is the sum of the given matrics.

$$A+B=\begin{pmatrix}22+20 & 15+38 & 14+15 & 23+40\\ 50+18 & 62+12 & 21+17 & 30+80\\ 53+81 & 80+47 & 32+52 & 40+18\end{pmatrix}$$

$$=\begin{pmatrix}42 & 53 & 29 & 63\\ 68 & 74 & 38 & 110\\ 134 & 127 & 84 & 58\end{pmatrix}$$

Example 3.59: $\boldsymbol{If\ A=\begin{bmatrix}1 & 3 & -2\\ 5 & -4 & 6\\ -3 & 2 & 9\end{bmatrix}, \mathrm{B}=\begin{bmatrix}1 & 8\\ 3 & 4\\ 9 & 6\end{bmatrix}, find\ A+B}$

It is not possible to add A and B because they have different orders.

Example 3.60: $\boldsymbol{If\ A=\begin{bmatrix}7 & 8 & 6\\ 1 & 3 & 9\\ -4 & 3 & -1\end{bmatrix}, \mathrm{B}=\begin{bmatrix}4 & 11 & -3\\ -1 & 2 & 4\\ 7 & 5 & 0\end{bmatrix}, find\ 2A+B}$

Since A and B have same order $3\times 3, 2A+B$ *is defined*

$$2A+B=2\begin{pmatrix}7 & 8 & 6\\ 1 & 3 & 9\\ -4 & 3 & -1\end{pmatrix}+\begin{pmatrix}4 & 11 & -3\\ -1 & 2 & 4\\ 7 & 5 & 0\end{pmatrix}$$

$$=\begin{pmatrix}14 & 16 & 12\\ 2 & 6 & 18\\ -8 & 6 & -2\end{pmatrix}+\begin{pmatrix}4 & 11 & -3\\ -1 & 2 & 4\\ 7 & 5 & 0\end{pmatrix}$$

$$=\begin{pmatrix}14+4 & 16+11 & 12-3\\ 2-1 & 6+2 & 18+4\\ -8+7 & 6+5 & -2+0\end{pmatrix}=\begin{pmatrix}18 & 27 & 9\\ 1 & 8 & 22\\ -1 & 11 & -2\end{pmatrix}$$

Example 3.61: $\boldsymbol{If\ \mathrm{A}==\begin{bmatrix}5 & 4 & -2\\ \frac{1}{2} & \frac{3}{4} & \sqrt{2}\\ 1 & 9 & 4\end{bmatrix}, \mathrm{B}=\begin{bmatrix}-7 & 4 & -3\\ \frac{1}{4} & \frac{7}{2} & 3\\ 5 & -6 & 9\end{bmatrix}, find\ 4A-3B.}$

Since A and B have same order $3\times 3, 4A-3B$ *is defined*

$$4A-3B=4\begin{pmatrix}5 & 4 & -2\\ \frac{1}{2} & \frac{3}{4} & \sqrt{2}\\ 1 & 9 & 4\end{pmatrix}-3\begin{pmatrix}-7 & 4 & -3\\ \frac{1}{4} & \frac{7}{2} & 3\\ 5 & -6 & 9\end{pmatrix}$$

$$=\begin{pmatrix}20 & 16 & -8\\ 2 & 3 & 4\sqrt{2}\\ 4 & 36 & 16\end{pmatrix}+\begin{pmatrix}21 & -12 & 9\\ -\frac{3}{4} & -\frac{21}{2} & -9\\ -15 & 18 & -27\end{pmatrix}=\begin{pmatrix}41 & 4 & 1\\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9\\ -11 & 54 & -11\end{pmatrix}$$

Example 3.62: *Find the value of a, b, c, d, x, y form the following matrix equation*

$$\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$$

First , we add the two matrics on both left, right hands sides to get

$$\begin{pmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{pmatrix} = \begin{pmatrix} 2+0 & 2a+1 \\ b-5 & 4c+0 \end{pmatrix}$$

equating the corresponding elements of the two matrics, we have

$$d+3=2 \Longrightarrow d=2-3=-1$$

$$8+a=2a+1 \Longrightarrow 8-1=2a-a \Longrightarrow a=7$$

$$3b-2=b-5 \Longrightarrow 3b-b=2-5 \Longrightarrow 2b=-3 \Longrightarrow b=\frac{-3}{2}$$

substituting a value

$$a-4=4c-0 \Longrightarrow 7-4=4c \Longrightarrow 4c=3 \Longrightarrow c=\frac{3}{4}$$

$$therefore, a=7, b=-\frac{3}{7}, c=\frac{3}{4}, d=-1$$

Example 3.63: *If* $A = \begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$

compute the following i) $3A+2B-C$ ii) $\frac{1}{2}A-\frac{3}{2}B$.

$i)\ 3A+2B-C$

$$= 3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} + 2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix} + \begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix} + \begin{pmatrix} -5 & -3 & 0 \\ 1 & 7 & -2 \\ -1 & -4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 3+16-5 & 24-12-3 & 9-8+0 \\ 9+4+1 & 15+22+7 & 0-6-2 \\ 24+0-1 & 21+2-4 & 18+10-3 \end{pmatrix} = \begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

$ii)\ \frac{1}{2}A-\frac{3}{2}B$

$$= \frac{1}{2}(A-3B) = \frac{1}{2}\left(\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} - 3\begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} + \begin{bmatrix} -24 & 18 & 12 \\ -6 & -33 & 9 \\ 0 & -3 & -15 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{pmatrix} 1-24 & 8+18 & 3+12 \\ 3-6 & 5-33 & 0+9 \\ 8+0 & 7-3 & 6-15 \end{pmatrix} \quad = \frac{1}{2}\begin{pmatrix} -23 & 26 & 15 \\ -3 & -28 & 9 \\ 8 & 4 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{23}{2} & 13 & \frac{15}{2} \\ -\frac{3}{2} & -14 & \frac{9}{2} \\ 4 & 2 & -\frac{9}{2} \end{pmatrix}$$

1. $\boldsymbol{If\ A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix},\ B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix},\ then\ verify\ that}$

$\boldsymbol{i) A + B = B + A \quad ii)\ A + (-A) = (-A) + A = 0}$

$i) A + B = B + A \quad A\ and\ B\ are\ same\ order$

$$A + B = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+5 & 9+7 \\ 3+3 & 4+3 \\ 8+1 & -3+0 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$$

$$B + A = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 5+1 & 7+9 \\ 3+3 & 3+4 \\ 1+8 & 0-3 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$$

$$A + B = B + A$$

$ii)\ A + (-A) = (-A) + A = 0$

$LHS\colon\ A + (-A)$

$$= \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$RHS: \ (-A)+A$

$$= \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$\therefore \text{LHS} = \text{RHS}$

2. $\boldsymbol{If\,A} = \begin{bmatrix} \mathbf{4} & \mathbf{3} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{-8} \\ \mathbf{1} & \mathbf{0} & \mathbf{-4} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{9} & \mathbf{2} \\ \mathbf{-7} & \mathbf{1} & \mathbf{-1} \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} \mathbf{8} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \\ \mathbf{2} & \mathbf{4} & \mathbf{-1} \end{bmatrix},$

$\boldsymbol{then\ verify\ that\ A+(B+C)=(A+B)+C}$

$$B+C = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix} = \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$A+(B+C) = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \ldots \ldots (1)$$

$$A+B = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix}$$

$$(A+B)+C = \begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \ldots \ldots (2)$$

$\therefore \quad from\ (1) and\ (2) \quad \text{LHS} = \text{RHS}$

3. $\boldsymbol{Find\ X\ and\ Y\ if\ X+Y=\begin{bmatrix}7 & 0\\ 3 & 5\end{bmatrix}\ and\ X-Y=\begin{bmatrix}3 & 0\\ 0 & 4\end{bmatrix}}$

$Adding\ (1)\ and\ (2)$

$$X+\cancel{Y}=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}\ \dots\ (1)$$

$$X-\cancel{Y}=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\ \dots\ (2)$$

$$2X=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\Longrightarrow 2X=\begin{pmatrix}10 & 0\\ 3 & 9\end{pmatrix}\Longrightarrow X=\begin{pmatrix}5 & 0\\ \frac{3}{2} & \frac{9}{2}\end{pmatrix}$$

$Subracting\ (1)\ and\ (2)$

$$\cancel{X}+Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}\ \dots\ (1)$$

$$(-)\quad (+)\quad (-)$$

$$\cancel{X}-Y=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\ \dots\ (2)$$

$$2Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}-\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\Longrightarrow 2Y=\begin{pmatrix}4 & 0\\ 3 & 1\end{pmatrix}\Longrightarrow y=\begin{pmatrix}2 & 0\\ \frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

4. $\boldsymbol{If\ A=\begin{bmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{bmatrix}, B=\begin{bmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{bmatrix}\ find\ the\ value\ of}$
$\boldsymbol{i)\ B-5A\ ii)\ 3A-9B}$

$$B-5A=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-5\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-\begin{pmatrix}0 & 20 & 45\\ 40 & 15 & 35\end{pmatrix}$$

$$=\begin{pmatrix}7-0 & 3-20 & 8-45\\ 1-40 & 4-15 & 9-35\end{pmatrix}=\begin{pmatrix}7 & -17 & -37\\ -39 & -11 & -26\end{pmatrix}$$

$$3A-9B=3\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}-9\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}=\begin{pmatrix}0 & 12 & 27\\ 24 & 9 & 21\end{pmatrix}-\begin{pmatrix}63 & 27 & 72\\ 9 & 36 & 81\end{pmatrix}$$

$$=\begin{pmatrix}0-63 & 12-27 & 27-72\\ 24-9 & 9-36 & 21-81\end{pmatrix}=\begin{pmatrix}-63 & -15 & -45\\ 15 & -27 & -60\end{pmatrix}$$

5. $\boldsymbol{Find\ the\ value\ of\ x, y, z\ if\ (i)\begin{bmatrix}x-3 & 3x-z\\ x+y+7 & x+y+z\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 6\end{bmatrix}}$

$$\begin{bmatrix}x-3 & 3x-z\\ x+y+7 & x+y+z\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 6\end{bmatrix}$$

$$x-3=1\Longrightarrow x=1+3$$

$$\therefore x=4$$

$$3x-z=0\Longrightarrow 3(4)-z=0\Longrightarrow 12-z=0$$

$$\therefore z=12$$

$$x+y+7=1\Longrightarrow x+y=1-7$$

$$x+y=-6\Longrightarrow 4+y=-6$$

$$y=-6-4\Longrightarrow \therefore y=-10$$

$$\boldsymbol{(ii)\begin{bmatrix} x & y-z & z+3 \end{bmatrix} + \begin{bmatrix} y & 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 16 \end{bmatrix}}$$

$$\begin{bmatrix} x & y-z & z+3 \end{bmatrix} + \begin{bmatrix} y & 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 16 \end{bmatrix}$$

$$\begin{bmatrix} x+y & y-z+4 & z+3+3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 16 \end{bmatrix}$$

$$\begin{bmatrix} x+y & y-z+4 & z+6 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 16 \end{bmatrix}$$

$$x + y = 4,\ \ y - z + 4 = 8,\ z + 6 = 16$$

$$x + 14 = 4,\ y - 10 + 4 = 8,\ z = 16 - 6$$

$$x = 4 - 14, \quad y - 6 = 8, \quad z = 10$$

$$y = 8 + 6,$$

$$\therefore x = -10$$

$$y = 14$$

6. *Find x and y if* $\boldsymbol{x\begin{bmatrix} 4 \\ -3 \end{bmatrix} + y\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}}$

$$x\begin{bmatrix} 4 \\ -3 \end{bmatrix} + y\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \Longrightarrow \begin{bmatrix} 4x \\ -3x \end{bmatrix} + \begin{bmatrix} -2y \\ 3y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4x - 2y \\ -3x + 3y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \Longrightarrow 4x - 2y = 4 \qquad and\ -3x + 3y = 6$$

$$\div 2 \qquad\qquad \div 3$$

$$2x - y = 2 \ldots (1) \qquad -x + y = 2 \ldots (2)$$

$Solve\ \ (1)\ and\ (2)$

$$(1) \Longrightarrow 2x - \cancel{y} = 2$$

$$(2) \Longrightarrow \underline{-x + \cancel{y} = 2}$$

$$x = 4$$

$$sub\ x = 4\ in\ (2)\ -x + y = 2$$

$$-4 + y = 2 \Longrightarrow y = 2 + 4$$

$$y = 6$$

$$\therefore x = 4, y = 6$$

7. *Find the non − zero values of x sarisfying the matrix equation*

$$\boldsymbol{x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}}$$

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

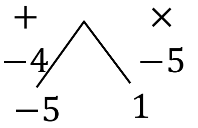
$$\begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix} + \begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix} = \begin{pmatrix} 2x^2 + 16 & 48 \\ 20 & 12x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2 + 16 & 12x \\ 3x + 8 & x^2 + 8x \end{pmatrix} = \begin{pmatrix} 2x^2 + 16 & 48 \\ 20 & 12x \end{pmatrix}$$

$$12x = 48 \Longrightarrow x = \frac{\cancel{48}^{\,4}}{\cancel{12}} \Longrightarrow x = 4$$

8. $\boldsymbol{Solve\ for\ x, y \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} -2x \\ -y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}}$

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} -2x \\ -y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + \begin{bmatrix} -4x \\ -2y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

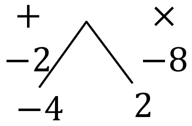


$$\begin{bmatrix} x^2 - 4x \\ y^2 + 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \Rightarrow \quad x^2 - 4x = 5 \ \ and\ y^2 - 2y = 8$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \Rightarrow x - 5 = 0, x + 1 = 0$$

$$x = 5, x = 1 \Rightarrow x = 5, -1$$



$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0 \Rightarrow y - 4 = 0, y + 2 = 0$$

$$y = 4, y = -2 \Rightarrow y = 4, -2$$

EXERCISE 3.18

Example 3.64: $\boldsymbol{If\ A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix} find\ AB.}$

Here order of A is 2×3 *matrix and order of B is* 3×3 *matrix, hence AB is of order* 2×3.

$$Given\ A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}_{2 \times 3}, B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}_{3 \times 3}$$

$$AB = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix} = \begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$$

Example 3.65: $\boldsymbol{If\ A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} find\ AB\ and\ BA. Check\ if\ AB = BA.}$

Here orderof A is 2×2 *matrix and B is* 2×2 *matrix, hence AB is defined and is of order* 2×2.

$$AB = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+1 & 0+3 \\ 2+3 & 0+9 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 5 & 9 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+0 & 2+0 \\ 2+3 & 1+9 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 5 & 10 \end{pmatrix}$$

$$Therefore, AB \neq BA$$

Example 3.66: If $A = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$.

Show that and B satisfy commutative property with respect to matrix multiplication

To show that $AB = BA$

$$L.H.S: AB = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4+4 & 4\sqrt{2} - 4\sqrt{2} \\ 2\sqrt{2} - 2\sqrt{2} & 4+4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$RHS : BA = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4+4 & -4\sqrt{2} + 4\sqrt{2} \\ -2\sqrt{2} + 2\sqrt{2} & 4+4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

Hence $LHS = RHS$ (ie) $AB = BA$

Example 3.67: Solve $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$:

$$\underset{2\times 2}{\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}} \times \underset{2\times 1}{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \Longrightarrow \begin{pmatrix} 2x + y \\ x + 2y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$2x + y = 4 \ldots (1) \text{ and } x + 2y = 5 \ldots (2)$$

Solve (1) and (2)

$$\begin{array}{l} \cancel{2x} \;+\; y \;=\; 4 \\ (-) \quad (-) \qquad (-) \\ \cancel{2x} \;+\; 4y = 10 \\ \hline \quad -3y = -6 \end{array} \Longrightarrow y = \frac{\cancel{-6}\,2}{\cancel{-3}\,1} \Longrightarrow y = 2$$

Sub $y = 2$ in (1) $2x + y = 4$

$$2x + 2 = 4 \Longrightarrow 2x = 4 - 2 \Longrightarrow x = \frac{\cancel{2}}{\cancel{2}}$$

$$x = 1$$

$\therefore x = 1, y = 2.$

Example 3.68: If $A = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ show that $(AB)C = A(BC)\cdot$

L.H.S: $(AB)C$

$$AB = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}_{1\times 3} \times \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}_{3\times 2}$$

$$\Rightarrow AB = \begin{bmatrix} 1-2+2 & -1-1+6 \end{bmatrix}$$

$$AB = \begin{pmatrix} 1 & 4 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 1 & 4 \end{pmatrix}_{1\times 2} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = \begin{bmatrix} 1+8 & 2-4 \end{bmatrix}$$

$$(AB)C = \begin{pmatrix} 9 & -2 \end{pmatrix} \quad \ldots.(1)$$

R.H.S: $A(BC)$

$$BC = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}_{3\times 2} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}_{2\times 2} = \begin{pmatrix} 1-2 & 2+1 \\ 2+2 & 4-1 \\ 1+6 & 2-3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}_{1\times 3} \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}_{3\times 2}$$

$$A(BC) = \begin{bmatrix} -1-4+14 & 3-3-2 \end{bmatrix}$$

$$A(BC) = \begin{pmatrix} 9 & -2 \end{pmatrix} \ldots (2)$$

From (1) and (2), $(AB)C = A(BC)$

Example 3.69: If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}, C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ verify that $A(B+C) = AB + AC\cdot$

L.H.S: $A(B+C)$

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1-7 & 2+8 \\ -4+3 & 2+2 \end{pmatrix}$$

$$B + C = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix} \ldots (1)$$

$R.H.S : AB + AC$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \times \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$\therefore AB + AC = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix} = \begin{pmatrix} -3-4 & 4+12 \\ -13+16 & 4+0 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix} \quad \ldots(2)$$

$From\ (1)\ and\ (2), A(B+C) = AB + AC$

Hence proved.

Example 3.70: If $\boldsymbol{A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}}$ **and** $\boldsymbol{B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}}$ **show that** $\boldsymbol{(AB)^T = B^T A^T}$**.**

$L.H.S: (AB)^T$

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}_{2\times3} \times \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}_{3\times2} = \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix} \Rightarrow (AB)^T = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}^T \Rightarrow (AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix} \ldots.(1)$$

$R.H.S: (B^T A^T)$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}, \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}_{2\times3} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}_{3\times2} = \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix} \ldots.(2)$$

$From\ (1)\ and\ (2), (AB)^T = B^T A^T.$

Hence proved.

1. *Find the order of the product matrix AB if*

	(i)	(ii)	(iii)	(iv)	(v)
Orders of A	3×3	4×3	4×2	4×5	1×1
Orders of B	3×3	3×2	2×2	5×1	1×3

$i)\ A\ is\ of\ odrer = 3\times3$

$B\ is\ of\ order = 3\times3$

$\therefore Order\ of\ AB = 3\times3$

$ii)\ A \rightarrow 4\times3, B \rightarrow 3\times2$

$\therefore Order\ of\ AB = 4\times2$

$iii)\ A \rightarrow 4\times2, B \rightarrow 2\times2$

$\therefore Order\ of\ AB = 4\times2$

$iv)\ A \rightarrow 4\times5, B \rightarrow 5\times1$

$\therefore Order\ of\ AB = 4\times1$

$v)\ A \rightarrow 1\times1, B \rightarrow 1\times3$

$\therefore Order\ of\ AB = 1\times3$

2. If A is of order $p \times q$ and B is of order $q \times r$ what is the order of AB and BA?

$Given: A \text{ is of order } p \times q$

$B \text{ is of order } q \times r$

$\therefore Order\ of\ AB = p \times r$

$Order\ of\ BA\ is\ not\ defined$

$(\because No.of\ columns\ in\ B\ \&\ No.of\ rows\ in\ A\ are\ not\ equal.)$

3. A has 'a' rows and '$a + 3$' columns. B has 'b' rows and '$17 - b$' columns, and if both products AB and BA exist, find a, b?

$Given:\ Order\ of\ A\ is\ a \times (a+3)\ and\ Order\ of\ B\ is\ b \times (17 - b)$

$Product\ AB\ exist\ \therefore a + 3 = b$

$(No.of\ columns\ in\ A = No.of\ rows\ in\ B)$

$a - b = -3\ \ldots(1)$

$Product\ BA\ exist\ \therefore 17 - b = a$

$(No.of\ columns\ in\ B = No.of\ rows\ in\ A)$

$a + b = 17\ \ldots(2)$

$Solving\ (1)\ \&\ (2)$

$$\begin{array}{l} a - \cancel{b} = -3 \\ \underline{a + \cancel{b} = 17} \\ 2a = 14 \implies a = 7 \end{array}$$

$Sub\ a = 7\ in\ (2)\ \ a + b = 17$

$7 + b = 17 \implies b = 17 - 7 \implies b = 10$

$\therefore a = 7, b = 10$

4. $A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$ find AB, BA and check if $AB = BA$?

$Given\ A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$

$$AB = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} (2 \times 1) + (5 \times 2) & (2 \times -3) + (5 \times 5) \\ (4 \times 1) + (3 \times 2) & (4 \times -3) + (3 \times 5) \end{pmatrix}$$

$$= \begin{pmatrix} 2 + 10 & -6 + 25 \\ 4 + 6 & -12 + 15 \end{pmatrix} = \begin{pmatrix} 12 & 19 \\ 10 & 3 \end{pmatrix} \ldots.(1)$$

$$BA = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} (1 \times 2) + (-3 \times 4) & (1 \times 5) + (-3 \times 3) \\ (2 \times 2) + (5 \times 4) & (2 \times 5) + (5 \times 3) \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 - 12 & 5 - 9 \\ 4 + 20 & 10 + 15 \end{pmatrix} \implies BA = \begin{pmatrix} -10 & -4 \\ 24 & 25 \end{pmatrix} \ldots(2)$$

$\therefore From\ (1)\ \&\ (2)\ AB \neq BA$

5. Given $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

verify that $A(B+C) = AB + AC$

Given: $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

$L.H.S: A(B+C)$

$$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} (1\times 2) + (3\times -1) & (1\times 2) + (3\times 6) & (1\times 4) + (3\times 5) \\ (5\times 2) + (-1\times -1) & (5\times 2) + (-1\times 6) & (5\times 4) + (-1\times 5) \end{pmatrix}$$

$$= \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix} = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad \ldots(1)$$

$R.H.S: AB + AC$

$$= \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix} + \begin{pmatrix} 1-12 & 3+3 & 2+9 \\ 5+4 & 15-1 & 10-3 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix} + \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad \ldots(2)$$

$\therefore$ From (1) & (2) $LHS = RHS$

6. Show that the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ **satisfy commutative property** $AB = BA$

Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1-6 & 2+2 \\ 3-3 & -6+1 \end{pmatrix}$$

$$AB = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1-6 & 2-2 \\ -3+3 & -6+1 \end{pmatrix}$$

$$AB = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\therefore AB = BA$$

$$\therefore Commutative\ property\ is\ true.$$

7. $\boldsymbol{Let\ A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\ show\ that}$

$\boldsymbol{i)\ A(BC) = (AB)C \quad ii)\ (A - B)C = AC - BC\ iii)\ (A - B)^T = A^T - B^T}$

$Given:\ A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$i)\ To\ show: A(BC) = (AB)C$

$$BC = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix} = \begin{pmatrix} 8+14 & 0+20 \\ 8+21 & 0+30 \end{pmatrix} = \begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix} \ldots (1)$$

$$AB = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 4+2 & 0+10 \\ 4+3 & 0+15 \end{pmatrix} = \begin{pmatrix} 6 & 10 \\ 7 & 15 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 6 & 10 \\ 7 & 15 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 12+10 & 0+20 \\ 14+15 & 0+30 \end{pmatrix} = \begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix} \ldots (2)$$

$\boldsymbol{ii)\ To\ show\ (A - B)C = AC - BC}$

$$A - B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 1-4 & 2-0 \\ 1-1 & 3-5 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}$$

$$(A - B)C = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -6+2 & 0+4 \\ 0-2 & 0-4 \end{pmatrix}$$

$$(A - B)C = \begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix} \ldots (1)$$

$$AC = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2+2 & 0+4 \\ 2+3 & 0+6 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 5 & 6 \end{pmatrix}$$

$$BC = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$$

$$\therefore AC - BC = \begin{pmatrix} 4 & 4 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix} \ldots (2)$$

$$\therefore From\ (1)\ and\ (2), L.H.S = R.H.S$$

iii) To show: $(A-B)^T = A^T - B^T$

$$(A-B)^T = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}^T = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix} \ldots (1)$$

$$A^T - B^T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ 0 & 5 \end{pmatrix} = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix} \ldots (2)$$

$\therefore$ *From* (1) & (2) $(A-B)^T = A^T - B^T$

8. If $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}, B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$ then show that $A^2 + B^2 = I$.

Given : $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}, B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$

To show: $A^2 + B^2 = I$

$$A^2 = A \times A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix} = \begin{bmatrix} \cos^2\theta + 0 & 0+0 \\ 0 & 0 + \cos^2\theta \end{bmatrix}$$

$$A^2 = \begin{pmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix}$$

$$B^2 = B \times B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix} = \begin{bmatrix} \sin^2\theta + 0 & 0+0 \\ 0 & 0 + \sin^2\theta \end{bmatrix}$$

$$B^2 = \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$\therefore A^2 + B^2 = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathrm{I}$$

Hence proved.

9. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ prove that $AA^T = I$

Given $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ *and* $A^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

To prove: $AA^T = I$

$$L.H.S: A.A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{bmatrix} (\cos^2\theta + \sin^2\theta) & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\,\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathrm{I}$$

Hence proved.

10. $\boldsymbol{Verify\ that\ A^2 = I\ when\ A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}}$

$Given : A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$

$To\ prove : A^2 = I$

$$A^2 = A \times A = \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix} = \begin{bmatrix} 25 - 24 & -20 + 20 \\ 30 - 30 & -24 + 25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$Hence\ proved.$

11. $\boldsymbol{If\ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\ and\ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\ show\ that\ A^2 - (a+d)A = (bc - ad)I_2.}$

$Given: A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$To\ prove: A^2 - (a+d)A = (bc - ad)I$

$$A^2 = A \times A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}$$

$$(a+d)A = (a+d)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + ad & ab + bd \\ ca + cd & ad + d^2 \end{pmatrix}$$

$$A^2 - (a+d)A = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} - \begin{pmatrix} a^2 + ad & ab + bd \\ ca + cd & ad + d^2 \end{pmatrix}$$

$$= \begin{pmatrix} bc - ad & 0 \\ 0 & bc - ad \end{pmatrix}$$

$$= (bc - ad)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (bc - ad)(I) = R.H.S$$

$Hence\ proved.$

12. $\boldsymbol{If\ A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}\ verify\ that\ (AB)^T = B^T A^T}$

$Given\ A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

$To\ verify: (AB)^T = B^T A^T$

$$AB = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 5 + 2 + 45 & 35 + 4 - 9 \\ 1 + 2 + 40 & 7 + 4 - 8 \end{pmatrix} = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$\therefore (AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \ldots (1)$$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \text{ and } A^T \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix} = \begin{pmatrix} 5 + 2 + 45 & 1 + 2 + 40 \\ 35 + 4 - 9 & 7 + 4 - 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \ldots (2)$$

$\therefore$ *From* (1) & (2), $(AB)^T = B^T A^T$

Hence proved.

13. $\boldsymbol{If\ A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\ show\ that\ A^2 - 5A + 7I_2 = 0}$

Given: $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$L.H.S: A^2 - 5A + 7I_2$

$$A^2 = A \times A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$$\therefore A^2 = \begin{pmatrix} 8 & 5 \\ -3 & 3 \end{pmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5 \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

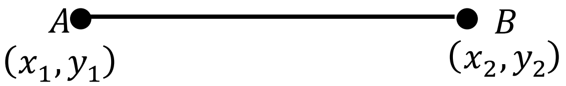
$$= \begin{pmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

EXERCISE 5.1

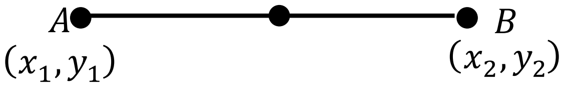
Distance between two points $A(x_1,y_1)$ *and* $B(x_2,y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



The midpoint of the line segment joining the points $A(x_1,y_1)$ and $B(x_2,y_2)$

$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

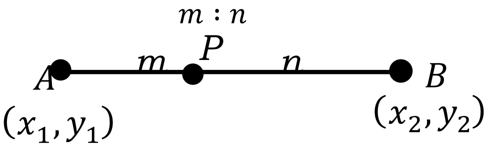


Section formula

The point P which divides the line segment joining the two points $A(x_1,y_1)$ *and* $B(x_2,y_2)$ *internally in the ratio* $m : n$ *is*

Internal division :

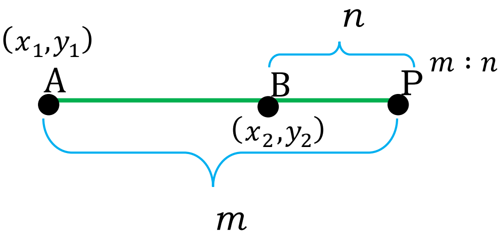
$$P(x,y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$



The point P which divides the line segment joining the two points $A(x_1,y_1)$ *and* $B(x_2,y_2)$ *externally in the ratio* $m : n$ *is*

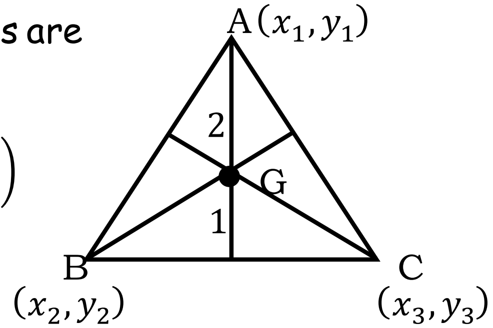
External division **:**

$$P(x,y) = \left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right]$$



The centroid of the triangle whose vertices are (x_1,y_1) , (x_2,y_2) and (x_3,y_3)

$$\textbf{centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

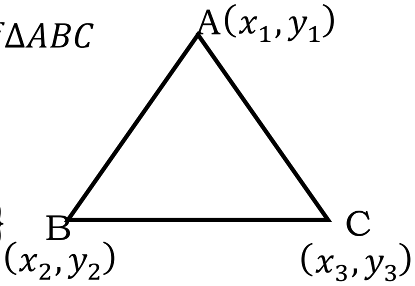


Area of a triangle

The vertices $A(x_1,y_1)$ *,*$B(x_2,y_2)$ *and* $C(x_3,y_3)$ *of* ΔABC

$$\Delta = \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$$

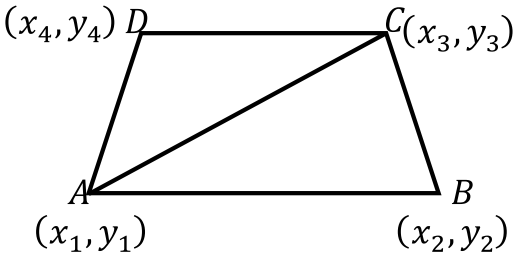
$$\Delta = \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\}$$



$$\Delta = \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}\ sq.units$$

Area of the quadrilateral ABCD

$A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ *and* $D(x_4,y_4)$
are the vertices of a quadrilateral ABCD



Area of the quadrilateral ABCD

$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\left\{ \begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \nwarrow\!\!\!\nearrow & & \nwarrow\!\!\!\nearrow & & \nwarrow\!\!\!\nearrow & & \nwarrow\!\!\!\nearrow & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix} \right\}$$

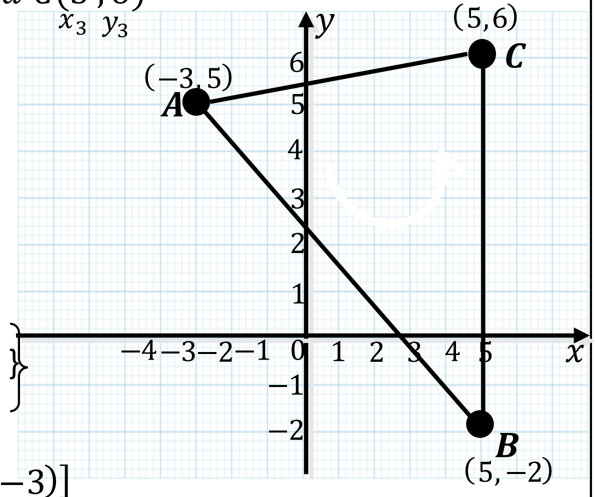
$$\Delta = \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\right\}$$

Area of the quadrilateral ABCD

$$= \frac{1}{2}\{(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\} sq.units$$

Example 5.1 : Find the area of the triangle whose vertices are $\boldsymbol{(-3,5),(5,6)}$ ***and*** $\boldsymbol{(5,-2)}$

Let the vertices be $A(\underset{x_1}{-3},\underset{y_1}{5}), B(\underset{x_2}{5}\,,\underset{y_2}{-2})$, *and* $C(\underset{x_3}{5}\,,\underset{y_3}{6})$



$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\left\{ \begin{matrix} -3 & & 5 & & 5 & & -3 \\ & \nwarrow\!\!\!\nearrow & & \nwarrow\!\!\!\nearrow & & \nwarrow\!\!\!\nearrow & \\ 5 & & -2 & & 6 & & 5 \end{matrix} \right\}$$

$$= \frac{1}{2}\left\{ 6 + 30 + 25 - \{25 - 10 - 18 \right\}$$

$$= \frac{1}{2}[61 - (25 - 28)] = \frac{1}{2}[61 - (-3)]$$

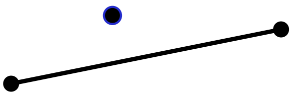
$$= \frac{1}{2}[61 + 3] = \frac{1}{\cancel{2}}[\overset{32}{\cancel{64}}] = 32 \; Sq.units$$

IDENTIFY POINTS, LINES, & PLANES

ON THE SAME LINE

Collinear

Non Collinear



Example 5.2

Show that the points $P(-1.5,3), Q(6,-2)$ and $R(-3,4)$ are collinear.

Let the vertices be $P(\underset{x_1}{-1.5},\underset{y_1}{3}), Q(\underset{x_2}{6},\underset{y_2}{-2})$ and $R(\underset{x_3}{-3},\underset{y_3}{4})$

$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{Bmatrix}$$

$$= \frac{1}{2}\left\{3 + 24 - 9 - \{18 \cancel{+ 6} \cancel{- 6}\}\right\}$$

$$= \frac{1}{2}[27 - 9 - 18] = \frac{1}{2}[27 - 27] = \frac{1}{2}(0) = 0$$

Hence, the given three points are collinear.

Example 5.3 If the area of the triangle formed by the vertices $A(-1,2), B(k,-2)$ and $C(7,4)$ (taken in order) is 22 sq.units, find the value of k.

Area of triangle ABC is 22 sq.units

Let the vertices be $A(\underset{x_1}{-1},\underset{y_1}{2}), B(\underset{x_2}{k},\underset{y_2}{-2})$ and $C(\underset{x_3}{7},\underset{y_3}{4})$

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = 22$$

$$\frac{1}{2}\begin{Bmatrix} -1 & k & 7 & -1 \\ 2 & -2 & 4 & 2 \end{Bmatrix} = 22$$

$$\frac{1}{2}\left\{2 + 4k + 14 - \{2k - 14 - 4\}\right\} = 22$$

$$\frac{1}{2}[4k + 16 - (2k - 18)] = 22 \implies 4k + 16 - 2k + 18 = 44$$

$$2k + 34 = 44 \implies 2k = 44 - 34 \implies 2k = 10$$

$$\boxed{k = 5}$$

Example 5.4 If the points $P(-1,-4), Q(b,c)$ and $R(5,-1)$ are collinear and if $2b + c = 4$ then find the values of b and c.

Since the points $P(\underset{x_1}{-1},\underset{y_1}{-4}), Q(\underset{x_2}{b},\underset{y_2}{c})$ and $R(\underset{x_3}{5},\underset{y_3}{-1})$ are collinear

The given points are collinear, then area of $\Delta PQR = 0$

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = 0$$

$$\frac{1}{2}\left\{\begin{matrix} -1 & b & 5 & -1 \\ -4 & c & -1 & -4 \end{matrix}\right\} = 0$$

$$\frac{1}{2}\left\{-c-b-20-\{-4b+5c+1\}\right\} = 0$$

$$-c-b-20+4b-5c-1=0$$

$$3b-6c-21=0 \Longrightarrow 3b-6c=21 \quad \div 3$$

$$b-2c=7\ldots(1)$$

$Given\ that\ 2b+c=4\ \ldots(2)$

$\boldsymbol{Solve\ (1)\ and\ (2)}$

$$(1) \Longrightarrow b-\cancel{2c}=7$$

$$(2)\times 2 \Longrightarrow 4b+\cancel{2c}=8$$

$$5b=15 \quad \Longrightarrow \quad b=\frac{\overset{3}{\cancel{15}}}{\cancel{5}}$$

$$\boxed{b=3}$$

$Sub\ b=3\ in\ equation\ (1)\ b-2c=7$

$$3-2c=7$$

$$-2c=7-3 \Longrightarrow -2c=4$$

$$c=-\frac{4}{2} \quad \Longrightarrow \boxed{c=-2}$$

Example 5.5 $\boldsymbol{The\ floor\ of\ a\ hall\ is\ covered\ with\ identical\ tiles\ which\ are\ in\ the\ shapes\ of\ triangles. One\ such\ triangle\ has\ the\ vertices\ at\ (-3,2),(-1,-1)\ and\ (1,2). If\ the\ floor\ of\ the\ hall\ is\ completely\ covered\ by\ 110\ tiles, find\ the\ area\ of\ the\ floor.}$

$Vertices\ of\ one\ triangular\ tile\ are\ at\ A(\underset{x_1}{-3},\underset{y_1}{2}), B(\underset{x_2}{-1},\underset{y_2}{-1})\ and\ C(\underset{x_3}{1},\underset{y_3}{2})$

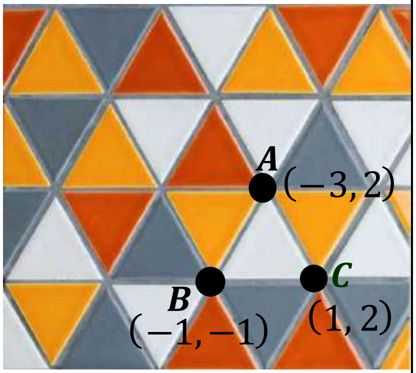
$$Area\ of\ this\ tile\ =\frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$$

$$\Delta=\frac{1}{2}\left\{\begin{matrix} -3 & -1 & 1 & -3 \\ 2 & -1 & 2 & 2 \end{matrix}\right\}$$

$$=\frac{1}{2}\left\{3-\cancel{2}+\cancel{2}-\{-2-1-6\}\right\} \quad =\frac{1}{2}[3-(-9)]$$

$$=\frac{1}{2}[3+9]=\frac{1}{\cancel{2}}[\overset{6}{\cancel{12}}]$$

$Area\ of\ this\ tile\ =6\ sq.units$



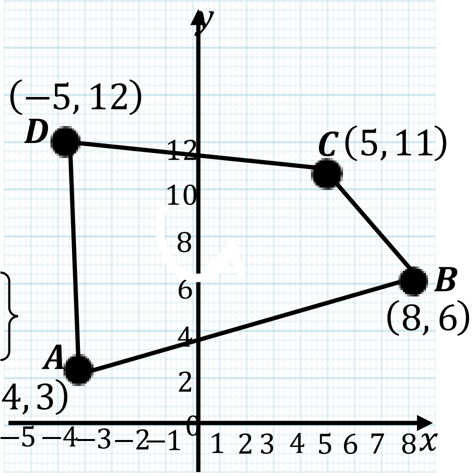
Since the floor is covered by 110 *triangle shaped identical tiles*

$$Area\ of\ floor = 110 \times 6\ sq.units$$

$$Area\ of\ floor = 660\ sq.units$$

Example 5.6 Find the area of the quadrilateral whose vertices are $(8,6), (5,11), (-5,12)$ and $(-4,3)$

Let the given points be $A(\underset{x_1}{-4}, \underset{y_1}{3}), B(\underset{x_2}{8}, \underset{y_2}{6}), C(\underset{x_3}{5}, \underset{y_3}{11})$, *and* $D(\underset{x_4}{-5}, \underset{y_4}{12})$



Area of the quadrilateral ABCD

$$= \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$$

$$= \frac{1}{2}\left\{\begin{matrix} -4 & 8 & 5 & -5 & -4 \\ 3 & 6 & 11 & 12 & 3 \end{matrix}\right\}$$

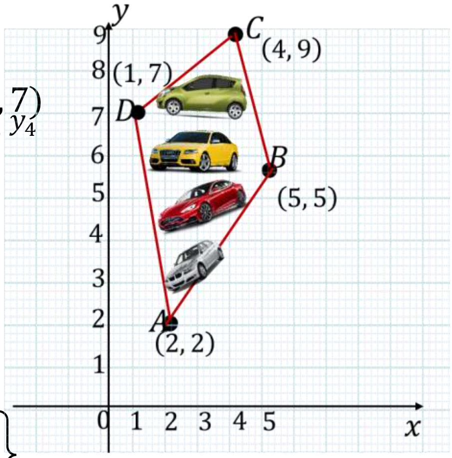
$$= \frac{1}{2}\left\{-24 + 88 + 60 - 15 - (24 + 30 - 55 - 48)\right\}$$

$$= \frac{1}{2}\{148 - 39 - (54 - 103)\}$$

$$= \frac{1}{2}\{109 - (-49)\} = \frac{1}{2}(109 + 49) = \frac{1}{\cancel{2}}(\cancel{158})^{79}$$

$\therefore$ *The area of the quadrilateral ABCD is* 79 *sq.units*

Example 5.7 The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost Rs.1300 per square feet. What will be the total cost for making the parking lot?



The parking lot is a quadrilateral whose vertices are at $A(\underset{x_1}{2}, \underset{y_1}{2}), B(\underset{x_2}{5}, \underset{y_2}{5}), C(\underset{x_3}{4}, \underset{y_3}{9})$ *and* $D(\underset{x_4}{1}, \underset{y_4}{7})$

$$Area\ of\ parking\ lot = \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$$

$$= \frac{1}{2}\left\{\begin{matrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{matrix}\right\}$$

$$= \frac{1}{2}\left\{10 + 45 + 28 + 2 - (10 + 20 + 9 + 14)\right\}$$

$$= \frac{1}{2}\{85 - 53\} = \frac{1}{\cancel{2}}\{\overset{16}{\cancel{32}}\}$$

Area of parking lot = 16 *sq.feets*

$$\text{Construction rate per square feet} = Rs.\,1300$$

$$\text{Total cost for constructing the parking lot} = 16 \times 1300 = Rs.\,20800$$

1. Find the area of the triangle formed by the points

(*i*) $(1,-1), (-4,6)$ and $(-3,-5)$

Let the vertices be $A(\underset{x_1}{-4},\underset{y_1}{6}), B(\underset{x_2}{-3},\underset{y_2}{-5})$, and $C(\underset{x_3}{1},\underset{y_3}{-1})$

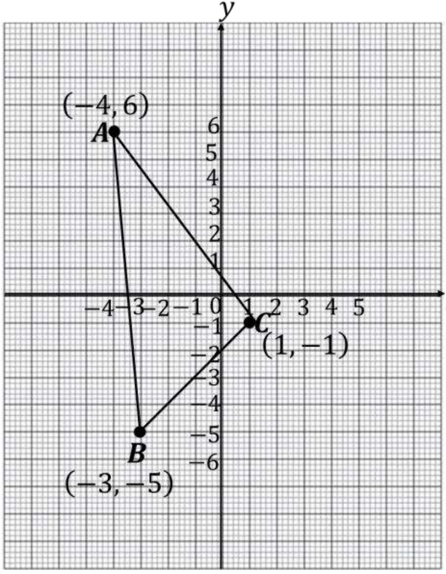
$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} -4 & -3 & 1 & -4 \\ 6 & -5 & -1 & 6 \end{Bmatrix}$$

$$= \frac{1}{2}\left\{20 + 3 + 6 - \{-18 - 5 + 4\}\right\}$$

$$= \frac{1}{2}[29 - (-23 + 4)] \quad = \frac{1}{2}[29 - (-19)]$$

$$= \frac{1}{2}[29 + 19] = \frac{1}{\cancel{2}}[\overset{24}{\cancel{48}}] = 24 \ Sq.units$$



1. Find the area of the triangle formed by the points

(*ii*) $(-10,-4), (-8,-1)$ and $(-3,-5)$

Let the vertices be $A(\underset{x_1}{-8},\underset{y_1}{-1}), B(\underset{x_2}{-10},\underset{y_2}{-4})$, and $C(\underset{x_3}{-3},\underset{y_3}{-5})$

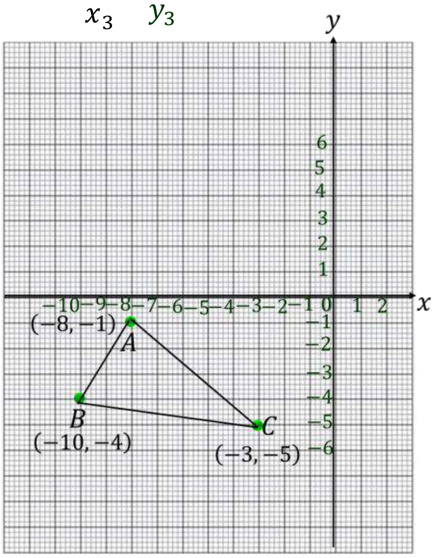
$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} -8 & -10 & -3 & -8 \\ -1 & -4 & -5 & -1 \end{Bmatrix}$$

$$= \frac{1}{2}\left\{32 + 50 + 3 - \{10 + 12 + 40\}\right\}$$

$$= \frac{1}{2}[85 - 62] = \frac{1}{\cancel{2}}[\overset{11.5}{\cancel{23}}]$$

$$= 11.5 \, Sq.units$$



2. Determine whether the set of points are collinear or not.

(*i*) $\left(-\frac{1}{2}, 3\right), (-5, 6)$ and $(-8, 8)$

(*i*) Let $A\left(\underset{x_1}{-\frac{1}{2}},\underset{y_1}{3}\right), B(\underset{x_2}{-5},\underset{y_2}{6})$ and $C(\underset{x_3}{-8},\underset{y_3}{8})$ be the given points.

$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$-\frac{1}{\cancel{2}} \times \cancel{6} = -3$$

$$-\frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} = -4$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} -\frac{1}{2} & -5 & -8 & -\frac{1}{2} \\ 3 & 6 & 8 & 3 \end{Bmatrix}$$

$$= \frac{1}{2}\left\{-3 - 40 - 24 - \{-15 - 48 - 4\}\right\} \quad = \frac{1}{2}[-67 - (-67)]$$

$$= \frac{1}{2}[-67 + 67] = \frac{1}{2}[0] = 0$$

Hence, the given three points are collinear.

2. Determine whether the set of points are collinear or not.

(ii) $(a, b + c), (b, c + a)$ and $(c, a + b)$

Let the vertices be $A(\underset{x_1}{a}, \underset{y_1}{b + c}), B(\underset{x_2}{b}, \underset{y_2}{c + a})$, *and* $C(\underset{x_3}{c}, \underset{y_3}{a + b})$

$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} a & b & c & a \\ b + c & c + a & a + b & b + c \end{Bmatrix}$$

$$= \frac{1}{2}\left\{a(c + a) + b(a + b) + c\,(b + c) - \{b(b + c) + c\,(c + a) + a(a + b)\}\right\}$$

$$= \frac{1}{2}\left\{ac + a^2 + ab + b^2 + bc + c^2 - \{b^2 + bc + c^2 + ac + a^2 + ab\}\right\}$$

$$= \frac{1}{2}\left\{\cancel{a}c + \cancel{a}^2 + \cancel{a}b + \cancel{b}^2 + \cancel{b}c + \cancel{c}^2 - \cancel{b}^2 - \cancel{b}c - \cancel{c}^2 - a\cancel{c} - \cancel{a}^2 - \cancel{a}b\right\}$$

$$= \frac{1}{2}[0] = 0$$

Hence, the given three points are collinear.

3. Vertices of the triangles taken in order and their areas are given. find the value of p

Vertices	*Area(in sq.units)*
(i) $(0, 0), (p, 8), (6, 2)$	20

Let the vertices be $A(\underset{x_1}{0}, \underset{y_1}{0})$, $B(\underset{x_2}{p}, \underset{y_2}{8})$, *and* $C(\underset{x_3}{6}, \underset{y_3}{2})$

The area of $\Delta ABC = 20$ *sq. units*

$$\frac{1}{2}\left\{\begin{matrix} 0 & p & 6 & 0 \\ 0 & 8 & 2 & 0 \end{matrix}\right\} = 20$$

$$\frac{1}{2}\left[0 + 2p + 0 - \{0 + 48 + 0\}\right] = 20$$

$$\frac{1}{2}\{2p - 48\} = 20 \Rightarrow 2p - 48 = 40$$

$$2p = 40 + 48 \Rightarrow 2p = 88 \Rightarrow p = \frac{88}{2} \Rightarrow p = 44$$

3. *Vertices of the triangles taken in order and their areas are given. find the value of p*

Vertices	*Area(in sq. units)*
(*ii*) $(p\,,p), (5\,,6), (5, -2)$	32

Let the vertices be $A(\underset{x_1}{p}\,,\underset{y_1}{p})\,, B(\underset{x_2}{5}\,,\underset{y_2}{6})$ *, and* $C(\underset{x_3}{5}\,,\underset{y_3}{-2})$

The area of $\Delta ABC = 32$ *sq. units*

$$\frac{1}{2}\left\{\begin{matrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{matrix}\right\} = 32$$

$$\frac{1}{2}\left[6p - 10 + 5p - \{5p + 30 - 2p\}\right] = 32$$

$$\frac{1}{2}[11p - 10 - (3p + 30] = 32 \Rightarrow \frac{1}{2}[11p - 10 - 3p - 30] = 32$$

$$8p - 40 = 64 \Rightarrow 8p = 104 \Rightarrow p = \frac{104}{8} \Rightarrow p = 13$$

4. *Find the value of* k *for which the given points are collinear.*
(*i*)(2 , 3), (4 , *a*) and (6 , −3)

Since the points $A(\underset{x_1}{2}\,,\underset{y_1}{3}), \mathrm{B}(\underset{x_2}{4}\,,\underset{y_2}{a})$ *and* $\mathrm{C}\,(\underset{x_3}{6}\,,\underset{y_3}{-3})$ *are collinear*

The given points are collinear, then area of $\Delta ABC = 0$

$$\frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\} = 0$$

$$\frac{1}{2}\left\{\begin{matrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{matrix}\right\} = 0$$

$$\frac{1}{2}\left\{2a - 12 + 18 - \{12 + 6a - 6\}\right\} = 0$$

$$2a + 6 - (6a + 6) = 0 \Rightarrow 2a + \cancel{6} - 6a - \cancel{6} = 0$$

$$-4a = 0 \Rightarrow a = 0$$

4. *Find the value of* k *for which the given points are collinear.*
$(ii)(a\,,2-2a),(-a+1\,,2a)$ and $(-4-a\,,6-2a)$

Since the points $A(\underset{x_1}{a}\,,\underset{y_1}{2-2a}), B(\underset{x_2}{-a+1}\,,\underset{y_2}{2a})$ *and* $C(\underset{x_3}{-4-a}\,,\underset{y_3}{6-2a})$ *are collinear*

The given points are collinear, then area of $\Delta ABC = 0$

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = 0$$

$$\frac{1}{2}\begin{Bmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{Bmatrix} = 0$$

$$\Big\{a(2a)+(-a+1)(6-2a)+(-4-a)\,(2-2a)-\{(-a+1)(2-2a)+2a\,(-4-a)+(6-2a)\,a\}\Big\} = 0$$

$$2a^2-\cancel{6a}+2a^2+6-\cancel{2a}-8+\cancel{8a}-2a+2a^2-[-2a+\cancel{2a^2}+2-2a-8a-\cancel{2a^2}+6a-2a^2]=0$$

$$6a^2-2-\cancel{2a}+\cancel{2a}-2+2a+8a-6a+2a^2=0$$

$$8a^2+4a-4=0$$

$$\div 4$$

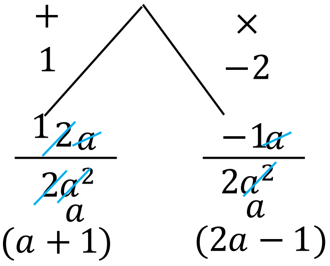
$$2a^2+a-1=0$$

$$(a+1)(2a-1)=0$$

$$a+1=0\,,\quad 2a-1=0$$

$$a=-1\,,\qquad 2a=1$$

$$a=\frac{1}{2}$$



5. *Find the area of the quadrilateral whose vertices are*
$(i)(-9,-2),(-8,-4),(2,2)$ *and* $(1,-3)$

y

Let the given points be $A(\underset{x_1}{-9}\,,\underset{y_1}{-2}), B(\underset{x_2}{-8}\,,\underset{y_2}{-4}), C(\underset{x_3}{1}\,,\underset{y_3}{-3})$, *and* $D(\underset{x_4}{2}\,,\underset{y_4}{2})$

Area of the quadrilateral ABCD

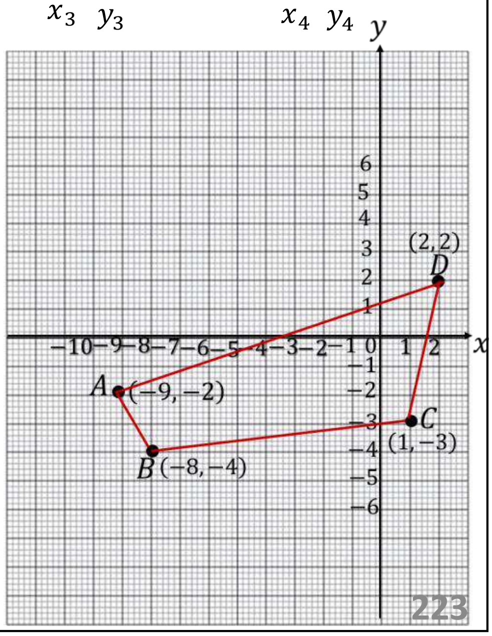
$$=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$=\frac{1}{2}\begin{Bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{Bmatrix}$$

$$=\frac{1}{2}\Big\{36+24+2-4-(16-4-6-18)\Big\}$$

$$=\frac{1}{2}\{62-4-(16-28)\}=\frac{1}{2}\{58-(-12)\}$$

$$=\frac{1}{2}\{58+12\}=\frac{1}{\cancel{2}}\{\cancel{70}^{35}\}=35\ Sq.units$$



5. Find the area of the quadrilateral whose vertices are
(ii) (−9, 0), (−8, 6), (−1, −2) and (−6, −3)

Let the given points be $A(\underset{x_1}{-8}, \underset{y_1}{-6}), B(\underset{x_2}{-9}, \underset{y_2}{0}), C(\underset{x_3}{-6}, \underset{y_3}{-3}), \text{and } D(\underset{x_4}{-1}, \underset{y_4}{-2})$

Area of the quadrilateral ABCD

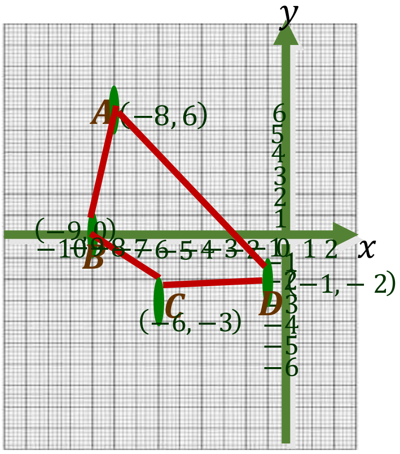
$$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -8 & -9 & -6 & -1 & -8 \\ -6 & 0 & -3 & -2 & -6 \end{Bmatrix}$$

$$= \frac{1}{2}\{0 + 27 + 12 - 6 - (-54 + 0 + 3 + 16)\}$$

$$= \frac{1}{2}\{39 - 6 - (-54 + 19)\} = \frac{1}{2}\{33 - (-35)\}$$

$$= \frac{1}{2}\{33 + 35\} = \frac{1}{\cancel{2}}\{\cancel{68}\}^{34} = 34\ Sq.units$$



6. Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (−4, −2), (−3, k), (3, −2) and (2, 3)

Let the given points be $A(\underset{x_1}{-4}, \underset{y_1}{-2}), B(\underset{x_2}{-3}, \underset{y_2}{k}), C(\underset{x_3}{3}, \underset{y_3}{-2}), \text{and } D(\underset{x_4}{2}, \underset{y_4}{3})$

Area of the quadrilateral $ABCD = 28\ sq.units$

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix} = 28$$

$$\frac{1}{2}\begin{Bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{Bmatrix} = 28$$

$$\frac{1}{2}\{-4k + 6 + 9 - 4 - (6 + 3k - 4 - 12)\} \rightarrow 28$$

$$-4k + 11 - (6 + 3k - 16) = 56 \Rightarrow -4k + 11 - (3k - 10) = 56$$

$$-4k + 11 - 3k + 10 = 56 \Rightarrow -7k + 21 = 56$$

$$-7k = 56 - 21 \Rightarrow -7k = 35 \Rightarrow k = -\frac{35}{7} \Rightarrow \boxed{k = -5}$$

7. If the points A(−3, 9), B (a, b) and C(4, −5) are collinear and if $a + b = 1$, then find a and b.

Since the points $A(\underset{x_1}{-3}, \underset{y_1}{9}), \mathrm{B}(\underset{x_2}{a}, \underset{y_2}{b}) \text{and } \mathrm{C}(\underset{x_3}{4}, \underset{y_3}{-5})$ *are collinear*

The given points are collinear, then area of $\Delta ABC = 0$

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = 0$$

$$\frac{1}{2}\left\{\begin{matrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{matrix}\right\} = 0$$

$$\frac{1}{2}\left\{-3b - 5a + 36 - \{9a + 4b + 15\}\right\} = 0$$

$$-3b - 5a + 36 - 9a - 4b - 15 = 0$$

$$-14a - 7b + 21 = 0 \implies -14a - 7b = -21$$

$$14a + 7b = 21$$

$$\div 7$$

$$2a + b = 3 \ldots (1)$$

$$Given\ that\ a + b = 1 \ldots (2)$$

$$Solve\ (1)\ and\ (2)$$

$$\begin{array}{rl} (1) \implies & 2a + \cancel{b} = 3 \\ & (-)\ \ (-)\ \ \ (-) \\ (2) \implies & \ \ a + \cancel{b} = 1 \\ \hline & a = 2 \end{array}$$

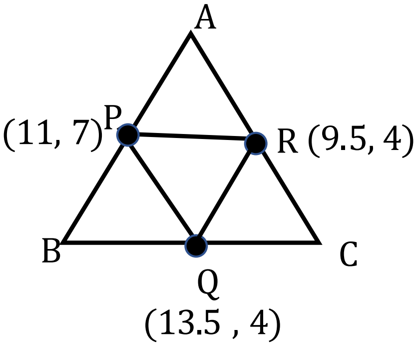
$$Sub\ a = 2\ in\ equation\ (2)\ a + b = 1$$

$$2 + b = 1$$

$$b = 1 - 2 \implies b = -1$$

8. Let $P(11, 7), Q(13.5, 4)$ and $R(9.5, 4)$ be the mid − points of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

$P(11, 7), Q(13.5, 4),$ and $R(9.5, 4)$ are the mid points of the sides of a ΔABC



$$Mid\ point = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$Midpoint\ of\ A(x_1, y_1)\ B(x_2, y_2) = (11,7)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (11,7)$$

$$\frac{x_1 + x_2}{2} = 11, \quad \frac{y_1 + y_2}{2} = 7$$

$$x_1 + x_2 = 22 \ldots (1)\ y_1 + y_2 = 14 \ \ldots (2)$$

$$Midpoint\ of\ B(x_2, y_2)\ and\ C(x_3, y_3) = (13.5,4)$$

$$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (13.5,4)$$

$$\frac{x_2 + x_3}{2} = 13.5, \quad \frac{y_2 + y_3}{2} = 4$$

$x_2 + x_3 = 27, \ldots (3) \qquad y_2 + y_3 = 8 \ \ldots (4)$

$Midpoint\ of\ A(x_1, y_1), C(x_3, y_3) = (9.5,4)$

$$\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) = (9.5,4)$$

$$\frac{x_1 + x_3}{2} = 9.5, \ \frac{y_1 + y_3}{2} = 4$$

$$x_1 + x_3 = 19 \ \ldots (5) \qquad y_1 + y_3 = 8 \ \ldots (6)$$

$Solve\ (1)\ and\ (3)$

$$\begin{array}{rl} (1) \Longrightarrow & x_1 + \cancel{x_2} = 22 \\ & (-)\ (-)\quad (-) \\ (3) \Longrightarrow & x_3 + \cancel{x_2} = 27 \\ \hline & x_1 - x_3 = -5 \ \ldots (7) \end{array}$$

$Solve\ (5)\ and\ (7)$

$$\begin{array}{rl} (5) \Longrightarrow & x_1 + \cancel{x_3} = 19, \\ (7) \Longrightarrow & x_1 - \cancel{x_3} = -5 \\ \hline & 2x_1 = 12 \ \Longrightarrow x_1 = \dfrac{12}{2} \\ & x_1 = 6 \end{array}$$

$\text{Sub } x_1 = 6 \text{ in } (5)\ x_1 + x_3 = 19$

$6 + x_3 = 19 \Longrightarrow x_3 = 19 - 6$

$$\boxed{x_3 = 13}$$

$\text{Sub } x_3 = 13 \text{ in } (3) x_2 + x_3 = 27$

$x_2 + 13 = 27 \Longrightarrow x_2 = 27 - 13$

$$\boxed{x_2 = 14}$$

$Solve\ (2)\ and\ (4)$

$$\begin{array}{rl} (1) \Longrightarrow & y_1 + \cancel{y_2} = 14 \\ & (-)\ (-)\quad (-) \\ (3) \Longrightarrow & y_3 + \cancel{y_2} = 8 \\ \hline & y_1 - y_3 = 6 \ldots (8) \end{array}$$

$Solve\ (6)\ and\ (8)$

$$\begin{array}{rl} (5) \Longrightarrow & y_1 + \cancel{y_3} = 8 \\ (7) \Longrightarrow & y_1 - \cancel{y_3} = 6 \\ \hline & 2y_1 = 14 \ \Longrightarrow y_1 = \dfrac{14}{2} \\ & y_1 = 7 \end{array}$$

$\text{Sub } y_1 = 7 \text{ in } (6)\ y_1 + y_3 = 8$

$7 + y_3 = 8 \Longrightarrow y_3 = 8 - 7$

$$y_3 = 1$$

$\text{Sub } y_1 = 7 \text{ in } (6)\ y_1 + y_3 = 8$

$7 + y_3 = 8 \Longrightarrow y_3 = 8 - 7$

$$\boxed{y_3 = 1}$$

$\text{Sub } y_1 = 7 \text{ in } (2)\ y_1 + y_2 = 14$

$7 + y_2 = 14 \Longrightarrow y_2 = 14 - 7$

$$\boxed{y_2 = 7}$$

$\therefore\ the\ vertices\ of\ A(\underset{x_1}{6}, \underset{y_1}{7}), B(\underset{x_2}{14}, \underset{y_2}{7})\ and\ C(\underset{x_3}{13}, \underset{y_3}{1})$

$$\Delta = \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$$

$$\Delta = \frac{1}{2}\left\{\begin{array}{cccc} 6 & 14 & 13 & 6 \\ 7 & 7 & 1 & 7 \end{array}\right\}$$

$$= \frac{1}{2}\left\{42 + 14 + 91 - \{98 + 91 + 6\}\right\} = \frac{1}{2}\{147 - 195\}$$

Area of a ΔPQR $P(\underset{x_1}{11},\underset{y_1}{7}), Q(\underset{x_2}{13.5},\underset{y_2}{4})$ *and* $R(\underset{x_3}{9.5},\underset{y_3}{4})$

$$\Delta = \frac{1}{2}\left\{\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array}\right\}$$

$$\Delta = \frac{1}{2}\left\{\begin{array}{cccc} 11 & 13.5 & 9.5 & 11 \\ 7 & 4 & 4 & 7 \end{array}\right\}$$

$$= \frac{1}{2}\left\{44 + 54 + 66.5 - \{94.5 + 38 + 44\}\right\}$$

$$= \frac{1}{2}\{164.5 - 176.5\} = \frac{1}{2}\{-12\} = -6$$

$$= 6\ sq.units$$

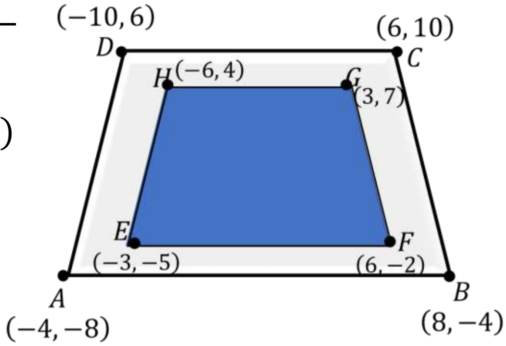
$$Area\ of\ \Delta ABC = 24 = 4 \times 6$$

$$Area\ of\ \Delta ABC = 4 \times Area\ of\ \Delta PQR$$

9. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.

Area of the patio

$= Area\ of\ the\ quadrilateral\ ABCD -$
$Area\ of\ the\ quadrilateral\ EFGH$



$A(\underset{x_1}{-4},\underset{y_1}{-8}), B(\underset{x_2}{8},\underset{y_2}{-4}), C(\underset{x_3}{6},\underset{y_3}{10}),$ *and* $D(\underset{x_4}{-10},\underset{y_4}{6})$

Area of the quadrilateral ABCD

$$= \frac{1}{2}\left\{\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{array}\right\}$$

$$= \frac{1}{2}\left\{\begin{array}{ccccc} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{array}\right\}$$

$$= \frac{1}{2}\left\{16 + 80 + 36 + 80 - (-64 - 24 - 100 - 24)\right\} = \frac{1}{2}\{212 - (-212)\}$$

$$= \frac{1}{2}\{212 + 212\} = \frac{1}{\cancel{2}}\{\cancel{424}\}212$$

$$Area\ of\ the\ quadrilateral\ ABCD = 212\ Sq.units$$

$E(\underset{x_1}{-3},\underset{y_1}{-5}), F(\underset{x_2}{6},\underset{y_2}{-2}), G(\underset{x_3}{3},\underset{y_3}{7}), and\ H(\underset{x_4}{-6},\underset{y_4}{4})$

$$Area\ of\ the\ quadrilateral\ EFGH = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{Bmatrix}$$

$$= \frac{1}{2}\{6+42+12+30-(-30-6-42-12)\}$$

$$= \frac{1}{2}\{90-(-90)\} = \frac{1}{2}\{90+90\} = \frac{1}{\cancel{2}}\{\overset{90}{\cancel{180}}\}$$

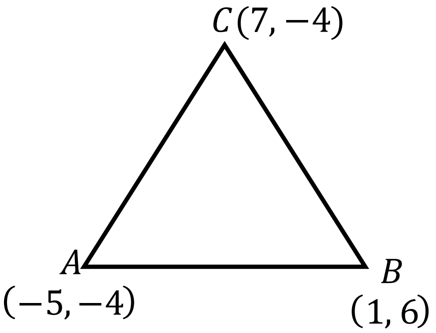
$Area\ of\ the\ quadrilateral\ EFGH = 90\ Sq.units$

$Area\ of\ the\ patio = Area\ of\ the\ quadrilateral\ ABCD -$
$Area\ of\ the\ quadrilateral\ EFGH$

$$= 212 - 90 = 122\ Sq.units$$

10. A triangular shaped glass with vertices at A(−5, −4), B(1, 6) and C(7, −4) has to be painted. If one bucket of paint covers 6 squar feets, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

$A(\underset{x_1}{-5},\underset{y_1}{-4}), B(\underset{x_2}{1},\underset{y_2}{6}), and\ C(\underset{x_3}{7},\underset{y_3}{-4})$



$$\Delta = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$\Delta = \frac{1}{2}\begin{Bmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{Bmatrix}$$

$$= \frac{1}{2}\{-30-4-28-\{-4+42+20\}$$

$$= \frac{1}{2}[-62-(-4+62)] = \frac{1}{2}[-62-58] \quad = \frac{1}{\cancel{2}}[\overset{-60}{\cancel{-120}}] = 60\ Sq.units$$

$$No.of\ paint\ cans = \frac{Area\ of\ a\ \Delta ABC}{Area\ covered\ by\ a\ bucket\ of\ paint}$$

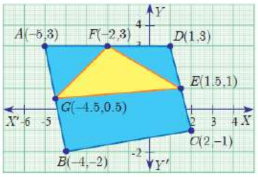
$$No.of\ paint\ cans = \frac{60}{6} = 10$$

10. In the figure, find the area of (i) triangle AGF (ii) triangle FED (iii) quadrilateral BCEG

(i) Area of triangle AGF

$A(\underset{x_1}{-5},\underset{y_1}{3}), G(\underset{x_2}{-4.5},\underset{y_2}{0.5}), F(\underset{x_3}{-2},\underset{y_3}{3})$

$$Area\ of\ \Delta\,AGF = \frac{1}{2}\left\{\begin{array}{cccc} -5 & -4.5 & -2 & -5 \\ 3 & 0.5 & 3 & 3 \end{array}\right\}$$



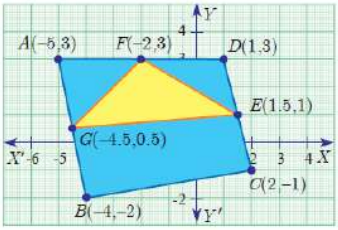
$$= \frac{1}{2}\left\{-2.5-13.5-6-\{-13.5-1-15\right\}$$

$$= \frac{1}{2}\{-22-(-29.5)\} = \frac{1}{2}\{-22+29.5\}$$

$$= \frac{1}{2}\{7.5\} = 3.75\ sq.units$$

(ii) $Area\ of\ triangle\ FED$

$$\underset{x_1\ \ y_1}{F(-2,3)},\ \underset{x_2\ \ y_2}{E(1.5,1)},\ \underset{x_3\ y_3}{D(1,3)}$$



$$Area\ of\ \Delta\,FED = \frac{1}{2}\left\{\begin{array}{cccc} -2 & 1.5 & 1 & -2 \\ 3 & 1 & 3 & 3 \end{array}\right\}$$

$$= \frac{1}{2}\left\{-2+4.5+3-\{4.5+1-6\right\}$$

$$= \frac{1}{2}\{5.5-(5.5-6)\}$$

$$= \frac{1}{2}\{5.5-(-0.5)\} = \frac{1}{2}\{5.5+0.5\}$$

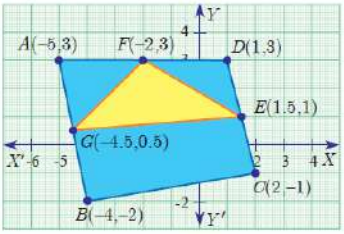
$$= \frac{1}{2}\{6\} = 3\ sq.units$$

(iii) $Area\ of\ the\ quadrilateral\ BCEG$

$$\underset{x_1\ \ y_1}{B(-4,-2)},\ \underset{x_2\ \ y_2}{C(2,-1)},\ \underset{x_3\ \ y_3}{E(1.5,1)},\ \underset{x_4\ \ \ y_4}{G(-4.5,0.5)}$$

$$Area\ of\ the\ quadrilateral\ BCEG = \frac{1}{2}\left\{\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{array}\right\}$$

$$= \frac{1}{2}\left\{\begin{array}{ccccc} -4 & 2 & 1.5 & -4.5 & -4 \\ -2 & -1 & 1 & 0.5 & -2 \end{array}\right\}$$



$$= \frac{1}{2}\left\{4+2+7.5+9-(-4-1.5-4.5-2)\right\}$$

$$= \frac{1}{2}\{15.75-(-12)\} = \frac{1}{2}\{15.75+12\}$$

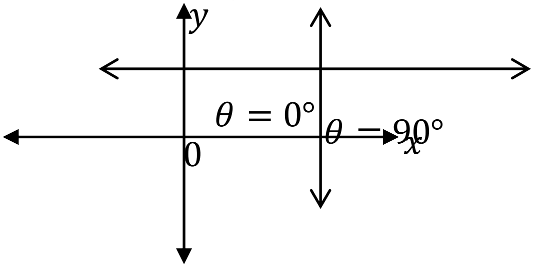
$$= \frac{27.75}{2} = 13.875\ sq.units$$

EXERCISE 5.2

Inclination of a line

The inclination of a line or the angle of inclination of a line is the angle which a straight line makes with the positive direction of X axis measured in the counter – clockwise direction to the part of the line above the X axis.

The inclination of X axis and every line parallel to X axis is $0°$.
The inclination of Y axis and every line parallel to Y axis is $90°$.



Given	Slope
Angle of inclination	$m = tan\theta$
Two points(x_1,y_1) and (x_2,y_2)	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$St. line\ ax + by + c = 0$	$m = -\frac{a}{b}$

Slope of a Straight line

Example 5.8 (i) What is the slope of a line whose inclination is $\mathbf{30°}$?

(ii) What is the inclination of a line whose slope is $\sqrt{3}$

(i) Given that $\theta = 30^\circ$

$m = tan\,\theta \implies m = tan\,30^\circ$

$$\boxed{m = \frac{1}{\sqrt{3}}}$$

(ii) Given that the slope of the straight line,

$m = \sqrt{3} \implies \tan\theta = \sqrt{3}$

where $m = tan\theta$

$\theta = 60^\circ$

$\boxed{\therefore \text{The angle of inclination of the straight line is } 60^\circ}$

Example 5.9 Find the slope of a line joining the given points

(i) $(-6,1)$ and $(-3,2)$ (ii) $\left(-\frac{1}{3},\frac{1}{2}\right)$ and $\left(\frac{2}{7},\frac{3}{7}\right)$ (iii) $(14,10)$ and $(14,-6)$

Slope of a line joining two given points is $m=\dfrac{y_2-y_1}{x_2-x_1}$

$$\underset{x_1\,,y_1}{(-6,1)},\ \underset{x_2\,,y_2}{(-3\,,2)}$$

$$m=\frac{2-1}{-3+6}=\frac{1}{3}$$

(ii) $\underset{x_1\,,y_1}{\left(-\frac{1}{3},\frac{1}{2}\right)}$ *and* $\underset{x_2\,,y_2}{\left(\frac{2}{7},\frac{3}{7}\right)}$

$$m=\frac{y_2-y_1}{x_2-x_1}\Longrightarrow m=\frac{\frac{3}{7}\times\frac{1}{2}}{\frac{2}{7}\times\frac{1}{3}}\Longrightarrow m=\frac{\frac{6-7}{14}}{\frac{6+7}{21}}\Longrightarrow m=\frac{\frac{-1}{14}}{\frac{13}{21}}$$

$$m=\frac{-1}{\underset{2}{\cancel{14}}}\times\frac{\cancel{21}^{\,3}}{13}\Longrightarrow m=\frac{-3}{26}$$

$$(iii)\ \underset{x_1\,,y_1}{(14{,}10)}\ and\ \underset{x_2\,,y_2}{(14,-6)}$$

$$m=\frac{y_2-y_1}{x_2-x_1}\Longrightarrow m=\frac{-6-10}{14-14}\Longrightarrow m=\frac{-16}{0}$$

$$\boxed{m=\infty}$$

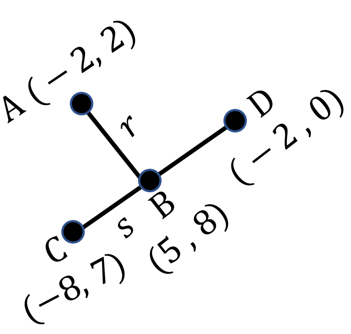
Example 5.10 The line r passes through the points $(-2,2)$ and $(5{,}8)$ and the line s passes through the points $(-8,7)$ and $(-2,0)$. Is the line r perpendicular to s?

$$A\underset{x_1\ y_1}{(-2,2)},B\underset{x_2 y_2}{(5{,}8)}$$

$$Slope\ of\ r=\frac{y_2-y_1}{x_2-x_1}=\frac{8-2}{5+2}\Longrightarrow\quad m_1=\frac{6}{7}$$

$$C\underset{x_1\ \ y_1}{(-8,7)},D\underset{x_2\,y_2}{(-2{,}0)}$$

$$Slope\ of\ CD=\frac{y_2-y_1}{x_2-x_1}=\frac{0-7}{-2+8}\Longrightarrow\quad m_2=-\frac{7}{6}$$



$$m_1\times m_2=\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{-7}}{\cancel{6}}$$

$$m_1\times m_2=-1$$

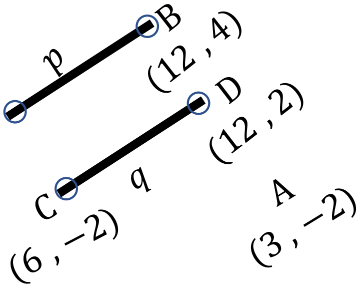
Hence the given lines r and s are perpendicular .

$$\boxed{\begin{array}{l}Let\ slope\ of\ r=m_1\\ and\ slope\ of\ q=m_2\end{array}}$$

***Example* 5.11 *The line p passes through the points* (3 ,– 2),(12, 4) *and the line q passes through the points* (6,–2) *and* (12 ,2). *Is p parallel to q* ?**

$$A(\underset{x_1}{3}\ ,\underset{y_1}{-2}),\ B(\underset{x_2}{12}\ ,\underset{y_2}{4})$$

$$Slope\ of\ p = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4+2}{12-3} = \frac{\overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}} \Rightarrow \quad m_1 = \frac{2}{3}$$



$$C(\underset{x_1}{6},\underset{y_1}{-2}),\ D(\underset{x_2}{12},\ \underset{y_2}{2})$$

$$Slope\ of\ q = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2+2}{12-6} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} \Rightarrow m_2 = \frac{2}{3}$$

Let slope of $p = m_1$
and slope of $q = m_2$

$$\therefore m_1 = m_2$$

Hence the given lines p and q are parallel

***Example* 5.11: *The line p passes through the points* (3 ,–2),(12, 4) *and the line q passes through the points* (6,–2) *and* (12 ,2). *Is p parallel to q* ?**

$$(\underset{x_1}{3},\underset{y_1}{-2})\,and\ (\underset{x_2}{12},\underset{y_2}{4})$$

$$Slope\ of\ p = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$$

$$Slope\ of\ p\ = \frac{3}{5}$$

$$(\underset{x_1}{6},\underset{y_1}{-2})\,and\ (\underset{x_2}{12},\underset{y_2}{2})$$

$$Slope\ of\ q = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2\ +2}{12-6} = \frac{4}{6} = \frac{2}{3}$$

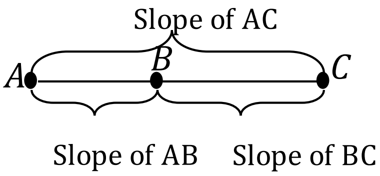
$$Slope\ of\ q = \frac{2}{3} \qquad \therefore Slope\ of\ p \neq Slope\ of\ q$$

Hence the given lines p and q are not parallel

Example* 5.12: *Show that the points* (–2,5), (6, –1) *and* (2 ,2) *are collinear

$$A(\underset{x_1}{-2},\underset{y_1}{5}),\ B(\underset{x_2}{6},\underset{y_2}{-1})$$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{6+2} = \frac{-\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = -\frac{3}{4}$$



$$Slope\ of\ AB\ = -\frac{3}{4}$$

$$B(\underset{x_1}{6},\underset{y_1}{-1}),\ C(\underset{x_2}{2}\ ,\underset{y_2}{2})$$

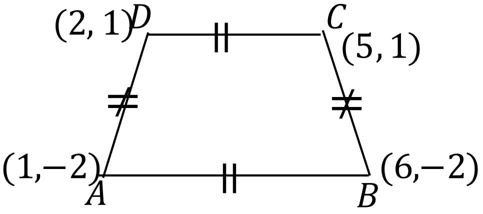
$$Slope\ of\ BC\ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2+1}{2-6} = -\frac{3}{4}$$

$$Slope\ of\ BC\ = -\frac{3}{4}$$

$$Slope\ of\ AB\ = \ Slope\ of\ BC$$

$$Hence\ A, B, C\ are\ collinear$$

$$B(\underset{x_1}{6}, \underset{y_1}{-2}), C(\underset{x_2}{5}, \underset{y_2}{1})$$



$$Slope\ of\ BC\ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+2}{5-6} = \frac{3}{-1}$$

$$Slope\ of\ BC = -3$$

$$\text{Let } A(\underset{x_1}{1}, \underset{y_1}{-2}),\ D(\underset{x_2}{2}, \underset{y_2}{1})$$

$$Slope\ of\ AD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+2}{2-1} = \frac{3}{1} = 3$$

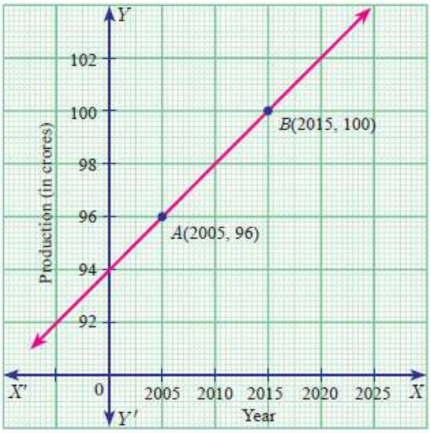
$$Slope\ of\ AD = 3 \quad \therefore Slope\ of\ BC \neq Slope\ of\ AD$$

$The\ slope\ of\ AB\ and\ CD\ are\ equal\ so\ AB, CD\ are\ parallel.$

Similarly the lines AD and BC are not parallel, since their slopes are not equal.

So, we can deduce that the quadrilateral ABCD is a trapezium.

Example 5.14: *Consider the graph representing growth of population (in crores). Find the slope of the line AB and hence estimate the population in the year 2030?*



The points $A(2005, 96)$ *and* $B(2015,100)$ *are on the line AB.*

$$A(\underset{x_1}{2005}, \underset{y_1}{96}), B(\underset{x_2}{2015}, \underset{y_2}{100})$$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{100 - 96}{2015 - 2005}$$

$$= \frac{\cancel{4}^{\,2}}{\cancel{10}_{\,5}} = \frac{2}{5}$$

$$Slope\ of\ AB = \frac{2}{5}$$

Let the growth of population in 2030 be k crores.

Assuming that the point C $(2030, k)$ *is on AB,*

$$Slope\ of\ AB\ = \ Slope\ of\ AC$$

$$\underset{x_1}{A}(2005, \underset{y_1}{96}), C(\underset{x_2}{2030}, \underset{y_2}{k})$$

$$Slope\ of\ AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k\ -\ 96}{2030 - 2005}$$

$$Slope\ of\ AC = \frac{k - 96}{25}$$

$$\frac{k - 96}{\underset{5}{\cancel{25}}} = \frac{2}{\cancel{5}} \Rightarrow \frac{k - 96}{5} = 2 \Rightarrow k - 96 = 10$$

$$k = 10 + 96 \Rightarrow k = 106$$

$Hence\ the\ estimated\ population\ in\ 2030\ is\ 106\ Crores.$

Example 5.15: *Without using Pythagoras theorem, show that the vertices* (1, −4) (2, −3) *and* (4, −7) *form a right angled triangle.*

$The\ vertices\ are\ A(1, -4), B(2, -3)\ and\ C(4, -7)$

$$\text{Let } A(\underset{x_1}{1}, \underset{y_1}{-4}), B(\underset{x_2}{2}, \underset{y_2}{-3})$$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 + 4}{2 - 1} = \frac{1}{1} = 1$$

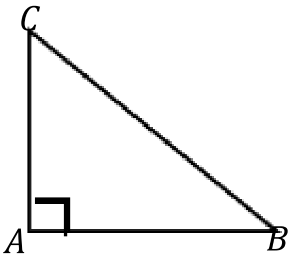
$$Slope\ of\ AB = 1$$

$$B(\underset{x_1}{2}, \underset{y_1}{-3}), C(\underset{x_2}{4}, \underset{y_2}{-7})$$

$$Slope\ of\ BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 + 3}{4 - 2} = -\frac{\overset{2}{\cancel{4}}}{\cancel{2}} = -2$$

$$Slope\ of\ BC = -2$$

$$A(\underset{x_1}{1}, \underset{y_1}{-4}), C(\underset{x_2}{4}, \underset{y_2}{-7})$$



$$Slope\ of\ AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 + 4}{4 - 1} = \frac{\cancel{-3}}{\cancel{3}}\ -1$$

$$Slope\ of\ AC = -1$$

$$Slope\ of\ AB \times Slope\ of\ AC = 1 \times -1$$

$$= -1$$

$AB\ is\ perpendicular\ to\ AC$

$Therefore, \Delta ABC\ is\ a\ right\ angled\ triangle.$

Example 5.16: *Prove analytically that the line segment joining the mid – points of two sides of a triangle is parallel to the third side and is equal to half of its length.*

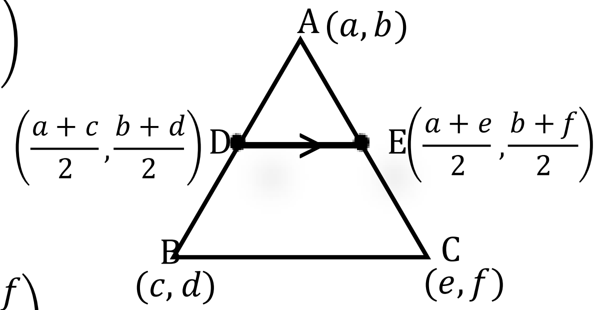
The vertices are $A(a,b)$, $B(c,d)$ *and* $C(e,f)$

Let D and E are the midpoint of the sides AB and AC

$$D = \text{midpoint of } A(\underset{x_1}{a},\underset{y_1}{b}) \text{ and } B(\underset{x_2}{c},\underset{y_2}{d})$$

$$= \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{a+c}{2},\frac{b+d}{2}\right)$$

$$D = \left(\frac{a+c}{2},\frac{b+d}{2}\right)$$



$$E = \text{midpoint of } A(\underset{x_1}{a},\underset{y_1}{b}) \text{ and } C(\underset{x_2}{e},\underset{y_2}{f})$$

$$= \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{a+e}{2},\frac{b+f}{2}\right)$$

$$E = \left(\frac{a+e}{2},\frac{b+f}{2}\right)$$

$$D\left(\underset{x_1}{\frac{a+c}{2}},\underset{y_1}{\frac{b+d}{2}}\right), E\left(\underset{x_2}{\frac{a+e}{2}},\underset{y_2}{\frac{b+f}{2}}\right)$$

$$\text{Slope of } DE = \frac{y_2-y_1}{x_2-x_1} = \frac{\frac{b+f}{2}-\frac{b+d}{2}}{\frac{a+e}{2}-\frac{a+c}{2}} = \frac{\frac{\cancel{b}+f-\cancel{b}-d}{\cancel{2}}}{\frac{\cancel{a}+e-\cancel{a}-c}{\cancel{2}}}$$

$$\text{Slope of } DE = \frac{f-d}{e-c}$$

$$B(\underset{x_1}{c},\underset{y_1}{d}), C(\underset{x_2}{e},\underset{y_2}{f})$$

$$\text{Slope of } BC = \frac{y_2-y_1}{x_2-x_1} = \frac{f-d}{\mathrm{e}-c}$$

$$\text{Slope of } BC = \frac{f-d}{\mathrm{e}-c} \qquad \therefore \text{Slope of } AB = \text{Slope of } AC$$

Therefore, DE is parallel to BC.

$$D\left(\underset{x_1}{\frac{a+c}{2}},\underset{y_1}{\frac{b+d}{2}}\right) \text{ and } E\left(\underset{x_2}{\frac{a+e}{2}},\underset{y_2}{\frac{b+f}{2}}\right)$$

$$DE = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$DE = \sqrt{\left(\frac{a+e}{2} - \frac{a+c}{2}\right)^2 + \left(\frac{b+f}{2} - \frac{b+d}{2}\right)^2}$$

$$= \sqrt{\left(\frac{\cancel{a}+e-\cancel{a}-c}{2}\right)^2 + \left(\frac{\cancel{b}+f-\cancel{b}-d}{2}\right)^2}$$

$$= \sqrt{\left(\frac{e-c}{2}\right)^2 + \left(\frac{f-d}{2}\right)^2} = \sqrt{\frac{(e-c)^2}{4} + \frac{(f-d)^2}{4}}$$

$$= \sqrt{\frac{1}{4}\left((e-c)^2 + (f-d)^2\right)}$$

$$DE = \frac{1}{2}\sqrt{(e-c)^2 + (f-d)^2}$$

$$B(\underset{x_1}{c}, \underset{y_1}{d}) and\ C(\underset{x_2}{e}, \underset{y_2}{f})$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(e-c)^2 + (f-d)^2}$$

$$\therefore DE = \frac{1}{2}BC$$

1. *What is the slope of a line whose inclination with positive direction of x − axis is* (*i*) 90 ° (*ii*) 0 °

(*i*) *Given that* $\theta = 90^\circ$

$$m = tan\,\theta \Rightarrow m = tan\,90^\circ$$

$$\boxed{m = \infty}$$

(*ii*) *Given that* $\theta = 0^\circ$

$$m = tan\,\theta \Rightarrow m = \tan 0^\circ$$

$$\boxed{m = 0}$$

2. *What is the inclination of a line whose slope is* (*i*) 0 (*ii*) 1

(*i*) *Given that the slope* $m = 0$

$\tan\theta = 0$ *where* $m = tan\theta$

$\theta = 0^\circ$

$$\boxed{\therefore \text{The angle of inclination of the straight line is } 0^\circ}$$

(*ii*) *Given that the slope* $m = 1$

$\tan\theta = 1 \quad where\ m = tan\theta$

$\theta = 45^\circ$

$\boxed{\therefore \text{The angle of inclination of the straight line is } 45^\circ}$

$Ex: 3$ Find the slope of a line joining the given points
(i) $(5, \sqrt{5})$ with the origin (ii) $(sin\theta, -cos\theta)$ and $(-sin\theta, cos\theta)$

Slope of a line joining two given points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$(i)\ \underset{x_1\ ,y_1}{(5, \sqrt{5})}, \underset{x_2\ ,y_2}{(0\ ,0)}$

$$m = \frac{0 - \sqrt{5}}{0 - 5} = \frac{-\sqrt{5}}{-5}$$

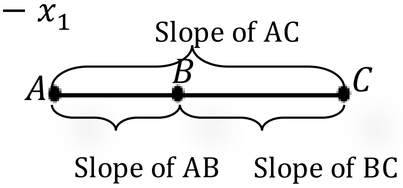
$$= \frac{\sqrt{5}}{5} = \frac{\cancel{\sqrt{5}}}{\cancel{\sqrt{5}} \times \sqrt{5}} = \frac{1}{\sqrt{5}}$$

$(ii)\ \underset{x_1\ ,y_1}{(sin\theta, -cos\theta)}\ and\ \underset{x_2\ ,y_2}{(-sin\theta, cos\theta)}$

$$m = \frac{cos\theta + cos\theta}{-sin\theta - sin\theta} = \frac{\cancel{2}cos\theta}{-\cancel{2}sin\theta} = -cot\theta$$

5. *Show that the given points are collinear*: $(-3, -4)$, $(7, 2)$ *and* $(12, 5)$

Slope of a line joining two given points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



$\underset{x_1\ ,y_1}{A(-3, -4)}, \underset{x_2, y_2}{B(7, 2)}$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 + 4}{7 + 3} = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}} = \frac{3}{5}$$

$$Slope\ of\ AB = \frac{3}{5}$$

$\underset{x_1\ ,y_1}{B(7, 2)}, \underset{x_2, y_2}{C(12\ ,5)}$

$$Slope\ of\ BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{12 - 7} = \frac{3}{5}$$

$$Slope\ of\ BC = \frac{3}{5}$$

Slope of AB = Slope of BC

Hence A, B, C are collinear

6. *If the three points* $(3, -1), (a\ ,3\ )$ *and* $(1, -3)$ *are collinear, find the value of a.*

Let $A(3, -1), B(a, 3), C(1, -3)$

Slope of AB = Slope of BC

$\underset{x_1\quad y_1}{A(3, -1)}, \underset{x_2\quad y_2}{B(a\ , 3)}$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+1}{a-3} = \frac{4}{a-3}$$

$$Slope\ of\ AB = -\frac{4}{a-3}$$

$$B(\underset{x_1}{a}, \underset{y_1}{3}), C(\underset{x_2}{1}, \underset{y_2}{-3})$$

$$Slope\ of\ BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-3}{1-\ a} = -\frac{-6}{1-a}$$

$$Slope\ of\ BC = \frac{-6}{1-a}$$

$$Slope\ of\ AB\ =\ Slope\ of\ BC$$

$$\frac{4}{a-3} = \frac{-6}{1-a} \Rightarrow 4(1-a) = -6(a-3)$$

$$4 - 4a = -6a + 18 \Rightarrow\ -4a + 6a = 18 - 4$$

$$2a = 14 \Rightarrow a = \frac{14}{2}$$

$$\boxed{a = 7}$$

7. *The line through the points* $(-2, a)$ *and* $(9,3)$ *has slope* $-\frac{1}{2}$ *. Find the value of* a.

Let $A(\underset{x_1}{-2}, \underset{y_1}{a}), B(\underset{x_2}{9}, \underset{y_2}{3})$

$$Slope\ of\ AB = -\frac{1}{2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -\frac{1}{2}$$

$$\frac{3-a}{9\ +2} = -\frac{1}{2} \Rightarrow \quad \frac{3-a}{11} = -\frac{1}{2} \Rightarrow 6 - 2a\ = -11$$

$$-2a = -11 - 6 \Rightarrow -2a = -17$$

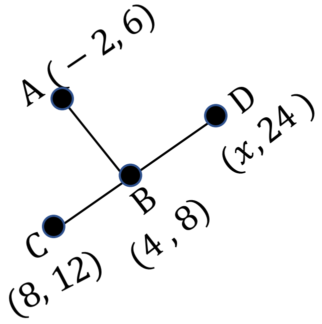
$$a = \frac{17}{2}$$

8. *The line through the points* $(-2,6)$ *and* $(4,8)$ *is perpendicular to the line through the points* $(8,12)$ *and* $(x,24)$ *. Find the value of* x.

$$A(\underset{x_1}{-2}, \underset{y_1}{6}), B(\underset{x_2}{4}, \underset{y_2}{8})$$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-\ 6}{4+2} = \frac{2}{6} = \frac{1}{3}$$

$$m_1 = \frac{1}{3}$$



$$C(\underset{x_1}{8},\underset{y_1}{12}), D(\underset{x_2}{x},\underset{y_2}{24})$$

$$Slope\ of\ CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24-12}{x-8} = \frac{12}{x-8}$$

$$m_2 = \frac{12}{x-8}$$

$Given\ lines\ are\ AB\ perpendicular\ to\ CD\ then \therefore m_1 \times m_2 = -1$

$$\frac{1}{\cancel{3}} \times \frac{\cancelto{4}{12}}{x-8} = -1$$

$$\frac{4}{x-8} = -1 \Rightarrow 4 = -1(x-8)$$

$$4 = -x + 8 \Rightarrow x = 8 - 4$$

$$\boxed{x = 4}$$

9. Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem (i) A (1,−4), B (2,−3) and C (4,−7)

$$\text{Let } A(\underset{x_1}{1},\underset{y_1}{-4}), B(\underset{x_2}{2},\underset{y_2}{-3})$$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3+4}{2-1} = \frac{\cancel{1}}{\cancel{1}} = 1$$

$$Slope\ of\ AB = 1$$

$$B(\underset{x_1}{2},\underset{y_1}{-3})\ and\ C(\underset{x_2}{4},\underset{y_2}{-7})$$

$$Slope\ of\ BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7+3}{4-2} = \frac{-\cancel{4}}{\cancel{2}} = -2$$

$$Slope\ of\ BC = -2$$

$$A(\underset{x_1}{1},\underset{y_1}{-4})\ and\ C(\underset{x_2}{4},\underset{y_2}{-7})$$

$$Slope\ of\ AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7+4}{4-1} = \frac{-\cancel{3}}{\cancel{3}} = -1$$

$Slope\ of\ AC = -1$

- ❖ $Slope\ of\ AB \times Slope\ of\ AC = 1 \times -1 = -1$
- ❖ $Hence\ AB\ perpendicular\ to\ AC$
- ❖ $Given\ points\ form\ a\ Right\ angled\ triangle$

To verify by using Pythagoras theorem

Let $A(\underset{x_1}{1},\underset{y_1}{-4}),\ B(\underset{x_2}{2},\underset{y_2}{-3}),$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-1)^2+(-3+4)^2} = \sqrt{1^2+1^2} = \sqrt{1+1}$$

$$AB = \sqrt{2} \Longrightarrow AB^2 = 2$$

Let $B(\underset{x_1}{2},\underset{y_1}{-3}),\ C(\underset{x_2}{4},\underset{y_2}{-7})$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4-2)^2+(-7+3)^2} = \sqrt{2^2+(-4)^2}$$

$$= \sqrt{4+16}$$

$$BC = \sqrt{20} \Longrightarrow BC^2 = 20$$

Let $A(\underset{x_1}{1},\underset{y_1}{-4}),\ C(\underset{x_2}{4},\underset{y_2}{-7}),$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4-1)^2+(-7+4)^2} = \sqrt{3^2+(-3)^2}$$

$$= \sqrt{9+9}$$

$$AC = \sqrt{18} \Longrightarrow AC^2 = 18$$

$$\boxed{\therefore \mathrm{AB}^2 + \mathrm{AC}^2 = \mathrm{BC}^2}$$

By Pythagoras theorem, the given points form a right angled triangle

(ii) $L\,(0{,}5)$, $M\,(9{,}12)$ and $N\,(3{,}14)$

$L\,(\underset{x_1}{0},\underset{y_1}{5}),\ M\,(\underset{x_2}{9},\underset{y_2}{-12})$

$$\textit{Slope of } LM = \frac{y_2-y_1}{x_2-x_1} = \frac{12-5}{9-0} = \frac{7}{9}$$

$$\textit{Slope of } LM = \frac{7}{9}$$

$M(\underset{x_1}{9},\underset{y_1}{12})$ and $N(\underset{x_2}{3},\underset{y_2}{14})$

$$\textit{Slope of } MN = \frac{y_2-y_1}{x_2-x_1} = \frac{14-12}{3-9} = \frac{\cancel{2}}{-\cancel{6}} = \frac{1}{-3}$$

$$\textit{Slope of } MN = -\frac{1}{3}$$

$L(\underset{x_1}{0},\underset{y_1}{5})$ and $N(\underset{x_2}{3},\underset{y_2}{14})$

$$\textit{Slope of } LN = \frac{y_2-y_1}{x_2-x_1} = \frac{14-4}{3-0} = \frac{\cancel{9}}{\cancel{3}} = 3$$

$$\textit{Slope of } LN = 3$$

$$\textit{Slope of } MN \times \textit{Slope of } LN = -\frac{1}{3} \times 3 = -1$$

- ❖ *Hence MN perpendicular to LN*

Given points form a Right angled triangle

To verify by using Pythagoras theorem

$$\text{L}\ (\underset{x_1}{0}, \underset{y_1}{5}), \text{M}\ (\underset{x_2}{9}, \underset{y_2}{12})$$

$$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9-0)^2 + (12-5)^2} = \sqrt{9^2 + 7^2} = \sqrt{81+49}$$

$$LM = \sqrt{130} \Longrightarrow \quad LM^2 = 130$$

$$\text{M}(\underset{x_1}{9}, \underset{y_1}{12}), \text{N}(\underset{x_2}{3}, \underset{y_2}{14})$$

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-9)^2 + (14-12)^2} = \sqrt{(-6)^2 + 2^2}$$

$$= \sqrt{36+4}$$

$$MN = \sqrt{40} \Longrightarrow MN^2 = 40$$

$$\text{L}\ (\underset{x_1}{0}, \underset{y_1}{5}), \text{N}\ (\underset{x_2}{3}, \underset{y_2}{14})$$

$$LN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-0)^2 + (14-5)^2} = \sqrt{3^2 + 9^2}$$

$$LN = \sqrt{9+81} \Longrightarrow LN = \sqrt{90} \Longrightarrow LN^2 = 90$$

$$\therefore \text{MN}^2 + \text{LN}^2 = \text{LM}^2$$

By Pythagoras theorem, the given points form a right angled triangle

10. *Show that the given points form a parallelogram* $A(2.5, 3.5)$, $B(10, -4)$, $C(2.5, -2.5)$ *and* $D(-5, 5)$

$$\text{A}(2.5,\ 3.5), B(10, -4), C(2.5, 2.5) \text{ and } D(-5, 5)$$

Slope of a line joining two given points $\text{A}(\underset{x_1}{2.5}, \underset{y_1}{3.5}), \text{B}(\underset{x_2}{10}, \underset{y_2}{-4})$

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 3.5}{10 + 2.5} = \frac{-\cancel{7.5}}{\cancel{7.5}} = -1$$

$$\text{Slope of } AB = -1$$

Slope of a line joining two given points $B(\underset{x_1}{10}, \underset{y_1}{-4})$ and $\text{C}(\underset{x_2}{2.5}, \underset{y_2}{-2.5})$

$$\text{Slope of } BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2.5 + 4}{2.5 - 10} = \frac{1.5}{-7.5}$$

$$= -\frac{\cancel{15}^{3}}{\cancel{75}_{15}} = \frac{\cancel{-3}^{1}}{\cancel{15}_{5}} = -\frac{1}{5} \qquad \therefore \text{Slope of } BC = \frac{-1}{5}$$

Slope of a line joining two given points $\text{C}(\underset{x_1}{2.5}, \underset{y_1}{-2.5}), \text{D}(\underset{x_2}{-5}, \underset{y_2}{5})$

$$\text{Slope of } CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 + 2.5}{-5 - 2.5} = \frac{7.5}{-7.5} = -1$$

$$Slope\ of\ CD = -1$$

Slope of a line joining two given points $A(\underset{x_1}{2.5}, \underset{y_1}{3.5})$ and $D(\underset{x_2}{-5}, \underset{y_2}{5})$

$$Slope\ of\ AD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3.5}{-5 - 2.5} = \frac{1.5}{-7.5} = \frac{\overset{3}{\cancel{-15}}}{\underset{15}{\cancel{75}}} = \frac{\overset{1}{\cancel{-3}}}{\underset{5}{\cancel{15}}} = -\frac{1}{5}$$

$$Slope\ of\ AD = -\frac{1}{5}$$

$Slope\ of\ AB = Slope\ of\ CD$ $\quad$ $Slope\ of\ BC = Slope\ of\ AD$

$\therefore AB\ perpendicular\ to\ CD\ then$ $\quad$ $\therefore AD\ perpendicular\ to\ BC\ then$

11. *If the points* $A(2,2), B(-2,-3), C(1,-3)$ *and* $D(x,y)$ *form a parallelogram then find the value of x and y.*

In a parallelogram slope of AB = slope of CD

Let $A(\underset{x_1}{2}, \underset{y_1}{2}), B(\underset{x_2}{-2}, \underset{y_2}{-3})$

$$Slope\ of\ AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{-2 - 2} = \frac{-5}{-4}$$

$$Slope\ of\ AB = \frac{5}{4}$$

$C(\underset{x_1}{1}, \underset{y_1}{-3})$ and $D(\underset{x_2}{x}, \underset{y_2}{y})$

$$Slope\ of\ CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y + 3}{x - 1}$$

$$Slope\ of\ CD = \frac{y + 3}{x - 1}$$

$$slope\ of\ AB = slope\ of\ CD$$

$$\frac{5}{4} = \frac{y + 3}{x - 1} \Longrightarrow 5(x - 1) = 4\,(y + 3)$$

$$5x - 5 = 4y + 12 \Longrightarrow 5x - 4y = 12 + 5$$

$$5x - 4\,y = 17\ldots(1)$$

Also slope of BC = slope of AD

Let $B(\underset{x_1}{-2}, \underset{y_1}{-3}), C(\underset{x_2}{1}, \underset{y_2}{-3})$

$$Slope\ of\ BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 + 3}{1 + 2} = \frac{0}{3} \quad \boxed{\therefore Slope\ of\ BC = 0}$$

Let $A(\underset{x_1}{2}, \underset{y_1}{2})$ *and* $D\,(\underset{x_2}{x}, \underset{y_2}{y})$

$$Slope\ of\ AD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - 2}{x - 2}$$

$$Slope\ of\ AD = \frac{y-2}{x-2}$$

$$slope\ of\ BC\ =\ slope\ of\ AD$$

$$\frac{y-2}{x-2} = 0 \Rightarrow y-2=0 \Rightarrow \boxed{y=2}$$

$$Sub\ y=2\ in\ (1)\ \ 5x-4y = 17$$

$$5x-4(2)\ =\ 17 \Rightarrow\ 5x-8 = 17$$

$$5x = 17+8 \Rightarrow\ \ 5x = 25$$

$$x = \frac{25}{5} \Rightarrow \boxed{x\ = 5}$$

12. Let A(3,–4), B(9,–4) , C(5,–7) and D(7,–7) . Show that ABCD is a trapezium.

Let $A(\underset{x_1}{3}, \underset{y_1}{-4}), B(\underset{x_2}{9}, \underset{y_2}{-4})$

$$Slope\ of\ AB = \frac{y_2-y_1}{x_2-x_1}\ = \frac{-4+4}{9\ -3} = \frac{0}{6} \qquad \therefore Slope\ of\ AB = 0$$

$B(\underset{x_1}{9}, \underset{y_1}{-4})\ and\ C(\underset{x_2}{5}, \underset{y_2}{-7})$

$$Slope\ of\ BC = \frac{y_2-y_1}{x_2-x_1} = \frac{-7+4}{5\ -9} = \frac{-3}{-4} \qquad \therefore Slope\ of\ BC = \frac{3}{4}$$

Let $C(\underset{x_1}{5}, \underset{y_1}{-7}), D(\underset{x_2}{7}, \underset{y_2}{-7})$

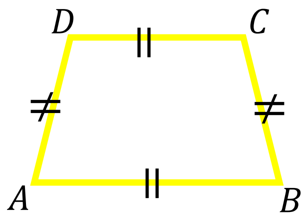
$$Slope\ of\ CD = \frac{y_2-y_1}{x_2-x_1} = \frac{-7+7}{7-5} = \frac{0}{2} \qquad \therefore Slope\ of\ CD = 0$$

$A(\underset{x_1}{3}, \underset{y_1}{-4})$ and $D(\underset{x_2}{7}, \underset{y_2}{-7})$

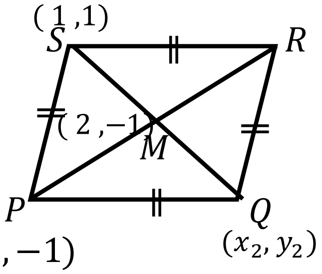
$$Slope\ of\ AD = \frac{y_2-y_1}{x_2-x_1} = \frac{-7+4}{7\ -3} = \frac{-3}{4} \qquad \therefore Slope\ of\ AD = -\frac{3}{4}$$

$$Slope\ of\ AB = Slope\ of\ CD \quad and\ Slope\ of\ BC \neq Slope\ of\ AD$$

$$Hence\ ABCD\ is\ a\ Trapezium$$



14. PQRS is a rhombus. Its diagonals PR and QS intersect at the point M and satisfy QS = 2PR. If the coordinates of S and M are (1,1) and (2,−1) respectively, find the coordinates of P.



Let S(1 ,1), Q(x_2, y_2)
$\quad x_1 \; y_1$

$$Midpoint\ of\ SQ = M(2,-1)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (2,-1) \Rightarrow \left(\frac{1 + x_2}{2}, \frac{1 + y_2}{2}\right) = (2,-1)$$

$$\frac{1 + x_2}{2} = 2,\ \frac{1 + y_2}{2} = -1 \Rightarrow\ 1 + x_2 = 4,\ 1 + y_2 = -2$$

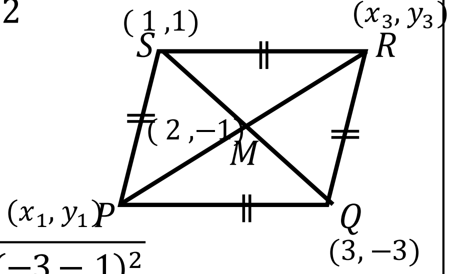
$$x_2 = 4 - 1,\ 1 + y_2 = -2 - 1 \Rightarrow\ x_2 = 3, y_2 = -3 \Rightarrow\ Q(3,-3)$$

P(x_1,y_1), R(x_3, y_3)

$$Midpoint\ of\ PR = (2,-1)$$

$$\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) = (2,-1) \Rightarrow \frac{x_1 + x_3}{2} = 2, \frac{y_1 + y_3}{2} = -1$$

$$x_1 + x_3 = 4, \ldots (1) \quad y_1 + y_3 = -2\ \ldots (2)$$



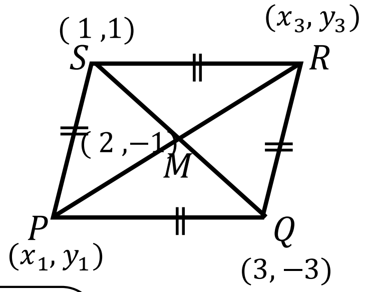
QS = 2PR

S(1 ,1), Q(3, −3)
$\quad x_1 \; y_1 \qquad x_2 \; y_2$

$$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-1)^2 + (-3-1)^2}$$

$$= \sqrt{(2)^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = \sqrt{4 \times 5}$$

$$SQ = 2\sqrt{5}$$



P(x_1,y_1), R(x_3, y_3)

$$PR = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$2\sqrt{5} = 2\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$2\sqrt{5} = 2\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

squaring on both sides

$$4 \times 5 = 4[(x_3 - x_1)^2 + (y_3 - y_1)^2]$$

$$5 = (x_3 - x_1)^2 + (y_3 - y_1)^2\ \ldots (3)$$

slope of PR

P(x_1,y_1), R(x_3, y_3)

$$slope\ of\ PR = \frac{y_3 - y_1}{x_3 - x_1}$$

Diagonals of a rhombus are at right angle

$$slope\ of\ PR \times slope\ of\ SQ = -1$$

$$\frac{y_3 - y_1}{x_3 - x_1} \times -2 = -1 \Rightarrow \frac{y_3 - y_1}{x_3 - x_1} \times 2 = 1$$

$$2(y_3 - y_1) = x_3 - x_1\ \ldots (4)$$

$sub\ x_3 - x_1 = 2(y_3 - y_1)\ in(1)$

$5 = [2(y_3 - y_1)]^2 + (y_3 - y_1)^2$

$5 = 4(y_3 - y_1)^2 + (y_3 - y_1)^2$

$5 = 5(y_3 - y_1)^2$

$1 = (y_3 - y_1)^2 \Longrightarrow y_3 - y_1 = 1 \ldots (5)$

$sub\ y_3 - y_1 = 1\ in\ (4) \quad 2(y_3 - y_1) = x_3 - x_1$

$2(1) = x_3 - x_1 \Longrightarrow x_3 - x_1 = 2 \ldots (6)$

$solve\ (1)\ and\ (6)$

$$\begin{array}{c} x_1 + x_3 = 4 \\ (-)\ (+)\ (-) \\ x_3 - x_1 = 2 \\ \hline 2x_1 = 2 \\ x_1 = 1 \end{array}$$

$solve\ (2)\ and\ (5)$

$$\begin{array}{c} y_1 + y_3 = -2 \\ (-)\ (+)\ (-) \\ y_3 - y_1 = 1 \\ \hline 2y_1 = -3 \\ y_1 = -\frac{3}{2} \end{array}$$

$\therefore The\ coordinats\ P\left(1, -\frac{3}{2}\right)$

$slope\ of\ SQ$

$S(\underset{x_1}{1}, \underset{y_1}{1}),\ Q(\underset{x_2}{3}, \underset{y_2}{-3})$

$slope\ of\ SQ = \frac{-3-1}{3-1}$

$slope\ of\ SQ = \frac{-4}{2}$

$slope\ of\ SQ = -2$

EXERCISE 5.3

5.17 Find the equation of a straight line passing through (5,7) and is
(i) parallel to x – axis (ii) parallel to y – axis

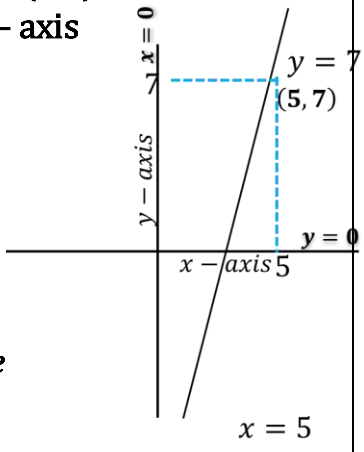
Equation of straight line passing through (5,7)

a) *Parallel to* $x - axis$

$$y = 7$$

b) *Parallel to* $y - axis$

$$\boxed{x = 5}$$



5.18 ***Find the equation of a straight line whose***

(i) ***slope is*** $\mathbf{5}$ ***and y intercept is*** $\mathbf{-9}$

(ii) ***inclination is*** $\mathbf{45^o}$ ***and y intercept is*** $\mathbf{11}$

Given: $slope: m = 5$ *and* y *intercept* $= -9$

The equation of the line is $y = mx + c$

$$y = (5)x - 9 \implies y = 5x - 9$$

$$5x - y - 9 = 0$$

(ii) ***inclination is*** $\mathbf{45^o}$ ***and y intercept is*** $\mathbf{11}$

Given: $\theta = 45^o$ *and* y *intercept* $= 11$

$$m = \tan\theta \implies m = \tan 45^o$$

$$m = 1$$

The required equation of the line is $y = mx + c$

$$y = (1)x + 11 \implies y = x + 11$$

$$x - y + 11 = 0$$

5.19 Calculate the slope and y intercept of the straight line $\mathbf{8x - 7y + 6 = 0}$

Given equation of a straight line is $8x - 7y + 6 = 0$

$$8x - 7y + 6 = 0 \implies 7y = 8x + 6$$

$$\div 7$$

$$y = \frac{8}{7}x + \frac{6}{7}$$

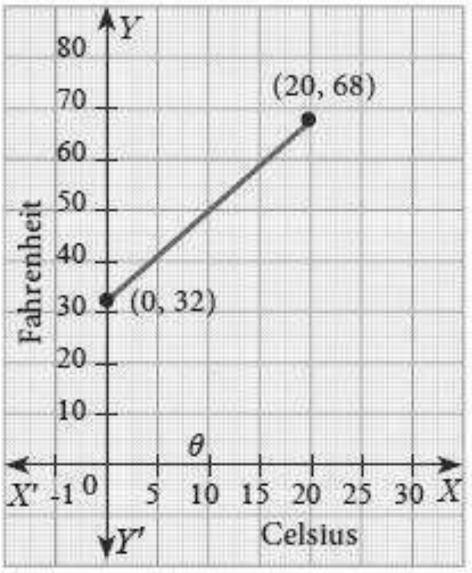
Compare with $y = mx + c$

$$Slope: m = \frac{8}{7} \quad and\ y - intercept, c = \frac{6}{7}$$

5.20 The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree)

(a) Find the slope and y intercept (b) Write an equation of the line

(c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius?



$(\boldsymbol{a})$ ***slope and y intercept***

Slope of a line joining two given points is

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$slope, m = \frac{68 - 32}{20 - 0} = \frac{\cancel{36}\ 9}{\cancel{20}\ 5} = \frac{9}{5}$$

$$\therefore\ slope, m = \frac{9}{5}$$

The line crosses the Y axis at $(0{,}32)$

$$\therefore\ y\ intercept = 32$$

$(\boldsymbol{b})$ ***Equation of the line***

Required equation of the line is $y = mx + c$

$$y = \left(\frac{9}{5}\right)x + 32\ \ldots(1) \implies y = \frac{9x}{5} + 32 \implies y = \frac{9x + 32 \times 5}{5}$$

$$y = \frac{9x + 160}{5} \implies 9x + 160 = 5y$$

$$9x - 5y + 160 = 0$$

(c) Mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius

In Celsius, the mean temperature of the earth is 25°*. To find the mean temperature in Fahrenheit, find the value of* y *when* $x = 25$

$$(1) \implies y = \left(\frac{9}{5}\right)x + 32$$

$$when\ x = 25$$

$$y = \left(\frac{9}{\cancel{5}}\right) \times \overset{5}{\cancel{25}} + 32 \implies y = (9 \times 5) + 32$$

$$y = 45 + 32 \implies \boxed{y = 77}$$

$$\therefore\ Mean\ temperature\ of\ the\ earth = 77^{o}$$

5.21 *Find the equation of a line passing through the point* $(3,-4)$ *and having slope* $-\frac{5}{7}$

$\therefore$ *Equation of the line with slope,* $m = \frac{-5}{7}$ *and point* $(\underset{x_1,y_1}{3,-4})$

$$y - y_1 = m(x - x_1)$$

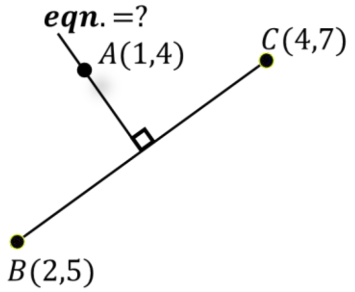
$$y-(-4) = -\frac{5}{7}\,(x-3) \Rightarrow y+4 = -\frac{5}{7}\,(x-3)$$

$$7y+28 = -5\,(x-3) \Longrightarrow 7y+28 = -5\,x+15$$

$$5x-15+7y+28 = 0 \implies 5x+7y+13 = 0$$

5.22 Find the equation of a line passing through the point A(1,4) and perpendicular to the line joining points (2,5) and (4,7)

$B(\underset{x_1,y_1}{2,5})$ *and* $C(\underset{x_2,y_2}{4,7})$



$$\text{Slope of line } BC = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{7-5}{4-2} = \frac{\cancel{2}^{\,1}}{\cancel{2}} = 1$$

$$m_1 = 1$$

Since Slope of required line(m) *is perpendicular to the line BC*

$$\therefore m_1 \times m = -1$$

$$1 \times m = -1 \Longrightarrow m = -1$$

$\therefore$ *Equation of the line passing through* $A(1,4)$ *is* $y - y_1 = m(x - x_1)$

$$y-4 = -1\,(x-1) \Longrightarrow y-4 = -x+1$$

$$x-1+y-4 = 0 \Longrightarrow x+y-5 = 0$$

5.23 *Find the equation of a straight line passing through* $(5,-3)$ *and* $(7,-4)$

Given points are $(\underset{x_1\;,y_1}{5,-3})$ *and* $(\underset{x_2\;,y_2}{7,-4})$

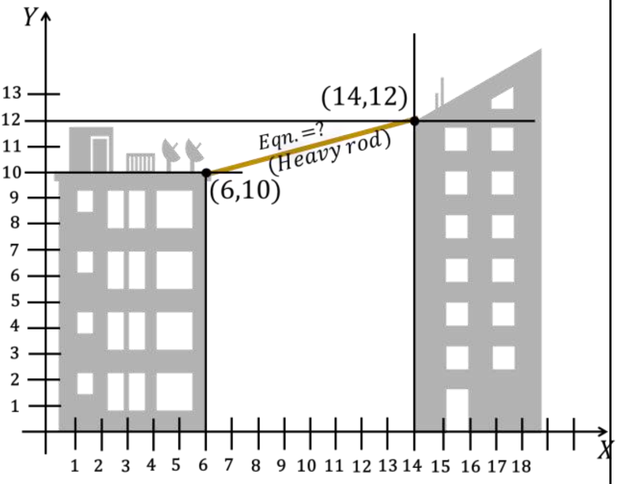
$\therefore$ *Equation of the line passing through two points* $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$$\frac{y+3}{-4+3} = \frac{x-5}{7-5} \Rightarrow \frac{y+3}{-1} = \frac{x-5}{2}$$

$$2y+6 = -x+5 \Longrightarrow x-5+2y+6 = 0$$

$$x+2y+1 = 0$$

5.24 Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6,10) to (14,12), find the equation of the rod joining the buildings ?



Given points are $\underset{x_1, y_1}{(6,10)}$ *and* $\underset{x_2, y_2}{(14,12)}$

$\therefore$ *Equation of the line passing through two points*

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-10}{12-10}=\frac{x-6}{14-6} \Rightarrow \frac{y-10}{\cancel{2}_1}=\frac{x-6}{\cancel{8}_4}$$

$$\frac{y-10}{1}=\frac{x-6}{4} \Rightarrow 4y-40=x-6$$

$$x-6-4y+40=0$$

$$x-4y+34=0$$

5.25. Find the equation of a line which passes through (5,7) and makes intercepts on the axes equal in magnitude but opposite in sign.

Given that intercept on the axes are equal in magnitude but opposite in direction

Here: x *intercept* $= a$ *and* y *intercept* $= -a$

$$b=-a$$

Equation of the line (intercepts form) is $\frac{x}{a}+\frac{y}{b}=1 \ldots (1)$

$$\frac{x}{a}+\frac{y}{-a}=1 \Rightarrow \frac{x}{a}-\frac{y}{a}=1$$ *. Since it passes through* $\underset{x\,,\,y}{(5,7)}$

$$\frac{5}{a}-\frac{7}{a}=1 \Rightarrow \frac{5-7}{a}=1 \Rightarrow \frac{-2}{a}=1 \Rightarrow a=-2$$

sub $a=-2$ *in* $b=-a$

$$b=-(-2) \Rightarrow b=2$$

sub $a=-2$ *and* $b=2$ *in* $(1)\,\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{-2}+\frac{y}{2}=1 \Rightarrow \frac{-x}{2}+\frac{y}{2}=1 \Rightarrow \frac{-x+y}{2}=1 \Rightarrow -x+y=2$$

$$x-y=-2$$

5.26. Find the intercepts made by the line $4x-9y+36=0$

Given equation is $4x-9y+36=0$

$$4x-9y=-36$$

$$\div -36 \quad \Rightarrow \frac{\cancel{4}x}{-\cancel{36}_{-9}}-\frac{\cancel{9}y}{-\cancel{36}_{-4}}=\frac{-\cancel{36}}{-\cancel{36}} \Rightarrow \frac{x}{-9}-\frac{y}{-4}=1$$

$Comparing\ with\ intercepts\ form\ \frac{x}{a} + \frac{y}{b} = 1$

$x - intercept\ is\ a = -9$

$y - intercept\ is\ b = 4$

27. A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' remaining after using the mobile phone for 'x' hours is assumed as $y = -0.25x + 1$.

(i) Draw a graph of the equation.

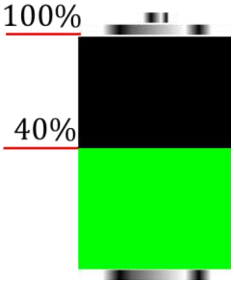
(ii) Find the number of hours elapsed if the battery power is 40%

(iii) How much time does it take so that the battery has no power?

$Given:\ y = -0.25\ x + 1 \dots (1)\ where,$

$x = Hours\ of\ using\ mobile\ phone$

$y = remianing\ battery\ power\ in\ percentage$



$x = 0\ ;\ y = -0.25(0) + 1 \implies y = 1$

$x = 1\ ;\ y = -0.25(1) + 1 = -0.25 + 1$

$y = 0.75$

$x = 2\ ;\ y = -0.25(2) + 1 = -0.5 + 1$

$y = 0.5$

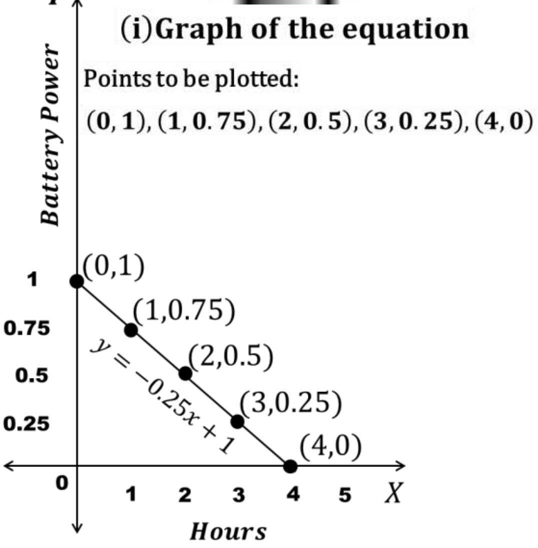
$x = 3\ ;\ y = -0.25(3) + 1 = -0.75 + 1$

$y = 0.25$

$x = 4\ ;\ y = -0.25(4) + 1 = -1 + 1$

$y = 0$

x	0	1	2	3	4
y	1	0.75	0.5	0.25	0



(ii) ***Find the number of hours elapsed if the battery power is*** 40%

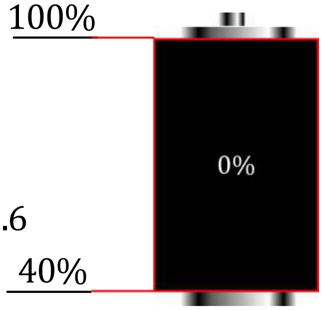
$when\ y = 40\% = \frac{4}{100} = 0.4$

$in\ y = -0.25x + 1$

$0.4 = -0.25x + 1$

$0.25x = 1 - 0.4 \implies 0.25x = 0.6$

$x = \frac{0.6}{0.25} \times \frac{100}{100} = \frac{\cancel{60}^{\ 12}}{\cancel{25}_{\ 5}}$



$$x = \frac{12}{5} \Rightarrow x = 2.4$$

Number of hours elapsed if the battery power is 40%

$$= 2.4\ hours$$

(*iii*) *How much time does it take so that the battery has no power*?

$$when\ y = 0\ \ in\ y = -0.25x + 1$$

$$0 = -0.25x + 1 \Rightarrow 0.25x = 1$$

$$x = \frac{1}{0.25} \Rightarrow x = \frac{1}{0.25} \times \frac{100}{100} \Rightarrow x = \frac{100}{25}$$

$$x = 4$$

Time taken so that the battery has no power $= 4\ hours$

5.28. *A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through* $(-3, 8)$. *Find its equation.*

Let x and y – intercepts of the straight line be a and b

$$a + b = 7$$

$$b = 7 - a$$

The equation of the straight line in intercepts form is $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{a} + \frac{y}{7-a} = 1 \ldots (1)$$

Since this line passes through $(-3\ , 8)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad x\ \ , y$

$$\frac{-3}{a} + \frac{8}{7-a} = 1 \Rightarrow \frac{-3(7-a) + 8a}{a(7-a)} = 1 \Rightarrow -21 + 3a + 8a = a(7-a)$$

$$-21 + 11a = 7a - a^2 \Rightarrow -21 + 11a - 7a + a^2 = 0 \Rightarrow -21 + 4a + a^2 = 0$$

$$a^2 + 4a - 21 = 0 \Rightarrow (a + 7)(a - 3) = 0$$

$$a + 7 = 0, a - 3 = 0 \Rightarrow a = -7, a = 3$$

$+ \quad \times$
$4 \quad -21$
$7 \quad -3$

Since a is positive, $\therefore a = 3$

sub $a = 3\ in\ (1)$ $\frac{x}{a} + \frac{y}{7-a} = 1$

$$\frac{x}{3} + \frac{y}{7-3} = 1 \Rightarrow \frac{x}{3} + \frac{y}{4} = 1 \Rightarrow \frac{4x + 3y}{12} = 1$$

$$4x + 3y = 12 \Rightarrow 4x + 3y - 12 = 0$$

5.29. A circular garden in bounded by East Avenue and Cross Road. Cross Road intersects North Street at D and East Avenue at E. AD is tangential to the circular garden at A (3,10) using the figure.

(a) Find the equation of
- (i) East Avenue
- (ii) North Street
- (iii) Cross Road

(b) Where does the Cross Road intersect the
- (i) East Avenue?
- (ii) North Street?

a (i) equation of East Avenue

From the figure,

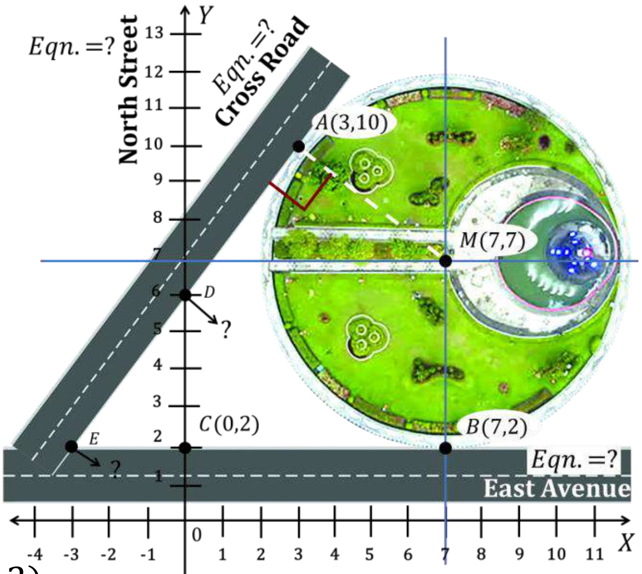
East avenue is the straight line joining $\underset{x_1, y_1}{C(0,2)}$ *and* $\underset{x_2, y_2}{B(7,2)}$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{2 - 2} = \frac{x - 0}{7 - 0}$$

$$\frac{y - 2}{0} = \frac{x}{7} \Rightarrow y - 2 = 0$$

$$y = 2$$



a (ii) equation of North Street

From the figure,

North street lie vertically above $C(0,2)$

$(i.e)$ *Exactly on* y *axis*

$\therefore$ *equation of North street is* $x = 0$ $[\because$ *it is a* $y -$ *axis*$]$

a (iii) equation of Cross Road

slope of the line joining two points $\underset{x_1, y_1}{M(7,7)}$, $\underset{x_2, y_2}{A(3,10)}$ *is*

slope of the line MA,

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 7}{3 - 7} = \frac{3}{-4} = \frac{-3}{4}$$

$\because$ *cross road is perpendicular to line MA*

$$\therefore \text{ slope of the cross road}, m_2 = -\frac{1}{m_1} = \frac{-1}{-3/4}$$

$$m_2 = \frac{4}{3}$$

$\therefore$ *equation of the cross road with slope,* $m = \frac{4}{3}$ *and point* $A(3,10)$ $\quad x_1, y_1$

$$y - y_1 = m(x - x_1)$$

$$y - 10 = \frac{4}{3}(x - 3) \Rightarrow 3(y - 10) = 4(x - 3) \Rightarrow 3y - 30 = 4x - 12$$

$$4x - 12 - 3y + 30 = 0 \Rightarrow 4x - 3y + 18 = 0$$

(b) Where does the Cross Road intersect the

(i) East Avenue?

if E *is* $(q, 2)$ *then* E *is a point on the Cross Road*

$\therefore$ *substituting* $x = q, y = 2$ *in eqn*(1)

$$(1) \Rightarrow 4x - 3y + 18 = 0$$

$$4(q) - 3(2) + 18 = 0 \Rightarrow 4q - 6 + 18 = 0$$

$$4q + 12 = 0 \Rightarrow k = \frac{\cancel{-12}^{\,-3}}{\cancel{4}} \Rightarrow k = -3$$

$\therefore$ *the point* E *is* $(-3,2)$

(b) Where does the Cross Road intersect the (ii) North Street?

if D *is* $(0, k)$ *then* D *is a point on the Cross Road*

$\therefore$ *substituting* $x = 0, y = k$ *in eqn*(1)

$$(1) \Rightarrow 4x - 3y + 18 = 0 \Rightarrow 4(0) - 3(k) + 18 = 0$$

$$0 - 3(k) + 18 = 0 \Rightarrow 3k = 18 \Rightarrow k = \frac{\cancel{18}^{\,6}}{\cancel{3}}$$

$$\boxed{k = 6}$$

$\therefore$ *the point* D *is* $(0,6)$

1. **Find the equation of a straight line passing through the mid-point of a line segment joining the points $(1, -5), (4, 2)$ and parallel to (i) X –axis (ii) Y -axis**

Mid point of the line joining the points $(1\,, -5)$ and $(4\,, 2)$ is
$\quad x_1 \;\; y_1 \qquad x_2 \;\; y_2$

$$Mid\ point = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{1 + 4}{2}, \frac{(-5) + 2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

Equation of straight line passing through $\left(\frac{5}{2}, \frac{-3}{2}\right)$ *and*

i) Parallel to $x - axis$

2. The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and interception on the Y - axis

Given equation of a straight line is $2(x-y)+5=0$

$$2x-2y+5=0\ \dots(1)$$

Slope of the line: $m = -\dfrac{coefficient\ of\ x}{coefficient\ of y} = -\dfrac{\cancel{2}^{1}}{-\cancel{2}} \Rightarrow \quad m = 1$

Inclination (angle) $m = \tan\theta$

$$\tan\theta = 1 \Rightarrow \quad \theta = 45^o$$

Intercept on $y -$ *axis*

Put $x = 0$ *in eqn.* (1) $2x-2y+5=0$

$$2(0)-2y+5=0 \Rightarrow -2y+5=0 \Rightarrow \cancel{-}2y = \cancel{-}5 \Rightarrow y = \frac{5}{2}$$

$$\therefore y - intercept\ = \frac{5}{2}$$

3. Find the equation of a line whose inclination is 30^o and making an intercept -3 on the Y-axis.

Given: $\theta = 30^o$

$$m = \tan\theta$$

$$m = \tan 30^o \Rightarrow \ m = \frac{1}{\sqrt{3}}$$

Given: $y\ intercept = -3$

The required equation of the line is $y = mx + c$

$$y = \frac{1}{\sqrt{3}}x - 3 \Rightarrow \ y = \frac{x - 3\sqrt{3}}{\sqrt{3}} \ \Rightarrow \ \sqrt{3}y = x - 3\sqrt{3}$$

$$x - \sqrt{3}y - 3\sqrt{3} = 0$$

4. *Find the slope and y intercept of* $\sqrt{3}x + \left(1-\sqrt{3}\right)y = 3$

Given line is: $\sqrt{3}x + \left(1-\sqrt{3}\right)y = 3$

Make the above line to the form $y = mx + c$

$$\left(1-\sqrt{3}\right)y = -\sqrt{3}x + 3$$

$$\div\ 1-\sqrt{3}$$

$$y = \boxed{-\frac{\sqrt{3}}{1-\sqrt{3}}}x + \boxed{\frac{3}{1-\sqrt{3}}}$$

Compare with $y = \boxed{m}x + \boxed{c}$

$$\therefore m = \frac{-\sqrt{3}}{1-\sqrt{3}} \quad and\ c = \frac{3}{1-\sqrt{3}}$$

5. Find the value of 'a', if the line through $(-2, 3)$ and $(8, 5)$ is perpendicular to $y = ax + 2$

Slope of the line joining two given points $\underset{x_1,y_1}{(-2,3)}, \underset{x_2,y_2}{(8,5)}$ *is*

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{5-3}{8-(-2)} = \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{10}}}$$

$$m_1 = \frac{1}{5}$$

slope of the line $y = ax + 2$ *is* $m_2 = a$

$[\because$ *compare with* $y = mx + c\ ]$

since the two lines are perpendicular $m_1 \times m_2 = -1$

$$\frac{1}{5} \times a = -1 \Rightarrow \quad a = -5$$

6. The hill in the form of a right triangle has its foot at (19, 3) . The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

To find : *Equation of the hill* (AC) *joining the foot and top*

$\therefore$ *Slope of line AC*

$$\theta = 45^o$$

$$m = \tan\theta$$

$$m = \tan 45^o \Rightarrow m = 1$$

$\therefore$ *Equation of AC whose slope 1 and passing through* $C(\underset{x}{19}, \underset{y}{3})$ *is*

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - 19) \ \Rightarrow\ y - 3 = x - 19$$

$$x - 19 - y + 3 = 0 \ \Rightarrow x - y - 16 = 0$$

7. *Find the equation of a line through the given pair of points*

$(i)\ \left(2,\frac{2}{3}\right)$ and $\left(-\frac{1}{2},-2\right)$ $\qquad (ii)\ (2,3)$ and $(-7,-1)$

Given points are $\underset{x_1,y_1}{\left(2,\frac{2}{3}\right)}$ *and* $\underset{x_2,y_2}{\left(-\frac{1}{2},-2\right)}$

$\therefore$ *Equation of the line passing through two points* $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{\frac{-1}{2}-2} \Rightarrow \frac{\frac{3y-2}{\cancel{3}}}{\frac{-6-2}{\cancel{3}}} = \frac{x-2}{\frac{-1-4}{2}} \Rightarrow \frac{3y-2}{-8} = \frac{x-2}{\frac{-5}{2}}$$

$$\frac{3y-2}{-8}=\frac{2(x-2)}{-5} \Rightarrow 15y-10=16(x-2) \Rightarrow 15y-10=16x-32$$

$$16x-32-15y+10=0$$

$$16x-15y-22=0$$

$(\boldsymbol{ii})(\mathbf{2},\mathbf{3})\ \boldsymbol{and}\ (-\mathbf{7},-\mathbf{1})$

$Given\ points\ are\ \underset{x_1\ ,y_1}{(2,3)}\ and\ \underset{x_2\ ,y_2}{(-7,-1)}$

$\therefore Equation\ of\ the\ line\ passing\ through\ two\ points\ \frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$$\frac{y-3}{-1-3}=\frac{x-2}{-7-2} \Rightarrow \frac{y-3}{-4}=\frac{x-2}{-9}$$

$$-9y+27=-4x+8 \Rightarrow -9y+27+4x-8=0$$

$$4x-9y+19=0$$

8. A cat is located at the point$(-\mathbf{6},-\mathbf{4})$ in xy plane. A bottle of milk is kept at $(\mathbf{5},\mathbf{11})$.The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

$\therefore Equation\ of\ the\ shortest\ distance\ between\underset{x_1\ ,y_1}{(-6,-4)}\ and\ \underset{x_2\ ,y_2}{(5,11)}$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y+4}{11+4}=\frac{x+6}{5+6}$$

$$\frac{y+4}{15}=\frac{x+6}{11} \Rightarrow 11y+44=15x+90$$

$$15x+90-11y-44=0$$

$$15x-11y-46=0$$

9.Find the equation of the median and altitude of ΔABC through A where the vertices are $A(6,2), B(-5,-1)$ and $C(1,9)$

Median is a straight line joining a vertex and the midpoint of the opposite side

Vertices of the ΔABC *are* $A(6,2), B(-5,-1)$ *and* $C(1,9)$

$$D = \text{midpoint of } B\underset{x_1\ y_1}{(-5,-1)}, \text{and } C\ \underset{x_2\ y_2}{(1,9)}$$

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+1}{2}, \frac{(-1)+9}{2}\right) = \left(\frac{\overset{-2}{\cancel{-4}}}{\cancel{2}}, \frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)$$

$$D = (-2,4)$$

Equation of median AD having points $\underset{x_1\ ,y_1}{A(6,2)}$ *and* $\underset{x_2\ ,y_2}{D(-2,4)}$ $(-5,-1)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{\cancel{2}} = \frac{x-6}{\cancel{-8}_{-4}} \Rightarrow \frac{y-2}{1} = \frac{x-6}{-4} \Rightarrow -4y+8 = x-6$$

$$x-6+4y-8=0 \Rightarrow x+4y-14=0$$

Equation of the altitude AE: $B\underset{x_1\ y_1}{(-5,-1)}$ *and* $C\underset{x_2\ y_2}{(1,9)}$

$$\text{Slope of } BC = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{9+1}{1+5} = \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}}} = \frac{5}{3}$$

$$\text{Slope of } BC = \frac{5}{3}$$

$$\text{slope of } BC \times \text{Slope of } AD = -1 \quad \therefore AD \perp BC$$

$$\frac{5}{3} \times \text{Slope of } AD = -1 \Rightarrow \text{Slope of } AD = -1 \times \frac{3}{5}$$

$$\text{Slope of } AD\ (m) = -\frac{3}{5}$$

Equation of altitude AE having point $A\underset{x_1\ y_1}{(6,2)}$ *and slope* $m = -\frac{3}{5}$

$$y-y_1 = m(x-x_1)$$

$$y-2 = -\frac{3}{5}(x-6) \Rightarrow 5y-10 = -3(x-6)$$

$$5y-10 = -3x+18 \Rightarrow 3x-18+5y-10=0$$

$$3x+5y-28=0$$

10. $\boldsymbol{Find\ the\ equation\ of\ the\ straight\ line\ which\ has\ slope\ -\frac{5}{4}\ and\ passing\ through\ the\ point\ (-1,2)}$

$Equation\ of\ the\ straight\ line\ having\ slope\ m = -\frac{5}{4}\ and\ passing\ through\ (-1\,,2)\ is\ y - y_1 = m(x - x_1)$

$$y - 2 = -\frac{5}{4}\,(x - (-1)) \Rightarrow\ y - 2 = -\frac{5}{4}\,(x + 1)$$

$$4y - 8 = -5(x + 1)\ \Rightarrow\ 4y - 8 = -5\,x - 5$$

$$5x + 5 + 4y - 8 = 0 \Rightarrow\ \ 5x + 4y - 3 = 0$$

11. You are downloading a song. The percent $\boldsymbol{y}$ (in decimal form) of mega bytes remaining to get downloaded in $\boldsymbol{x}$ seconds is given by $\boldsymbol{y\ =\ -0.1x + 1}$.

(i) graph the equation.

(ii) find the total MB of the song.

(iii) after how many seconds will 75% of the song gets downloaded?

(iv) after how many seconds the song will be downloaded completely?

$Given\colon y = -0.1\,x\ \ +1 \ldots\ldots(1)$

$where,$

$x = time\ (in\ seconds)$

$y = remianing\ data\ to\ be\ downloaded$

x	0	2	4	6	8	10
y	1	0.8	0.6	0.4	0.2	0

(ii) find the total MB of the song.

$When\ x = 0\ \ \ (\because x\ can't\ be\ -ve)$

$$(1) \Rightarrow y = -0.1x + 1$$

$$y = -0.1(0) + 1 \Rightarrow y = 1$$

$\therefore Total\ MB\ of\ the\ song = 1MB$

(iii) after how many seconds will 75% of the song gets downloaded?

$\therefore 25\%\ of\ MB\ to\ be\ downloaded$

$\therefore Put\ y = 0.25\ in\ eqn.(1)$

$$(1) \Rightarrow y = -0.1x + 1$$

$$0.25\ = -0.1x + 1\ \Rightarrow 0.1x\ = -0.25 + 1$$

$$0.1x\ = 0.75 \Rightarrow x = \frac{0.75}{0.1}\ = \frac{7.5}{1}$$

$$\boxed{x = 7.5}$$

$\therefore Required\ time\ = 7.5\ sec$

(iv) After how many seconds the song will be downloaded completely?

$$\therefore Put\ y = 0\ in\ eqn.\ (1)$$

$$(1) \Rightarrow y = -0.1x + 1$$

$$0 = -0.1x + 1 \Rightarrow -1 = -0.1x$$

$$x = \frac{1}{0.1} \quad \Rightarrow x = \frac{10}{1}$$

$$\boxed{x = 10}$$

$\therefore$ *Songs will be downloaded completely after* **10 sec**

12. ***Find the equation of the straight line whose intercepts on the x and y axes are given below*** **(*i*) 4, −6 (ii) $-5, \frac{3}{4}$**

(*i*) 4 *and* − 6

Given that x – intercept is $a = 4$ *and y – intercepts is* $b = -6$

The equation of the straight line in intercepts form is $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{4} + \frac{y}{-6} = 1 \Rightarrow \frac{x}{4} - \frac{y}{6} = 1 \Rightarrow \frac{3x - 2y}{12} = 1$$

$$3x - 2y = 12 \Rightarrow 3x - 2y - 12 = 0$$

(*ii*) − 5 *and* $\frac{3}{4}$

Given that x – intercept is $a = -5$ *and y – intercepts is* $b = \frac{3}{4}$

$$\frac{x}{-5} + \frac{y}{\frac{3}{4}} = 1 \Rightarrow \frac{-x}{5} + \frac{4y}{3} = 1 \Rightarrow \frac{-3x + 20y}{15} = 1 \Rightarrow -3x + 20y = 15$$

$$-3x + 20y = 15 \Rightarrow 3x - 20y + 15 = 0$$

13. Find the intercepts made by the following lines on the coordinate axes.
(*i*) $3x - 2y - 6 = 0$ (*ii*) $4x + 3y + 12 = 0$

(*i*) $3x - 2y - 6 = 0$

Given equation is $3x - 2y - 6 = 0$

$$3x - 2y = 6$$

Dividing both sides by 6

$$\frac{\cancel{3}x}{{}_2\cancel{6}} - \frac{\cancel{2}y}{{}_3\cancel{6}} = \frac{6}{6} \Rightarrow \frac{x}{2} - \frac{y}{3} = 1$$

$$\frac{x}{2} + \frac{y}{-3} = 1$$

$$Comparing\ with\ intercepts\ form\ \frac{x}{a}+\frac{y}{b}=1$$

$$x-\ intercept\ is\ a=2$$

$$y-\ intercept\ is\ b=-3$$

$(iii)\ 4x+3y+12=0$

$$Given\ equation\ is\ 4x+3y+12=0$$

$$4x+3y=-12$$

$$Dividing\ both\ sides\ by\ -12$$

$$\frac{\cancel{4x}}{\underset{-3}{\cancel{-12}}}+\frac{\cancel{3y}}{\underset{-4}{\cancel{-12}}}=\frac{\cancel{-12}}{\cancel{-12}}\ \Rightarrow \frac{x}{-3}+\frac{y}{-4}=1$$

$$Comparing\ with\ intercepts\ form\ \ \frac{x}{a}+\frac{y}{b}=1$$

$$x-\ intercept\ is\ \ a=-3$$

$$y-\ intercept\ is\ b=-4$$

14. Find the equation of the straight line
(i) passing through $(1,-4)$ and has intercepts which are in the ratio $2:5$
(ii) passing through $(-8,4)$ and making equal intercepts on the coordinate axes

$(i)\ passing\ through\ (1,-4)\ and\ has\ intercepts\ which\ are\ in\ the\ ratio\ 2:5$

$$Let\ x\ and\ y-intercepts\ of\ the\ straight\ line\ be\ a\ and\ b$$

$$a:b=2:5$$

Let $a=2k$ and $b=5k$

$$The\ equation\ of\ the\ straight\ line\ in\ intercepts\ form\ is\ \frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{2k}+\frac{y}{5k}=1\ \ldots\ (1)$$

$$Since\ this\ line\ passes\ through\ \underset{x\ ,\ y}{(1\ ,-4)}$$

$$\frac{1}{2k}+\frac{-4}{5k}=1\Rightarrow\frac{1}{2k}-\frac{4}{5k}=1\Rightarrow\frac{5-8}{10k}=1$$

$$\frac{-3}{10k}=1\ \Rightarrow\frac{-3}{10}=k\ \Rightarrow\ \boxed{k=-\frac{3}{10}}$$

$$substituting\ k=-\frac{3}{10}\ in\ (1)\ \ \frac{x}{2k}+\frac{y}{5k}=1$$

$$\frac{x}{\cancel{2}\left(-\frac{3}{\underset{5}{\cancel{10}}}\right)}+\frac{y}{\cancel{5}\left(-\frac{3}{\underset{2}{\cancel{10}}}\right)}=1\ \Rightarrow\frac{x}{-\frac{3}{5}}+\frac{y}{-\frac{3}{2}}=1\ \Rightarrow\frac{5x}{-3}+\frac{2y}{-3}=1$$

$$\frac{5x+2y}{-3}=1\Rightarrow\ 5x+2y=-3\Rightarrow\ 5x+2y+3=0$$

(ii) passing through $(-8, 4)$ and making equal intercepts on the coordinate axes

Given that intercept on the axes are equal

$$\text{Here: } x \text{ intercept} = a \text{ and } y \text{ intercept} = -a$$

$$b = a$$

$$\text{Equation of the line (intercepts form) is } \frac{x}{a} + \frac{y}{b} = 1 \ldots (1)$$

$$\frac{x}{a} + \frac{y}{a} = 1 \Rightarrow \frac{x}{a} + \frac{y}{a} = 1 \text{ . Since it passes through } (\underset{x}{-8}, \underset{y}{4})$$

$$\frac{-8}{a} + \frac{4}{a} = 1 \Rightarrow \frac{-8+4}{a} = 1 \Rightarrow \frac{-4}{a} = 1 \Rightarrow a = -4$$

$$\text{sub } a = -4 \text{ in } b = a$$

$$b = -4$$

$$\text{sub } a = -4 \text{ and } b = -4 \text{ in } (1) \frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-4} + \frac{y}{-4} = 1 \Rightarrow \frac{x+y}{-4} = 1 \Rightarrow x + y = -4$$

$$x + y + 4 = 0$$

EXERCISE 5.4

Example 5.30: Find the slope of the straight line $6x+8y+7=0$

Given line is $6x+8y+7=0$

Comparing with $ax+by+c=0$

$$Slope, m=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=\frac{-\cancel{6}^{\,3}}{\cancel{8}_{\,4}}\Longrightarrow m=-\frac{3}{4}$$

$\therefore$ Slope of the straight line is $-\frac{3}{4}$

Example 5.31: Find the slope of the line which is (i) parallel to $3x-7y=11$ (ii) perpendicular to $2x-3y+8=0$

(i) parallel to $3x-7y=11$

Given line is $3x-7y-11=0$

Comparing with $ax+by+c=0$

$$Slope, m=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=-\frac{3}{-7}\Longrightarrow\ m=\frac{3}{7}$$

Since parallel lines have same slope

$\therefore$ Slope of any line parallel to $3x-7y=11$ is $\frac{3}{7}$

(ii) perpendicular to $2x-3y+8=0$

Given line is $2x-3y+8=0$

Comparing with $ax+by+c=0$

$$Slope, m=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=-\frac{2}{-3}=\frac{2}{3}$$

Since product of slopes is -1 for perpendicular lines

slope of line perpendicular to $2x-3y+8=0$ is

$$=-\frac{1}{m}\quad=-\frac{1}{2/3}$$

$$\boxed{\therefore Slope\ =-\frac{3}{2}}$$

Example 5.32 Show that the straing lines $2x+3y-8=0$ and $4x+6y+18=0$ is parallel

Given pair of lines $2x+3y-8=0$

Comparing with $ax+by+c=0$

$$m_1=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=-\frac{2}{3}\Longrightarrow m_1=-\frac{2}{3}$$

$4x + 6y + 18 = 0$

Comparing with $ax + by + c = 0$

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{a}{b} = -\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} = -\frac{2}{3} \Rightarrow m_2 = -\frac{2}{3}$$

$$\therefore m_1 = m_2$$

The given two lines are parallel

Example 5.33 Show that the straing lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ is perpendicular

Given pair of lines $x - 2y + 3 = 0$

Comparing with $ax + by + c = 0$

$$m_1 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{a}{b} = -\frac{1}{-2} \Rightarrow m_1 = \frac{1}{2}$$

$6x + 3y + 8 = 0$

Comparing with $ax + by + c = 0$

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{a}{b} = -\frac{\overset{2}{\cancel{6}}}{\cancel{3}} = -2 \Rightarrow m_2 = -2$$

$$Find: m_1 \times m_2 = \frac{1}{\cancel{2}} \times \cancel{-2} = -1$$

$$\therefore m_1 \times m_2 = -1$$

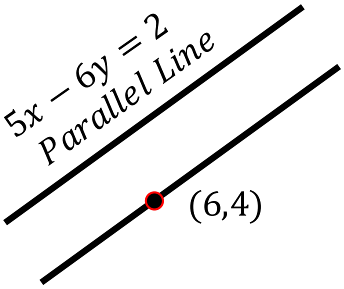
The given two lines are perpendicular

Example 5.34: Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point $(6,4)$

Given line: $3x - 7y - 12 = 0$

Comparing with $ax + by + c = 0$

$$m = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{a}{b} = -\frac{3}{-7} \Rightarrow m = \frac{3}{7}$$



For parallel lines, slopes are equal

$\therefore$ *Equation of the line passing through* $\underset{x_1,y_1}{(6,4)}$ *is*

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{3}{7}(x - 6) \Rightarrow 7(y - 4) = 3\,(x - 6)$$

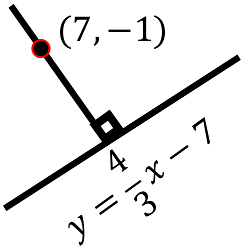
$$7y - 28 = 3\,x - 18 \Rightarrow 3\,x - 18 - 7y + 28 = 0$$

$$\boxed{\therefore 3\,x - 7y + 10 = 0}$$

Example 5.35: Find the equation of a straight line perpendicular to the line $\boldsymbol{y = \frac{4}{3}x - 7}$ and passing through the point $\boldsymbol{(7, -1)}$

$$\text{Given line: } y = \frac{4}{3}x - 7 \Rightarrow y = \frac{4x - 21}{3} \Rightarrow 3y = 4x - 21$$

$$4x - 3y - 21 = 0$$



Comparing with $ax + by + c = 0$

$$m = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{a}{b} = -\frac{4}{-3} = \frac{4}{3} \Rightarrow m = \frac{4}{3}$$

$\therefore$ Slope of the line perpendicular to above line $= -\frac{1}{m}$

$$= \frac{-1}{4/3} = -\frac{3}{4}$$

$\therefore$ Equation of the line passing through $\underset{x_1, y_1}{(7, -1)}$ is $y - y_1 = m(x - x_1)$

$$y - (-1) = -\frac{3}{4}(x - 7) \Rightarrow y + 1 = -\frac{3}{4}(x - 7)$$

$$4(y + 1) = -3(x - 7) \Rightarrow 4y + 4 = -3x + 21$$

$$3x - 21 + 4y + 4 = 0 \Rightarrow 3x + 4y - 17 = 0$$

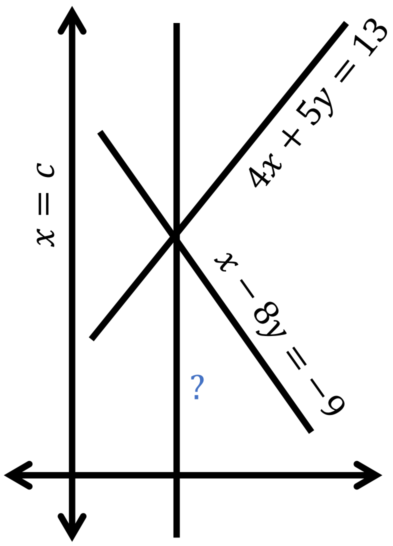
Example 5.36: Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $\boldsymbol{4x + 5y = 13, x - 8y + 9 = 0}$

Given: Required line is passing through the intersection of the lines

$$4x + 5y = 13 \ldots (1) \quad , x - 8y = -9 \ldots (2)$$

and parallel to the line Y axis

Solving (1) and (2)



$$(1) \Rightarrow \quad \cancel{4x} + 5y = 13$$

$$(2) \times 4 \Rightarrow \underset{(-)}{\cancel{4x}} \underset{(+)}{-} 32y \underset{(+)}{=} -36$$

$$37y = 49 \Rightarrow y = \frac{49}{37}$$

Sub $y = \frac{49}{37}$ in eqn(2)

$$x - 8\left(\frac{49}{37}\right) = -9 \Rightarrow x = -9 + \left(\frac{8 \times 49}{37}\right)$$

$$x = \frac{(-9 \times 37) + (8 \times 49)}{37} \Rightarrow x = \frac{-333 + 392}{37} \Rightarrow x = \frac{59}{37}$$

$$\therefore \text{Intersection point is } \left(\frac{59}{37},\frac{49}{37}\right)$$

$$\text{Equation of line parallel to Y axis is } x = c$$

$$\text{It passes through } (x,y) = \left(\frac{59}{37},\frac{49}{37}\right)$$

$$x = c = \frac{59}{37} \Longrightarrow \therefore \; c = \frac{59}{37}$$

$$\text{The equation of the line is } x = \frac{59}{37}$$

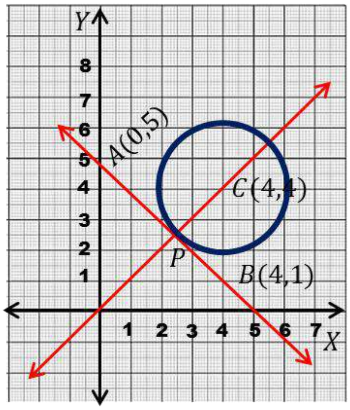
$$37x = 59 \Longrightarrow 37x - 59 = 0$$

Example 5.37 The line joining the points $A(0,5)$ and $B(4,1)$ is a tangent to a circle whose centre C is at the point $(4,4)$ find

(i) the equation of the line AB

(ii) the equation of the line through C which is perpendicular to the line AB

(iii) the coordinates of the point of contact of tangent line with the circle



(i) the equation of the line AB

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Longrightarrow \frac{y-5}{1-5} = \frac{x-0}{4-0}$$

$$4(y-5) = -4x \Longrightarrow y - 5 = -x$$

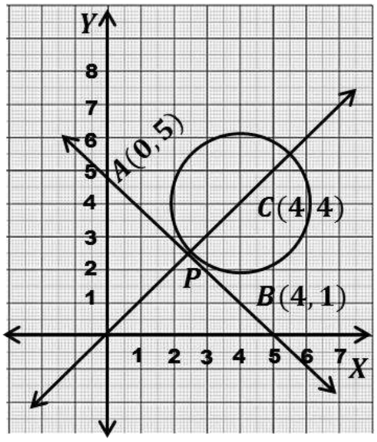
$$x + y - 5 = 0 \quad \ldots (1)$$

(ii) the equation of the line through C which is perpendicular to the line AB

$$(1) \Longrightarrow \; x + y - 5 = 0$$

$$\text{Comparing with } ax + by + c = 0$$

$$m = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -\frac{1}{1} = -1 \Longrightarrow m = -1$$



$$\therefore \text{slope of the line perpendicular to above line} =$$

$$-\frac{1}{m} = \frac{-1}{-1}$$

$$= 1$$

$$\therefore \text{Equation of the line passing through } \underset{x_1,y_1}{C(4,4)} \text{ is}$$

$$y - y_1 = m(x - x_1) \Longrightarrow y - 4 = 1\,(x - 4)$$

$$y - 4 = x - 4 \Longrightarrow x - y = 0 \quad \ldots (2)$$

(iii) the coordinates of the point of contact of tangent line with the circle

It is the intersetion of the lines $x+y-5=0$ *and* $x-y=0$

$$x-y=0 \Rightarrow x=y \ldots(3)$$

Substitute (3) *in* $x+y-5=0$

$$x+x-5=0 \Rightarrow 2x-5=0$$

$$x=\frac{5}{2} \Rightarrow x=y=\frac{5}{2}$$

$\therefore$ *The coordinate of the point of conact is* $P\left(\frac{5}{2},\frac{5}{2}\right)$

1. Find the slope of the following straight lines

(i) $5y-3=0$ **(ii)** $7x-\frac{3}{17}=0$

Given line is $5y-3=0$

Comparing with $ax+by+c=0$

$$Slope=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=\frac{-0}{5}$$

$$m=0$$

Given line is $7x-\frac{3}{17}=0$

Comparing with $ax+by+c=0$

$$Slope=-\frac{coefficient\ of\ x}{coefficient\ of\ y}=-\frac{a}{b}=\frac{-7}{0}$$

$$m=\infty\ undefined$$

2. Find the slope of the line which is

(i) parallel to $y=0.7x-11$

(ii) perpendicular to the line $x=-11$

Given line is $y=0.7x-11$

Comparing with $y=mx+c$ *whose slope is* $,m=0.7$

For parallel lines, slopes are equal

$\therefore$ *Slope of the line parallel to* $y=0.7x-11$ *is also* 0.7

$$\therefore m=0.7$$

Given line is $x=-11$

Comparing with $y=mx+c$ *whose slope is*$,m=0$

$\therefore$ *Slope of the line perpendicular to* $x=-11$ *is* $m_p=-\frac{1}{m}=-\frac{1}{0}$

$$\therefore m_p=\infty$$

3. Check whether the given lines are parallel or perpendicular

(i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ **and** $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$

(ii) $5x+23y+14=0$ **and** $23x-5y+9=0$

Given pair of lines $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$

Comparing with $ax+by+c=0$

$$m_1=-\frac{a}{b}=\frac{-1/3}{1/4}\Rightarrow m_1=-\frac{4}{3}$$

$$\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$$

Comparing with $ax+by+c=0$

$$m_2=-\frac{a}{b}=\frac{-2/3}{1/2}\Rightarrow m_2=-\frac{4}{3}$$

$$\therefore m_1=m_2$$

The given two lines are parallel

(ii) $5x+23y+14=0$ **and** $23x-5y+9=0$

Given pair of lines $5x+23y+14=0$

Comparing with $ax+by+c=0$

$$m_1=-\frac{a}{b}=-\frac{5}{23}\Rightarrow m_1=-\frac{5}{23}$$

$23x-5y+9=0$ *Comparing with* $ax+by+c=0$

$$m_2=-\frac{a}{b}=-\frac{23}{-5}\Rightarrow m_2=\frac{23}{5}$$

$$Find: m_1\times m_2=\left(-\frac{\cancel{5}}{\cancel{23}}\right)\times\left(\frac{\cancel{23}}{\cancel{5}}\right)=-1$$

$$\therefore m_1\times m_2=-1$$

The given two lines are perpendicular

4. If the straight lines $12y=-(p+3)x+12$ and $12x-7y=16$ are perpendicular then find 'p'

Given pair of lines $12y=-(p+3)x+12$

$$-(p+3)x-12y+12=0$$

Comparing with $ax+by+c=0$

$$m_1=-\frac{a}{b}=-\frac{-(p+3)}{-12}\Rightarrow m_1=\frac{-(p+3)}{12}$$

$$12x-7y=16 \Longrightarrow 12x-7y-16=0$$

$Comparing\ with\ ax+by+c=0$

$$m_2=-\frac{a}{b}=-\frac{12}{-7}\Longrightarrow m_2=\frac{12}{7}$$

$Since, the\ given\ two\ lines\ are\ perpendicular$

$$\therefore m_1\times m_2=-1$$

$$\left(\frac{-(p+3)}{\cancel{12}}\right)\times\left(\frac{\cancel{12}}{7}\right)=-1\Longrightarrow\left(\frac{-(p+3)}{7}\right)=-1$$

$$-(p+3)=-7\Longrightarrow p+3=7\Longrightarrow p=7-3$$

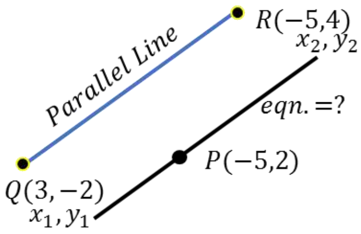
$$\boxed{p=4}$$

5. *Find the equation of a straight line passing through the point $P(-5,2)$ and parallel to the line joining the points $Q(3,-2)$ and $R(-5,4)$*

$$Slope\ of\ QR=\frac{y_2-y_1}{x_2-x_1}=\frac{4-(-2)}{-5-3}=\frac{6}{-8}=\frac{-3}{4}$$

$For\ parallel\ lines, slopes\ are\ equal\ m=-\frac{3}{4}$

$\therefore Equation\ of\ the\ line\ passing\ through\ P(-5,2)\ is$



$$y-y_1=m(x-x_1)$$

$$y-2=-\frac{3}{4}(x-(-5))\Longrightarrow 4(y-2)=-3\,(x+5)$$

$$4y-8=-3\,x-15\Longrightarrow 3\,x+15+4y-8=0$$

$$\boxed{\therefore 3\,x+4y+7=0}$$

6. *Find the equation of a straight line passing through the point $(6,-2)$ and perpendicular to the line joining the points $(6,7)$ and $(2,-3)$*

$$Slope\ of\ line\ joining(6,7)\ and\ (2,-3)=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-7}{2-6}=\frac{-10}{-4}=\frac{5}{2}$$

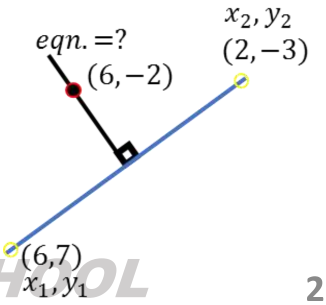
$$\therefore Slope\ of\ the\ line\ perpendicular\ to\ above\ line=-\frac{1}{m}=\frac{-1}{5/2}=\frac{-2}{5}$$

$\therefore Equation\ of\ the\ line\ passing\ through\ (6,-2)\ is\ y-y_1=m(x-x_1)$

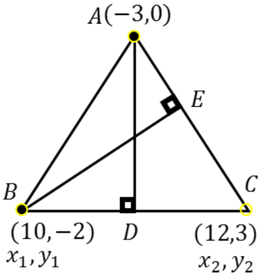
$$y-(-2)=-\frac{2}{5}(x-6)\Longrightarrow y+2=-\frac{2}{5}(x-6)$$

$$5(y+2)=-2\,(x-6)\Longrightarrow 5y+10=-2x+12$$

$$2x-12+5y+10=0\Longrightarrow 2x+5y-2=0$$



7. $\boldsymbol{A(-3,0), B(10,-2)}$ ***and*** $\boldsymbol{C(12,3)}$ ***are the vertices of*** $\boldsymbol{\Delta ABC}$***.Find the equation of the altitude through A and B.***



Vertices of the Δ *are* $A(-3,0), B(10,-2), C(12,3)$

Equation of altitude AD:

slope of line joining $=$ *B*$(10,-2)$ *and C*$(12,3)$ $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-(-2)}{12-10} = \dfrac{5}{2}$

$\therefore$ *slope of the line (AD) perpendicular to BC line* $= -\dfrac{1}{m} = \dfrac{-1}{5/2} = \dfrac{-2}{5}$

$\therefore$ *equation of the line AD is* $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{2}{5}\,(x-(-3)) \Longrightarrow y = -\frac{2}{5}\,(x+3)$$

$$5y = -2\,(x+3) \Longrightarrow 5y = -2x - 6$$

$$2x + 6 + 5y = 0 \Longrightarrow 2x + 5y + 6 = 0$$

Equation of the altitude BE:

Slope of line joining $=$ *A*$(-3,0)$ *and C*$(12,3)$ $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-0}{12-(-3)} = \dfrac{3}{15} = \dfrac{1}{5}$

$\therefore$ *slope of the line (BE) perpendicular to AC line* $= -\dfrac{1}{m} = \dfrac{-1}{1/5} = -5$

$\therefore$ *equation of the line BE is* $y - y_1 = m(x - x_1)$

$$y - (-2) = -5\,(x-10) \Longrightarrow y + 2 = -5\,(x-10)$$

$$y + 2 = -5\,x + 50 \Longrightarrow 5x - 50 + y + 2 = 0$$

$$\boxed{5x + y - 48 = 0}$$

8. ***Find the equation of the perpendiculat bisector of the line joining the points*** $\boldsymbol{A(-4,2)}$ ***and*** $\boldsymbol{B(6,-4)}$

Given: *AB and CD are perpendicular*

D is the midpoint of AB

Midpoint of line joining $\underset{x_1\ y_1}{A(-4,2)}$ and $\underset{x_2\ y_2}{B(6,-4)}$

$$\text{Mid point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-4+6}{2}, \frac{2-4}{2}\right) = \left(\frac{\overset{1}{\cancel{2}}}{\cancel{2}}, \frac{\overset{-1}{-\cancel{2}}}{\cancel{2}}\right)$$

Midpoint $= (1,-1)$

$$\text{slope of line joining } \underset{x_1,y_1}{A(4,2)} \text{ and } \underset{x_2,y_2}{B(6,-4)} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{6-(-4)} = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore \text{slope of the line } (CD) \text{ perpendicular to } AB \text{ line } = -\frac{1}{m} = \frac{-1}{-3/5} = \frac{5}{3}$$

$\therefore$ *Equation of the perpendicular bisector CD is* $y-y_1=m(x-x_1)$

$$y-(-1)=\frac{5}{3}\ (x-1) \Longrightarrow y+1=\frac{5}{3}\ (x-1)$$

$$3(y+1)=5\ (x-1) \Longrightarrow 3y+3=5x-5$$

$$5x-5-3y-3=0 \Longrightarrow 5x-3y-8=0$$

9. *Find the equation of a straight line through the intersection of lines* $\boldsymbol{7x+3y=10, 5x-4y=1}$ ***and parallel to the line*** $\boldsymbol{13x+5y+12=0}$

Required line is passing through the intersection of the lines

$$7x+3y=10 \quad \dots(1)$$

$$5x-4y=10 \quad \dots(2)$$

and parallel to the line $13x+5y+12=0$

Solving (1) *and* (2)

$$(1)\times 4 \Longrightarrow 28x+\cancel{12y}=40$$

$$(2)\times 3 \Longrightarrow \underline{15x-\cancel{12y}=3}$$

$$43x \qquad = 43$$

$$x=\frac{43}{43}=1 \Longrightarrow x=1$$

Sub $x=1$ *in eqn*(1)

$$7x+3y=10 \Longrightarrow 7(1)+3y=10$$

$$7+3y=10 \Longrightarrow 3y=10-7 \Longrightarrow 3y=3 \Longrightarrow y=1$$

$\therefore$ *Intersection point is* (1,1)

Condtion for Two lines to be parallel

Equations differ only in the constant term

Given Line: $13x+5y+12=0$

Parallel Line to be: $13x+5y+k=0 \dots(3)$

Since the eqn (3) *passes through* (1,1)

$$(3) \Longrightarrow 13x+5y+k=0$$

$$13(1)+5(1)+k=0 \Longrightarrow 13(1)+5(1)+k=0$$

$$18+k=0 \Longrightarrow k=-18$$

$\therefore$ *The required line equation is* $13x+5y-18=0$

10. *Find the equation of a straight line through the intersection of lines* $\boldsymbol{5x - 6y = 2, 3x + 2y = 10}$ *and parallel to the line* $\boldsymbol{4x - 7y + 13 = 0}$

Given: *Required line is passing through the intersection of the lines*

$$5x - 6y = 2 \ldots (1)$$

$$3x + 2y = 10 \ldots (2)$$

and perpendicular to the line $4x - 7y + 13 = 0$

Solving (1) *and* (2)

$$\begin{array}{rl} (1) \Longrightarrow & 5x - \cancel{6y} = 2 \\ (2) \times 3 \Longrightarrow & 9x + \cancel{6y} = 30 \\ \hline & 14x \qquad = 32 \end{array}$$

$$x = \frac{32}{14} = \frac{16}{7} \Longrightarrow \quad x = \frac{16}{7}$$

Sub $x = \dfrac{16}{7}$ *in eqn*(1)

$$(2) \Longrightarrow 3x + 2y = 10$$

$$3\left(\frac{16}{7}\right) + 2y = 10 \Longrightarrow \frac{48}{7} + 2y = 10$$

$$2y = 10 - \frac{48}{7} \Longrightarrow 2y = \frac{70 - 48}{7} \Longrightarrow 2y = \frac{22}{7} \Longrightarrow y = \frac{11}{7}$$

$$\therefore \textit{Intersection point is } \left(\frac{16}{7}, \frac{11}{7}\right)$$

Rule for Two lines to be perpendicular

i. Replace x coefficient by $[(-1) \times$ *y coefficient*$]$

ii. Replace y coefficient by just x coefficient

iii. Equation differs by constant term

Given Line: $4x - 7y + 13 = 0$

$\perp^r$ *Line to be*: $7x + 4y + k = 0 \ldots (3)$

Since the eqn (3) *passes through* $\left(\dfrac{16}{7}, \dfrac{11}{7}\right)$

$$(3) \Longrightarrow 7x + 4y + k = 0$$

$$7\left(\frac{16}{7}\right) + 4\left(\frac{11}{7}\right) + k = 0 \Longrightarrow 7\left(\frac{16}{7}\right) + 4\left(\frac{11}{7}\right) + k = 0$$

$$\left(\frac{7 \times 16}{7}\right) + \left(\frac{4 \times 11}{7}\right) + k = 0 \Longrightarrow \left(\frac{112}{7}\right) + \left(\frac{44}{7}\right) + k = 0$$

$$\frac{156}{7} + k = 0 \Rightarrow k = -\frac{156}{7}$$

$\therefore$ *The required line equation is* $7x + 4y - \frac{156}{7} = 0$

$$\frac{(7 \times 7)x + (4 \times 7)y - 156}{7} = 0 \Rightarrow \frac{49x + 28y - 156}{7} = 0$$

$$\boxed{49x + 28y - 156 = 0}$$

11. *Find the equation of a straight line joining the point of intersection of $3x + y + 2 = 0, x - 2y - 4 = 0$ to the point of intersection of $7x - 3y = 12$ and $2y = x + 3$*

Given: *Required line is passing through the intersection of the lines*

$$3x + y = -2 \ldots (1)$$
$$x - 2y = \ \ 4 \ldots (2)$$

Solving (1) *and* (2)

$$\begin{aligned} (1) \times 2 &\Rightarrow 6x + \cancel{2y} = -4 \\ (2) \times 1 &\Rightarrow \ \ x - \cancel{2y} = \ \ 4 \\ & \overline{7x \qquad\ \ = \ \ 0} \end{aligned}$$

$$x = \frac{0}{7} \Rightarrow x = 0$$

Sub $x = 0$ *in eqn*(1)

$$(1) \Rightarrow 3x + y = -2$$
$$3(0) + y = -2 \Rightarrow y = -2$$

$\therefore$ *point of intersection of eqn.*(1)&(2) *is* $P(0, -2)$

Now, to find the point of intersection of the lines $7x - 3y = -12 \ldots (3)$

$x - 2y = \ \ -3 \ldots (4)$

Solving (3) *and* (4)

$$\begin{aligned} (3) &\Rightarrow \quad \cancel{7x} - \ \ 3y = -12 \\ & \quad\ \ (-) \quad (+) \qquad (+) \\ (4) \times 7 &\Rightarrow \cancel{7x} - 14y = -21 \\ & \overline{\qquad\quad 11y = \ \ 9} \Rightarrow y = \frac{9}{11} \end{aligned}$$

Sub $y = \frac{9}{11}$ *in eqn*(4)

$$(4) \Rightarrow x - 2y = -3 \Rightarrow x - 2\left(\frac{9}{11}\right) = -3$$

$$x - \frac{18}{11} = -3 \Rightarrow x = -3 + \frac{18}{11}$$

$$x = \frac{-33 + 18}{11} \Rightarrow x = \frac{-15}{11}$$

$\therefore$ *point of intersection of eqn.* $(3)\&(4)$ *is* $Q\left(-\frac{15}{11},\frac{9}{11}\right)$

Equation of the required line joining

$$\underset{x_1\ \ y_1}{P(0,-2)} \text{ and } Q\left(\underset{x_2}{\frac{-15}{11}},\underset{y_2}{\frac{9}{11}}\right)$$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-(-2)}{\frac{9}{11}-(-2)}=\frac{x-0}{\frac{-15}{11}-0}\Longrightarrow \frac{y+2}{(9+22)/11}=\frac{x}{-15/11}$$

$$\frac{y+2}{31/11}=\frac{x}{-15/11}\Longrightarrow \frac{y+2}{31}=\frac{x}{-15}$$

$$-15(y+2)=31x \Longrightarrow -15y-30=31x \Longrightarrow 31x+15y+30=0$$

12. ***Find the equation of a straight line through the point of intersection of the lines*** $\boldsymbol{8x+3y=18, 4x+5y=9}$ ***and bisecting the line segment joining the points*** $\boldsymbol{(5,-4)}$ ***and*** $\boldsymbol{(-7,6)}$

Given: *Required line is passing through the intersection of the lines*

$$8x+3y=18\ldots(1)$$
$$4x+5y=\ \ 9\ldots(2)$$

Solving (1) *and* (2)

$$\begin{array}{rrcrcr}
(1)\Rightarrow & \cancel{8x} & + & 3y & = & 18\\
 & (-) & & (-) & & (-)\\
(2)\times 2\Rightarrow & \cancel{8x} & + & 10y & = & 18\\
\hline
 & & & -7y & = & 0
\end{array}$$

$$y=\frac{0}{-7}\Longrightarrow y=0$$

Sub $y=0$ *in eqn* (2)

$$(2)\Longrightarrow 4x+5y=9$$
$$4x+5(0)=9\Longrightarrow x=\frac{9}{4}$$

$\therefore$ *point of intersection of eqn.* $(1)\ \&\ (2)$ *is* $P\left(\frac{9}{4},0\right)$

Now, to find the mid point Q of the line joining

$$\underset{x_1\ \ y_1}{\mathrm{R}(5,-4)} \text{ and } \underset{x_2\ \ y_2}{\mathrm{S}(-7,6)} \Longrightarrow Q=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right) = \left(\frac{\overset{1}{\cancel{-2}}}{\cancel{2}}, \frac{\overset{1}{\cancel{2}}}{\cancel{2}}\right) = (-1,1)$$

$$\boxed{Q = (-1,1)}$$

Equation of the required line joining

$$P\left(\underset{x_1}{\frac{9}{4}}, \underset{y_1}{0}\right) \text{ and } Q(\underset{x_2}{-1}, \underset{y_2}{1})$$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{1-0} = \frac{x-\frac{9}{4}}{-1-\frac{9}{4}} \Rightarrow y = \frac{(4x-9)/\cancel{4}}{(-4-9)/\cancel{4}}$$

$$y = \frac{4x-9}{-13} \Rightarrow -13y = 4x-9 \Rightarrow 4x+13y-9=0$$

EXERCISE 6.1

Example 6.1: $\boldsymbol{\tan^2\theta - \sin^2\theta = \tan^2\theta\,.\sin^2\theta}$

$$L.H.S = \tan^2\theta - \sin^2\theta$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \frac{\sin^2\theta - \sin^2\theta \times \cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \tan^2\theta\,(1-\cos^2\theta)$$

$$= \tan^2\theta\,.\sin^2\theta \qquad = R.H.S$$

Hence Proved

$$\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$
$$\left[\because \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta\right]$$
$$[\because 1-\cos^2\theta = \sin^2\theta]$$

Example 6.2: **Prove that** $\boldsymbol{\dfrac{\sin A}{1+\cos A} = \dfrac{1-\cos A}{\sin A}}$

$$L.H.S = \frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A} = \frac{\sin A\,(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A\,(1-\cos A)}{1^2 - \cos^2 A} = \frac{\sin A\,(1-\cos A)}{1-\cos^2 A}$$

$$\boxed{[\because (a+b)(a-b) = a^2 - b^2]}$$
$$\boxed{[\because 1-\cos^2\theta = \sin^2\theta]}$$

$$= \frac{\cancel{\sin A}\,(1-\cos A)}{\sin A\,\cancel{\sin^2 A}} = \frac{1-\cos A}{\sin\ A} \qquad = R.H.S$$

Hence Proved

Example 6.3: **Prove that** $\boldsymbol{1 + \dfrac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta}$

$$[\because \operatorname{cosec}^2\theta - 1 = \cot^2\theta]$$
$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$L.H.S = 1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta}$$

$$= 1 + \frac{\operatorname{cosec}^2\theta - 1^2}{\operatorname{cosec}\theta + 1} = 1 + \frac{\cancel{(\operatorname{cosec}\theta + 1)}(\operatorname{cosec}\theta - 1)}{\cancel{\operatorname{cosec}\theta + 1}}$$

$$= 1 + (\operatorname{cosec}\theta - 1) \;\; \cancel{=1} + \operatorname{cosec}\theta \cancel{-1} \;\; = \operatorname{cosec}\theta \;\; = R.H.S$$

Hence Proved

Example 6.4: **Prove that** $\boldsymbol{\sec\theta - \cos\theta = \tan\theta\,.\sin\theta}$

$$L.H.S = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta \times \sin\theta}{\cos\theta}$$

$$= \tan\theta\,.\sin\theta \qquad = R.H.S$$

Hence Proved

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$
$$\left[\because \frac{1}{\cos\theta} = \sec\theta\right]$$
$$[\because 1-\cos^2\theta = \sin^2\theta]$$

Example 6.5: Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

$[\because (a+b)(a-b) = a^2 - b^2]$

$[\because 1 - \cos^2\theta = \sin^2\theta]$

$$L.H.S = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{1^2-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

$\because \frac{\cos\theta}{\sin\theta} = \cot\theta$

$$= \operatorname{cosec}\theta + \cot\theta \quad = R.H.S$$

Example 6.6: Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

$\because \frac{1}{\cos\theta} = \sec\theta$

$[\because 1 - \sin^2\theta = \cos^2\theta]$

$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$$L.H.S = \frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{1}{\cos\theta}}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} \times \frac{1}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\sin\theta.\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1-\sin^2\theta}{\sin\theta\cos\theta} = \frac{\overset{\cos\theta}{\cancel{\cos^2}\theta}}{\sin\theta\,\cancel{\cos\theta}}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta \qquad = R.H.S$$

Example 6.7: Prove that
$\sin^2 A\,.\cos^2 B + \cos^2 A.\sin^2 B + \cos^2 A.\cos^2 B + \sin^2 A.\sin^2 B = 1$

$$L.H.S = \sin^2 A\,.\cos^2 B + \cos^2 A.\sin^2 B + \cos^2 A.\cos^2 B + \sin^2 A.\sin^2 B$$

Rearranging for simplification

$$= \sin^2 A\,.\cos^2 B + \sin^2 A.\sin^2 B + \cos^2 A.\sin^2 B + \cos^2 A.\cos^2 B$$

Taking $\sin^2 A$ *and* $\cos^2 A$ *as common* $[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \sin^2 A\,(\cos^2 B + \sin^2 B) + \cos^2 A(\sin^2 B + \cos^2 B)$$

$$= \sin^2 A\,(1) + \cos^2 A\,(1) = \sin^2 A + \cos^2 A$$

$$= 1 \quad = R.H.S$$

Example 6.8: If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$**, then prove that** $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta \Rightarrow \quad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\sin\theta = (\sqrt{2}-1)\cos\theta \Longrightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{2}-1$$

$$\boxed{\because a^2-b^2 = (a+b)(a-b)}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}-1\times\sqrt{2}+1}{\sqrt{2}+1} \Longrightarrow \frac{\sin\theta}{\cos\theta} = \frac{(\sqrt{2})^2-1^2}{\sqrt{2}+1}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{2-1}{\sqrt{2}+1} \Longrightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{2}+1} \Longrightarrow (\sqrt{2}+1)\sin\theta = \cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta \Longrightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Example 6.9: Prove that
$$(\mathbf{cosec}\,\boldsymbol{\theta} - \mathbf{sin}\,\boldsymbol{\theta})(\mathbf{sec}\,\boldsymbol{\theta} - \mathbf{cos}\,\boldsymbol{\theta})(\mathbf{tan}\,\boldsymbol{\theta} + \mathbf{cot}\,\boldsymbol{\theta}) = \mathbf{1}$$

$$L.H.S = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \frac{1-\sin^2\theta}{\sin\theta} \times \frac{1-\cos^2\theta}{\cos\theta} \times \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta} = \frac{\cancel{\cos^2}\theta}{\cancel{\sin^2}\theta} \times \frac{\cancel{\sin^2}\theta}{\cancel{\cos^2}\theta}$$

$$= 1$$

$$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$
$$\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$
$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$
$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$[\because 1-\sin^2\theta = \cos^2\theta]$

$[\because 1-\cos^2\theta = \sin^2\theta]$

$[\because \sin^2\theta + \cos^2\theta = 1]$

Example 6.10: Prove that $\dfrac{\mathbf{sin}\,\boldsymbol{A}}{\mathbf{1}+\mathbf{cos}\,\boldsymbol{A}} + \dfrac{\mathbf{sin}\,\boldsymbol{A}}{\mathbf{1}-\mathbf{cos}\,\boldsymbol{A}} = \mathbf{2}\,\mathbf{cosec}\,\boldsymbol{A}$

$$L.H.S = \frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$$

$$= \frac{\sin A\,(1-\cos A) + \sin A\,(1+\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A - \cancel{\sin A\cos A} + \sin A + \cancel{\sin A\cos A}}{1^2 - \cos^2 A}$$

$$= \frac{2\sin A}{1-\cos^2 A} = \frac{2\cancel{\sin A}}{\cancel{\sin^2 A}}\ {}_{\sin A}$$

$$= \frac{2}{\sin A} = 2\times\frac{1}{\sin A} = 2\operatorname{cosec} A$$

$[\because (a+b)(a-b) = a^2-b^2]$

$[\because 1-\cos^2\theta = \sin^2\theta]$

$\left[\because \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$

Example 6.11: If $cosec\ \theta + cot\ \theta = P$, then prove that $cos\ \theta = \frac{P^2-1}{P^2+1}$

$[\because a^2 - b^2 = (a+b)(a-b)]$

Given: $cosec\ \theta + cot\ \theta = P \ldots (1)$

$$Identity: 1 + \cot^2\theta = cosec^2\theta$$

$$cosec^2\ \theta - \cot^2\theta = 1$$

$$(cosec\ \theta + \cot\theta)(cosec\ \theta - \cot\theta) = 1$$

$$cosec\ \theta - \cot\theta = \frac{1}{cosec\ \theta + \cot\theta}$$

$$cosec\ \theta - \cot\theta = \frac{1}{P} \Longrightarrow \quad cosec\ \theta - cot\ \theta = \frac{1}{P} \ldots (2)$$

Adding (1) and (2)

$$(1) \Longrightarrow \quad cosec\ \theta + \cancel{\cot\theta} = P$$

$$(2) \Longrightarrow \quad cosec\ \theta - \cancel{\cot\theta} = \frac{1}{P}$$

$$2cosec\ \theta = P + \frac{1}{P} \quad \Longrightarrow \quad 2cosec\ \theta = \frac{P^2+1}{P} \ldots (3)$$

Subtracting (2) from (1)

$$(1) \Longrightarrow \quad \cancel{cosec\ \theta} + \cot\theta = P$$

$$(-) \qquad (+) \qquad (-)$$

$$(2) \Longrightarrow \quad \cancel{cosec\ \theta} - \cot\theta = \frac{1}{P}$$

$$2\cot\ \theta = P - \frac{1}{P} \quad \Longrightarrow \quad 2\cot\ \theta = \frac{P^2-1}{P} \ldots (4)$$

Dividing (4) by (3)

$$\frac{(4)}{(3)} \Longrightarrow \frac{\cancel{2}\cot\theta}{\cancel{2}cosec\ \theta} = \frac{\frac{P^2-1}{\cancel{P}}}{\frac{P^2+1}{\cancel{P}}} \Longrightarrow \frac{\cot\theta}{cosec\ \theta} = \frac{P^2-1}{P^2+1}$$

$$\left[\because sin\ \theta = \frac{1}{cosec\ \theta}\right]$$

$$\left[\because cot\ \theta = \frac{cos\ \theta}{sin\ \theta}\right]$$

$$\cot\theta \times \frac{1}{cosec\ \theta} = \frac{P^2-1}{P^2+1} \Longrightarrow \frac{cos\ \theta}{sin\ \theta} \times sin\ \theta = \frac{P^2-1}{P^2+1}$$

$$cos\ \theta = \frac{P^2-1}{P^2+1} \qquad Hence\ Proved$$

Example 6.12: Prove that $tan^2A - tan^2B = \frac{sin^2A - sin^2B}{cos^2A\,.\,cos^2B}$

$$L.H.S = tan^2\ A - tan^2\ B$$

$$\left[\because tan\ \theta = \frac{sin\ \theta}{cos\ \theta}\right]$$

$$= \frac{sin^2\ A}{cos^2\ A} - \frac{sin^2\ B}{cos^2\ B} = \frac{sin^2\ A\, cos^2\ B - sin^2\ B\, cos^2\ A}{cos^2\ A\, cos^2\ B}$$

$$[\because 1 - sin^2\ \theta = cos^2\ \theta]$$

$$= \frac{\sin^2 A\,(1-\sin^2 B) - \sin^2 B\,(1-\sin^2 A)}{\cos^2 A\cos^2 B}$$

$$= \frac{\sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}}{\cos^2 A\cos^2 B} \quad = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B} = R.H.S$$

Hence Proved

Example 6.13: Prove that

$$\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$$

$$L.H.S = \left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right)$$

$$= \left(\frac{\cancel{(\cos A - \sin A)}(\cos^2 A + \sin^2 A + \cos A\sin A)}{\cancel{\cos A - \sin A}}\right) - \left(\frac{\cancel{(\cos A + \sin A)}(\cos^2 A + \sin^2 A - \cos A\sin A)}{\cancel{\cos A + \sin A}}\right)$$

$$\boxed{\begin{array}{l}[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)] \\ [\because a^3 + b^3 = (a+b)(a^2+b^2-ab)]\end{array}}$$

$$= (1 + \cos A\sin A) - (1 - \cos A\sin A)$$

$$= \cancel{1} + \cos A\sin A \cancel{-1} + \cos A\sin A \qquad \boxed{[\because \sin^2\theta + \cos^2\theta = 1]}$$

$$= 2\cos A\sin A \quad = R.H.S$$

Hence Proved

Example 6.14: Prove that $\dfrac{\sin A}{\sec A + \tan A - 1} + \dfrac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$

$$L.H.S = \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1}$$

$$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right] \quad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right] \quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{\sin A\,(\operatorname{cosec} A + \cot A - 1) + \cos A\,(\sec A + \tan A - 1)}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\sin A\operatorname{cosec} A + \sin A\cot A - \sin A + \cos A\sec A + \cos A\tan A - \cos A}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\cancel{\sin A}\times\frac{1}{\cancel{\sin A}} + \cancel{\sin A}\times\frac{\cos A}{\cancel{\sin A}} - \sin A + \cancel{\cos A}\times\frac{1}{\cancel{\cos A}} + \cancel{\cos A}\times\frac{\sin A}{\cancel{\cos A}} - \cos A}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{1 \cancel{+\cos A} \cancel{-\sin A} + 1 \cancel{+\sin A} \cancel{-\cos A}}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)} \qquad \boxed{\because \tan\theta = \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{2}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$=\frac{2}{\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1\right)\left(\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1\right)}$$

$$=\frac{2}{\left(\frac{1+\sin A-\cos A}{\cos A}\right)\left(\frac{1+\cos A-\sin A}{\sin A}\right)}$$

$$=2\times\frac{\cos A}{1+\sin A-\cos A}\times\frac{\sin A}{1+\cos A-\sin A}$$

$$=\frac{2\,sin\,A\,cos\,A}{[1+sin\,A-cos\,A][1+cos\,A-sin\,A]}$$

$$=\frac{2\sin A\cos A}{1\cancel{+\cos A}\cancel{-\sin A}\cancel{+\sin A}+\sin A\cos A-\sin^2 A\cancel{-\cos A}-\cos^2 A+\sin A\cos A}$$

$$=\frac{2\sin A\cos A}{1+\sin A\cos A-\sin^2 A-\cos^2 A+\sin A\cos A}$$

$$=\frac{2\sin A\cos A}{1+2\sin A\cos A-(\sin^2 A+\cos^2 A)}$$

$$=\frac{2\,sin\,A\,cos\,A}{\cancel{1}+2sin\,A\,cos\,A\cancel{-1}}=\frac{\cancel{2\,sin\,A\,cos\,A}}{\cancel{2sin\,A\,cos\,A}}$$

$=1 \quad = R.H.S$ *Hence Proved*

Example 6.15: Show that $\left(\frac{\mathbf{1+tan^2\,A}}{\mathbf{1+cot^2\,A}}\right)=\left(\frac{\mathbf{1-tan\,A}}{\mathbf{1-cot\,A}}\right)^{\mathbf{2}}$

$$L.H.S=\left(\frac{1+tan^2\,A}{1+cot^2\,A}\right)$$

$$=\frac{1+tan^2\,A}{1+\frac{1}{\tan^2 A}}=\frac{\cancel{1+tan^2\,A}}{\frac{\cancel{\tan^2 A+1}}{\tan^2 A}}=\frac{1}{\frac{1}{\tan^2 A}}=\tan^2 A=L.H.S$$

$$R.H.S=\left(\frac{1-tan\,A}{1-cot\,A}\right)^2=\left(\frac{1-tan\,A}{1-\frac{1}{\tan A}}\right)^2=\left(\frac{1-tan\,A}{\frac{\tan A-1}{\tan A}}\right)^2=\left(\frac{1-tan\,A}{\frac{-(1-\tan A)}{\tan A}}\right)^2$$

$$=\left(\frac{\cancel{1-tan\,A}}{\frac{-\cancel{(1-\tan A)}}{\tan A}}\right)^2=\left(\frac{1}{\frac{-1}{\tan A}}\right)^2=(-\tan A)^2$$

$$R.H.S=tan^2A$$

$L.H.S=R.H.S$ *Hence the result*

Example 6.16: Show that

$$\boldsymbol{\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}=\sin^2 A\cos^2 A}$$

$$\left[\because \cot\theta=\frac{\cos\theta}{\sin\theta}\right]$$

$$\left[\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right]$$

$$\boxed{[\because \sin^2\theta+\cos^2\theta=1]}$$

$$L.H.S=\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}$$

$$=\frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{(\sec A-\operatorname{cosec}A)(\sec^2 A+\operatorname{cosec}^2 A+\sec A\operatorname{cosec}A)}$$

$$=\frac{\frac{(\sin A\cos A+\cos^2 A+\sin^2 A)}{\sin A\cos A}\times(\sin A-\cos A)}{(\sec A-\operatorname{cosec}A)(\sec^2 A+\operatorname{cosec}^2 A+\sec A\operatorname{cosec}A)}$$

$$=\frac{(\sin A\cos A+1)\times\left(\frac{\sin A}{\sin A\cos A}-\frac{\cos A}{\sin A\cos A}\right)}{(\sec A-\operatorname{cosec}A)\left(\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}+\frac{1}{\cos A\sin A}\right)}$$

$$=\frac{(\sin A\cos A+1)\times\left(\frac{\cancel{\sin A}}{\cancel{\sin A}\cos A}-\frac{\cancel{\cos A}}{\sin A\cancel{\cos A}}\right)}{(\sec A-\operatorname{cosec}A)\left(\frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\sin^2 A\cos^2 A}\right)}$$

$$=\frac{(\sin A\cos A+1)\times\left(\frac{1}{\cos A}-\frac{1}{\sin A}\right)}{(\sec A-\operatorname{cosec}A)\left(\frac{1+\sin A\cos A}{\sin^2 A\cos^2 A}\right)}$$

$$\boxed{\left[\because \sec\theta=\frac{1}{\cos\theta}\ \Rightarrow \sec^2\theta=\frac{1}{\cos^2\theta}\right]}$$

$$\boxed{\left[\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}\ \Rightarrow \operatorname{cosec}^2\theta=\frac{1}{\sin^2\theta}\right]}$$

$$=\frac{\cancel{(\sin A\cos A+1)}\times\cancel{(\sec A-\operatorname{cosec}A)}}{\cancel{(\sec A-\operatorname{cosec}A)}\cancel{(1+\sin A\cos A)}}\times\sin^2 A\cos^2 A$$

$$\boxed{[\because \sin^2\theta+\cos^2\theta=1]}$$

$$=\sin^2 A\cos^2 A$$

$$\boxed{\left[\because \sec\theta=\frac{1}{\cos\theta}\right]\ \left[\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}\right]}$$

$$=R.H.S\quad \textit{Hence proved}$$

Example 6.17: $\boldsymbol{If\,\frac{\cos^2\theta}{\sin\theta}=p\ and\ \frac{\sin^2\theta}{\cos\theta}=q, then\ prove\ that}$

$$\boldsymbol{p^2q^2\left(p^2+q^2+3\right)=1}$$

$$Given:\ \frac{\cos^2\theta}{\sin\theta}=p\ldots(1)\ \ and\ \frac{\sin^2\theta}{\cos\theta}=q\ \ldots.(2)$$

$$p^2q^2(p^2+q^2+3)=\left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2+\left(\frac{\sin^2\theta}{\cos\theta}\right)^2+3\right]$$

$$=\left(\frac{\overset{\cos^2\theta}{\cancel{\cos^4\theta}}}{\cancel{\sin^2\theta}}\right)\left(\frac{\overset{\sin^2\theta}{\cancel{\sin^4\theta}}}{\cancel{\cos^2\theta}}\right)\left[\left(\frac{\cos^4\theta}{\sin^2\theta}\right)+\left(\frac{\sin^4\theta}{\cos^2\theta}\right)+3\right]$$

$$= (\cos^2\theta \times \sin^2\theta)\left[\left(\frac{\cos^4\theta}{\sin^2\theta}\right) + \left(\frac{\sin^4\theta}{\cos^2\theta}\right) + 3\right]$$

$$= \cancel{(\cos^2\theta \times \sin^2\theta)}\left[\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta\cos^2\theta}{\cancel{\sin^2\theta\cos^2\theta}}\right]$$

$$= \cos^6\theta + \sin^6\theta + 3\sin^2\theta\cos^2\theta$$

$$= (\cos^2\theta)^3 + (sin^2\theta)^3 + 3\sin^2\theta\cos^2\theta$$

$$\left[\begin{array}{l} \because (a+b)^3 = a^3 + b^3 + 3ab(a+b) \\ a^3 + b^3 = (a+b)^3 - 3ab(a+b) \end{array}\right]$$

$$= (\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta\,(\cos^2\theta + \sin^2\theta) + 3\sin^2\theta\cos^2\theta$$

$$= 1^3 - 3\cos^2\theta\sin^2\theta\,(1) + 3\sin^2\theta\cos^2\theta \qquad [\because sin^2\theta + cos^2\theta = 1]$$

$$= 1 \quad = R.H.S \quad Hence\ Proved$$

1. *Prove the following identities* i) *cot θ + tan θ = sec θ cosec θ*

$$L.H.S = cot\,\theta + tan\,\theta$$

$$= \frac{cos\,\theta}{sin\,\theta} + \frac{sin\,\theta}{cos\,\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\,.\cos\theta}$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right] \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta\,.\cos\theta} = \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$

$$\left[\because \frac{1}{cos\ \theta} = sec\ \theta\right] \qquad \left[\because \frac{1}{sin\ \theta} = cosec\ \theta\right]$$

$$= \sec\theta\,.\,cosec\ \theta \quad = RHS$$

Hence Proved

ii) tan⁴θ + tan²θ = sec⁴θ − sec²θ

$$[\because 1 + tan^2\theta = sec^2\ \theta]$$

$$[\because tan^2\theta = sec^2\theta - 1]$$

$$L.H.S = \tan^4\theta + \tan^2\theta$$

$$= \tan^2\theta\,(\tan^2\theta + 1) = (\sec^2\theta - 1)\,(\sec^2\theta)$$

$$= \sec^4\theta - \sec^2\theta \quad = R.H.S$$

Hence Proved

2. *Prove the following identities :* $i)\ \dfrac{1 - tan^2\,\theta}{cot^2\theta - 1} = tan^2\,\theta$

$$L.H.S = \frac{1 - \tan^2\theta}{\cot^2\theta - 1}$$

$$\left[\because cot\ \theta = \frac{1}{tan\ \theta}\right] \Longrightarrow \left[cot^2\ \theta = \frac{1}{tan^2\ \theta}\right]$$

$$= \frac{1 - \tan^2\theta}{\dfrac{1}{\tan^2\theta} - 1} = \frac{1 - \tan^2\theta}{\dfrac{1 - \tan^2\theta}{\tan^2\theta}} = \cancel{1 - \tan^2\theta} \times \frac{\tan^2\theta}{\cancel{1 - \tan^2\theta}}$$

$$= \tan^2\theta \quad = R.H.S\ Hence\ Proved$$

$$ii)\ \boldsymbol{\frac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta}$$

$$[\because 1-\sin^2\theta = \cos^2\theta]$$

$$\left[\because \frac{1}{\cos\theta} = \sec\theta\right]$$

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$L.H.S = \frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta\,(1-\sin\theta)}{1-\sin^2\theta} = \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cancel{\cos^2\theta}\ \mathbf{\cos\theta}} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta \quad = R.H.S$$

Hence Proved

$$\mathbf{3.}\ \textit{Prove the following identities:}\ \mathbf{i)}\ \boldsymbol{\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta}$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$[\because 1-\sin^2\theta = \cos^2\theta]$$

$$\left[\because \frac{1}{\cos\theta} = \sec\theta\right]$$

$$\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$

$$L.H.S = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1^2-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta \quad = R.H.S\ \textit{Hence Proved}$$

$$\mathbf{ii)}\ \boldsymbol{\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta}$$

$$\left[\because \boldsymbol{\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta}\right]$$

(***from the previous result***)

$$\left[\because \boldsymbol{\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta}\right]$$

(***from the previous result***)

$$LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec\theta + \tan\theta + \sec\theta - \tan\theta$$

$$= \sec\theta + \cancel{\tan\theta} + \sec\theta - \cancel{\tan\theta}$$

$$= 2\sec\theta \quad = RHS\ \textit{Hence Proved}$$

4. Prove the following identities i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

$L.H.S = \sec^6\theta$

$= (\sec^2\theta)^3 = (1+\tan^2\theta)^3 = (1+\tan^2\theta)^3$

$= 1^3 + (\tan^2\theta)^3 + 3(1)(\tan^2\theta)(1+\tan^2\theta)$ $[\because 1+tan^2\theta = sec^2\theta]$

$= 1 + \tan^6\theta + 3\tan^2\theta(1+\tan^2\theta)$ $[\because (a+b)^3 = a^3+b^3+3ab(a+b)]$

$= 1 + \tan^6\theta + 3\tan^2\theta\sec^2\theta$

$= R.H.S$ *Hence Proved*

ii) $(\sin\theta + \sec\theta)^2 + (\cos\theta + cosec\,\theta)^2 = 1 + (\sec\theta + cosec\,\theta)^2$

$L.H.S = (sin\,\theta + sec\,\theta)^2 + (cos\,\theta + cosec\,\theta)^2$ $[\because (a+b)^2 = a^2+b^2+2ab]$

$= sin^2\theta + sec^2\theta + 2\,sin\,\theta\,.\,sec\,\theta + cos^2\theta + cosec^2\theta + 2\,cos\,\theta\,.\,cosec\,\theta$

$$= (sin^2\theta + cos^2\theta) + sec^2\theta + 2\,sin\,\theta\,.\frac{1}{cos\,\theta} + cosec^2\theta + 2\,cos\,\theta\,.\frac{1}{sin\,\theta}$$

$$= 1 + sec^2\theta + \frac{2\,sin\,\theta}{cos\,\theta} + cosec^2\theta + \frac{2\,cos\,\theta}{sin\,\theta}$$

$$= 1 + sec^2\theta + cosec^2\theta + \frac{2\,sin\,\theta}{cos\,\theta} + \frac{2\,cos\,\theta}{sin\,\theta}$$

$$= 1 + sec^2\theta + cosec^2\theta + 2\left(\frac{sin\,\theta}{cos\,\theta} + \frac{cos\,\theta}{sin\,\theta}\right)$$

$$= 1 + sec^2\theta + cosec^2\theta + 2\left(\frac{sin^2\theta + cos^2\theta}{cos\,\theta\,.\,sin\,\theta}\right)$$

$$= 1 + sec^2\theta + cosec^2\theta + 2\left(\frac{1}{cos\,\theta\,.\,sin\,\theta}\right)$$

$$\left[\because sec\,\theta = \frac{1}{cos\,\theta}\right]$$
$$\left[\because cosec\,\theta = \frac{1}{sin\,\theta}\right]$$
$$[\because sin^2\theta + cos^2\theta = 1]$$
$$[\because sin^2\theta + cos^2\theta = 1]$$
$$\left[\because cosec\,\theta = \frac{1}{sin\,\theta}\right]$$
$$\left[\because sec\,\theta = \frac{1}{cos\,\theta}\right]$$

$= 1 + sec^2\theta + cosec^2\theta + 2\,sec\,\theta\,cosec\,\theta$ $[\because a^2+b^2+2ab = (a+b)^2]$

$= 1 + (sec\,\theta + cosec\,\theta)^2 = R.H.S$

Hence Proved

5. Prove the following identities i) $sec^4\theta\,(1 - sin^4\theta) - 2tan^2\theta = 1$

$L.H.S = sec^4\theta\,(1 - sin^4\theta) - 2tan^2\theta$ $[\because \boldsymbol{a^2 - b^2 = (a+b)(a-b)}]$

$= sec^4\theta\,[1^2 - (sin^2\theta)^2] - 2tan^2\theta$

$$= \frac{1}{cos^4\theta}(1 + sin^2\theta).(1 - sin^2\theta) - \frac{2sin^2\theta}{cos^2\theta}$$ $\left[\because \boldsymbol{sec\,\theta = \frac{1}{cos\,\theta}}\right]$

$$= \frac{1+\sin^2\theta}{\cancel{\cos^4\theta}_{\cos^2\theta}} . \cancel{\cos^2\theta} - \frac{2\sin^2\theta}{\cos^2\theta} = \frac{1+\sin^2\theta}{\cos^2\theta} - \frac{2\sin^2\theta}{\cos^2\theta} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\boxed{\left[\because 1-\sin^2\theta = \cos^2\theta\right]}$$

$$= \frac{1+\sin^2\theta - 2\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}^{\,1} = 1 \quad = R.H.S$$

Hence Proved

$$\textbf{ii)}\ \frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right] \quad \left[\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

$$L.H.S = \frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$$

$$= \frac{\cancel{\cos}\theta\left(\frac{1}{\sin\theta} - 1\right)}{\cancel{\cos}\theta\left(\frac{1}{\sin\theta} + 1\right)} = \frac{\frac{1}{\sin\theta} - 1}{\frac{1}{\sin\theta} + 1} = \frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1} \quad = \text{R.H.S}$$

Hence Proved

6. Prove the following identities : i) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

$$\boxed{[\because (a+b)(a-b) = a^2 - b^2]}$$

$$\boxed{[\because \sin^2\theta + \cos^2\theta = 1]}$$

$$L.H.S = \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)} = \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A + \cos^2 A - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)} = \frac{0}{(\cos A + \cos B)(\sin A + \sin B)}$$

$= 0 = R.H.S$ *Hence Proved*

$$\textbf{ii)}\ \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$$

$$\boxed{[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]}$$

$$\boxed{[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]}$$

$$L.H.S = \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$= \frac{\cancel{(\sin A + \cos A)}(\sin^2 A - \sin A . cosA + \cos^2 A)}{\cancel{sin\ \theta + cos\ \theta}} + \frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \sin A . cosA + \cos^2 A)}{\cancel{sin\ \theta - cos\ \theta}}$$

$\boxed{[\because sin^2\theta + cos^2\theta = 1]}$

$$= sin^2 A - sin\, A . cosA + cos^2 A + sin^2 A + sin\, A . cosA + cos^2 A$$

$$= \sin^2 A + \cos^2 A - \sin A . cosA + \sin^2 A + \cos^2 A + \sin A . cosA$$

$$= 1 - \cancel{\sin A . cosA} + 1 + \cancel{\sin A . cosA} = 2$$

$= R.H.S$ *Hence Proved*

7. i) $\boldsymbol{If\ sin\ \theta + cos\ \theta = \sqrt{3},\ then\ prove\ that\ tan\ \theta + cot\ \theta = 1}$

$Given: \sin\theta + \cos\theta = \sqrt{3}$

$\boxed{[\because (a+b)^2 = a^2 + b^2 + 2ab]}$

Squaring on both sides

$\boxed{[\because sin^2\theta + cos^2\theta = 1]}$

$$(\sin\theta + \cos\theta)^2 = 3$$

$\left[\because \frac{sin\ \theta}{cos\ \theta} = tan\ \theta\right]\left[\because \frac{cos\ \theta}{sin\ \theta} = cot\ \theta\right]$

$$sin^2\,\theta + cos^2\,\theta + 2\,sin\,\theta . cos\,\theta = 3$$

$$1 + 2\sin\theta . \cos\theta = 3 \Rightarrow 2\sin\theta . \cos\theta = 3 - 1$$

$$\cancel{2}\sin\theta . \cos\theta = \cancel{2} \Rightarrow sin\,\theta\ cos\ \theta = 1 \ldots (1)$$

$To\ prove : tan\,\theta + cot\,\theta = 1$

$[\because sin^2\theta + cos^2\theta = 1]$

$L.H.S = tan\,\theta + cot\,\theta$

$$= \frac{sin\ \theta}{cos\ \theta} + \frac{cos\ \theta}{sin\ \theta} = \frac{sin^2\,\theta + cos^2\,\theta}{sin\,\theta . cos\,\theta} = \frac{1}{sin\ \theta\ cos\ \theta} = \frac{1}{1} = 1$$

$= R.H.S$ *Hence Proved*

ii) $\boldsymbol{If\ \sqrt{3}\ sin\ \theta - cos\ \theta = 0, then\ show\ that\ tan\,3\theta = \frac{3\ tan\ \theta - tan^3\theta}{1 - 3tan^2\ \theta}}$

$Given: \sqrt{3}\ \sin\theta - cos\ \theta = 0$

$\left[\because \boldsymbol{tan\ 30^o = \frac{1}{\sqrt{3}}}\right]$

$$\sqrt{3}\ \sin\theta = cos\ \theta \Rightarrow \frac{sin\ \theta}{cos\ \theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^o$$

$To\ prove : tan\ 3\theta = \dfrac{3\ tan\ \theta - tan^3\theta}{1 - 3tan^2\ \theta}$

$$\boxed{\begin{aligned}\left(\frac{1}{\sqrt{3}}\right)^3 &= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\\ &= \frac{1}{3} \times \frac{1}{\sqrt{3}} = \frac{1}{3\sqrt{3}}\end{aligned}}$$

$L.H.S = tan\ 3\theta$

$\quad = tan\ 3(30^o) = tan\ 90^o$

$L.H.S = \infty$

$$R.H.S = \frac{3\ tan\ \theta - tan^3\theta}{1 - 3tan^2\ \theta} = \frac{3\ tan\ (30^o) - tan^3(30^o)}{1 - 3tan^2\ (30^o)} = \frac{3\left(\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\sqrt{3} \times \cancel{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{3}}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{1 - \cancel{3}\left(\frac{1}{\cancel{3}}\right)} = \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{1 - 1}$$

$$= \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{0} = \infty$$

$$L.H.S = R.H.S$$

8. i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2 + n^2)\cos^2\beta = n^2$

Given: $\frac{cos\ \alpha}{cos\ \beta} = m, \ \frac{cos\ \alpha}{sin\ \beta} = n$

Squaring on both sides

$$\frac{\cos^2\alpha}{\cos^2\beta} = m^2, \ \frac{\cos^2\alpha}{\sin^2\beta} = n^2$$

To prove : $(m^2 + n^2)\ cos^2\beta = n^2$

$$L.H.S = (m^2 + n^2)\ cos^2\beta = \left(\frac{cos^2\alpha}{cos^2\beta} + \frac{cos^2\alpha}{sin^2\beta}\right)cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right)\cos^2\beta \ = cos^2\alpha\left(\frac{sin^2\beta + cos^2\beta}{cos^2\beta\,.sin^2\beta}\right)cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cancel{\cos^2\beta}\,.\sin^2\beta}\right)\cancel{\cos^2\beta} = \frac{\cos^2\alpha}{\sin^2\beta} = n^2 = R.H.S$$

Hence Proved

8. i)if $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2 + n^2)\cos^2\beta = n^2$

Given: $\frac{cos\ \alpha}{cos\ \beta} = m, \ \frac{cos\ \alpha}{sin\ \beta} = n$

Squaring on both side

$$\frac{\cos^2\alpha}{\cos^2\beta} = m^2, \ \frac{\cos^2\alpha}{\sin^2\beta} = n^2 \ \Longrightarrow \ m^2 + n^2 = \frac{cos^2\alpha}{cos^2\beta} + \frac{cos^2\alpha}{sin^2\beta}$$

$$m^2 + n^2 = cos^2\alpha\left(\frac{1}{cos^2\beta} + \frac{1}{sin^2\beta}\right) \Longrightarrow \ m^2 + n^2 = \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\,.\sin^2\beta}\right)$$

$$m^2 + n^2 = cos^2\alpha\left(\frac{1}{cos^2\beta\ sin^2\beta}\right) \Longrightarrow \ m^2 + n^2 = \frac{cos^2\alpha}{cos^2\beta\,.sin^2\beta}$$

$$m^2 + n^2 = \frac{1}{cos^2\beta} \times \frac{cos^2\alpha}{sin^2\beta} \ \Longrightarrow \ m^2 + n^2 = \frac{1}{cos^2\beta} \times n^2$$

$$(m^2 + n^2)\ cos^2\beta = n^2$$

Hence Proved

ii)If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

$$[\because 1 - \cos^2\theta = \sin^2\theta]$$

Given: $x = \cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\ \cos\theta}$$

$$x = \frac{1}{\sin\theta\cos\theta}$$

$$y = \sec\theta - \cos\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta = \frac{1 - \cos^2\theta}{\cos\theta}$$

$$y = \frac{\sin^2\theta}{\cos\theta}$$

To prove : $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

$$L.H.S = \left(\frac{1}{\cancel{\sin^2\theta}\ \cos^2\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}\right)^{\frac{2}{3}} - \left(\frac{1}{\cancel{\sin\theta}\cos\theta} \times \frac{\overset{\sin^3\theta}{\cancel{\sin^4\theta}}}{\cos^2\theta}\right)^{\frac{2}{3}}$$

$$\because 1 + \tan^2\theta = \sec^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = (\sec^{\cancel{3}}\theta)^{\cancel{\frac{2}{3}}} - (\tan^{\cancel{3}}\theta)^{\cancel{\frac{2}{3}}}$$

$$= \sec^2\theta - \tan^2\theta = 1$$

$= R.H.S$ Hence Proved

9. i)If $\sin\theta + \cos\theta = p$ and $\sec\theta + \mathrm{cosec}\,\theta = q$, then prove that $q(p^2 - 1) = 2p$

$$\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

$$\left[\because \mathrm{cosec}\ \theta = \frac{1}{\sin\theta}\right]$$

$$\boxed{[\because (a+b)^2 = a^2 + b^2 + 2ab]}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

Given: $p = \sin\theta + \cos\theta$, $q = \sec\theta + \mathrm{cosec}\ \theta$

To prove : $q(p^2 - 1) = 2p$

$$L.H.S = q(p^2 - 1)$$

$$= (\sec\theta + \mathrm{cosec}\ \theta)[(\sin\theta + \cos\theta)^2 - 1]$$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)[\sin^2\theta + \cos^2\theta + 2\sin\theta\,.\cos\theta - 1]$$

$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\,.\sin\theta}\right)[\cancel{1} + 2\sin\theta\,.\cos\theta - \cancel{1}] = \left(\frac{\sin\theta + \cos\theta}{\cos\theta\,.\sin\theta}\right)[2\sin\theta\,.\cos\theta]$$

$$= \left(\frac{\sin\theta + \cos\theta}{\cancel{\cos\theta\,.\sin\theta}}\right)[2\cancel{\sin\theta\,.\cos\theta}] = (\sin\theta + \cos\theta)[2]$$

$$\boxed{[\because p = \sin\theta + \cos\theta]}$$

$= 2(\sin\theta + \cos\theta) = 2p = R.H.S$ Hence Proved

ii) If $\sin\theta(1+\sin^2\theta)=\cos^2\theta$, then prove that $\cos^6\theta-4\cos^4\theta+8\cos^2\theta=4$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$[\because \cos^2\theta = 1-\sin^2\theta]$

$[\because (a-b)^2 = a^2+b^2-2ab]$

Given: $\sin\theta\,(1+\sin^2\theta) = \cos^2\theta$

$$\sin\theta + \sin^3\theta = 1-\sin^2\theta$$

squaring on both side

$$(\sin\theta+\sin^3\theta)^2 = (1-\sin^2\theta)^2$$

$$\sin^2\theta + (\sin^3\theta)^2 + 2\sin\theta\sin^3\theta = 1^2 + (\sin^2\theta)^2 - 2(1)\sin^2\theta$$

$$\sin^2\theta + \sin^6\theta + 2\sin^4\theta = 1 + \sin^4\theta - 2\sin^2\theta$$

$$\sin^2\theta + \sin^6\theta + 2\sin^4\theta - \sin^4\theta + 2\sin^2\theta = 1$$

$$3\sin^2\theta + \sin^4\theta + \sin^6\theta = 1$$

$[\because (a-b)^3 = a^3-b^3-3a^2b+3ab^2]$

$$3\sin^2\theta + (\sin^2\theta)^2 + (\sin^2\theta)^3 = 1$$

$$3(1-\cos^2\theta) + (1-\cos^2\theta)^2 + (1-\cos^2\theta)^3 = 1$$

$[\because \sin^2\theta = 1-\cos^2\theta]$

$$3-3\cos^2\theta + 1 + \cos^4\theta - 2\cos^2\theta$$
$$+ 1^3 - (\cos^2\theta)^3 - 3(1)^2(\cos^2\theta) + 3(1)(\cos^2\theta)^2 = 1$$

$$3-3\cos^2\theta + 1 + \cos^4\theta - 2\cos^2\theta + 1 - \cos^6\theta + 3\cos^4\theta - 3\cos^2\theta = 1$$

$$5 - 8\cos^2\theta + 4\cos^4\theta - \cos^6\theta = 1$$

$$-8\cos^2\theta + 4\cos^4\theta - \cos^6\theta = 1-5$$

$$-8\cos^2\theta + 4\cos^4\theta - \cos^6\theta = -4$$

$$8\cos^2\theta - 4\cos^4\theta + \cos^6\theta = 4$$

$$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$$ *Hence Proved*

10) If $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1}{a}$, then prove that $\dfrac{a^2-1}{a^2+1} = \sin\theta$

$\left[\because \sec\theta = \dfrac{1}{\cos\theta}\right]$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

Given: $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1}{a} \Longrightarrow a = \dfrac{1+\sin\theta}{\cos\theta}$

$$a = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \Longrightarrow a = \sec\theta + \tan\theta$$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$[\because 1+\tan^2\theta = \sec^2\theta]$

$[\tan^2\theta = \sec^2\theta - 1]$

To prove : $\dfrac{a^2-1}{a^2+1} = \sin\theta$

$$L.H.S = \frac{a^2-1}{a^2+1} = \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{\cancel{1} + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta \cancel{-1}}{\sec^2\theta + \sec^2\theta \cancel{-1} + 2\sec\theta\tan\theta \cancel{+1}}$$

$$= \frac{2\,tan^2\,\theta + 2\,sec\,\theta\,tan\,\theta}{2\,sec^2\,\theta + 2\,sec\,\theta\,tan\,\theta} = \frac{\cancel{2}\,tan\,\theta\,\cancel{(tan\,\theta + sec\,\theta)}}{\cancel{2}\,sec\,\theta\,\cancel{(sec\,\theta + tan\,\theta)}}$$

$$= \frac{tan\,\theta}{sec\,\theta} = \frac{\frac{sin\,\theta}{\cancel{cos\,\theta}}}{\frac{1}{\cancel{cos\,\theta}}} = sin\,\theta \quad = R.H.S$$

$$\left[\because tan\,\theta = \frac{sin\,\theta}{cos\,\theta}\right]$$

$$\left[\because sec\,\theta = \frac{1}{cos\,\theta}\right]$$

Hence Proved

EXERCISE 6.2

Angle of depression and angle of elevation

Angle of elevation

If an object is above the horizontal line from our eyes we have to raise our head to view the object. In this process our eyes move through an angle formed by the line of sight and horizontal line which is called the angle of elevation.

Angle of depression :

If an object is below the horizontal line from the eye, we have to lower our head to view the object. In this process our eyes moves through an angle. This angle is called the angle of depression

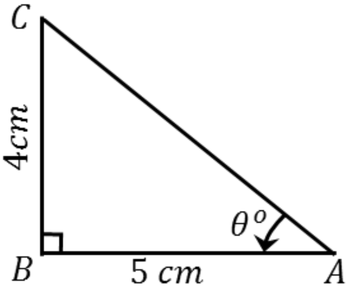
The angle of elevation of an object as seen by the observer is same as the angle of depression of the observer as seen from the object.

The angle of elevation = The angle of depression

$$\boxed{\theta_1 = \theta_2}$$

Example 6.19. Calculate the size of ∠BAC in the given triangle

(i) In Δ ABC,



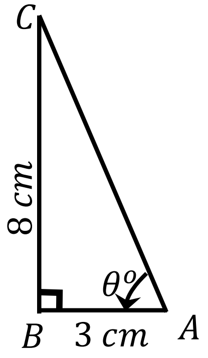
$$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{BC}{AB} = \frac{4}{5}$$

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) = \tan^{-1}(0.8)$$

$$\theta = 38.7 \qquad \boxed{[\because \tan 38.7^o = 0.8011]}$$

$$\therefore \angle BAC = 38.7^o$$

(ii) In Δ ABC,



$$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{BC}{AB} = \frac{8}{3}$$

$$\theta = \tan^{-1}\left(\frac{8}{3}\right)$$

$$= \tan^{-1}(2.66) \qquad [\because \tan 69.4^o = 2.6604]$$

$$= 69.4$$

$$\therefore \angle BAC = 69.4^o$$

Example 6.20. A tower stands vertically on the ground. From a point on the ground, which is 48m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

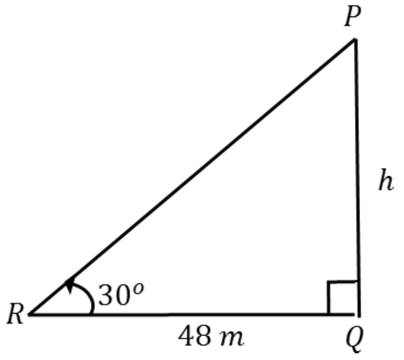
$Let, PQ = Height\ of\ the\ tower = h\ meters\ \ and\ QR = 48m$

$In\ \Delta PQR, \angle PRQ = 30^o$

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^o = \frac{PQ}{RQ} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$\frac{48}{\sqrt{3}} = h \Rightarrow h = \frac{48\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Longrightarrow h = \frac{\overset{16}{\cancel{48}}\sqrt{3}}{\cancel{3}}$$



$Height\ of\ the\ tower = 16\sqrt{3}\,m$

Example 6.20. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string withthe ground is 60°. Find the length of the string, assuming that there is no slack in the string.

$Let,\ AC = Length\ of\ the\ string$

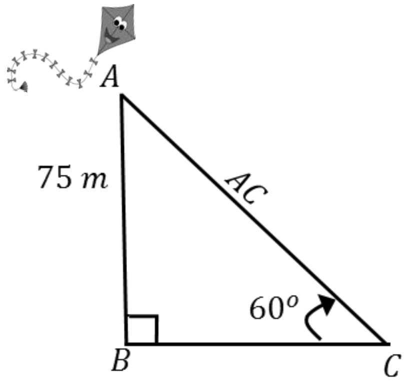
$AB = Height\ of\ the\ kite\ above\ the\ ground = 75\ meters$

$In\ \Delta\ ABC,\ \angle ACB = 60^o$

$$\sin\theta = \frac{opp}{hyp} \Longrightarrow \sin 60^o = \frac{AB}{AC} \Longrightarrow \frac{\sqrt{3}}{2} = \frac{75}{AC}$$

$$AC = \frac{75\times 2}{\sqrt{3}} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\overset{50}{\cancel{150}}\sqrt{3}}{\underset{3}{\cancel{3}}}$$

$$AC = 50\sqrt{3}$$



$\therefore Length\ of\ the\ string = 50\sqrt{3}$

Example 6.21. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the light−house is 200 m high, find the distance between the two ships.

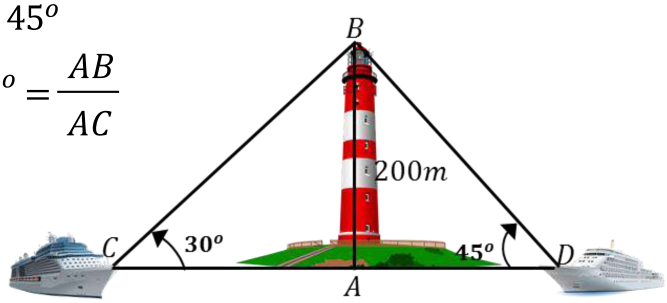
$Let, CD\ be\ the\ distance\ between\ two\ ships$

$AB = Height\ of\ the\ light\ house = 200\ m$

$\angle ACB = 30^o\ and\ \angle ADB = 45^o$

$$In\ \Delta\ BAC,\quad \tan\theta = \frac{opp}{adj} \Longrightarrow \tan 30^o = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{200}{AC} \Longrightarrow AC = 200\sqrt{3}$$



$In\ \Delta\ BAD,$

$$\tan\theta = \frac{opp}{adj} \Longrightarrow \tan 45^o = \frac{AB}{AD}$$

$$1 = \frac{200}{AD} \implies AD = 200$$

$$CD = CA + AD = 200\sqrt{3} + 200$$

$$= 200(\sqrt{3} + 1) = 200(1.732 + 1)$$

$$= 200(2.732) = 546.4$$

$\therefore$ *Distance between two ships* $= 546.4$ *meters*

Example 6.22. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3} = 1.732$)

Let, $AC =$ *Height of the tower* $= h$ *meters*

$AB =$ *Height of the building* $= 30\ m$

$\angle CPB = 60^o$ *and* $\angle APB = 45^o$

In $\Delta\, ABP$, $\tan\theta = \frac{opp}{adj} \implies \tan 45^o = \frac{AB}{BP}$

$$1 = \frac{30}{BP} \implies BP = 30$$

In $\Delta\, CBP$, $\tan\theta = \frac{opp}{adj} \implies \tan\theta = \frac{BC}{BP}$

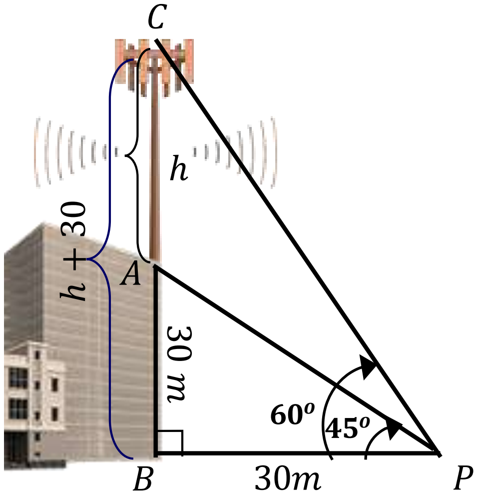
$$\tan 60^o = \frac{h+30}{30} \implies \sqrt{3} = \frac{h+30}{30}$$

$$\sqrt{3} \times 30 = 30 + h \implies h = 30\sqrt{3} - 30$$

$$h = 30(\sqrt{3} - 1) = 30(1.732 - 1) = 30(0.732)$$

$$\boxed{h = 21.96}$$

$\therefore$ *Height of the tower* $= 21.96\ m$



Example 6.23. A TV tower stands vertically on a bank of a canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is 58°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. ($\tan 58° = 1.6003$)

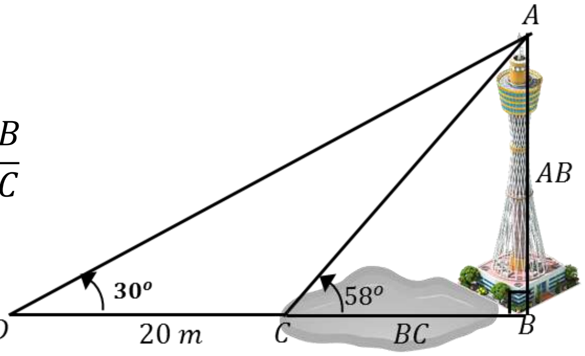
Let, $AB =$ *Height of the TV tower*

$BC =$ *Width of the canal*

$CD =$ *Distance between the two points* $= 20\ m$

In $\Delta\, ABC$, $\tan\theta = \frac{opp}{adj} \implies \tan 58^o = \frac{AB}{BC}$

$$1.6003 = \frac{AB}{BC} \dots (1)$$



$$In\ \Delta\, ABD,\ \tan\theta = \frac{opp}{adj} \Rightarrow\ \tan 30^o = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+20} \dots (2)$$

Dividing (1) *by* (2)

$$\frac{1.6003}{\frac{1}{\sqrt{3}}} = \frac{\frac{AB}{BC}}{\frac{AB}{BC+20}} \Rightarrow \frac{1.6003}{\frac{1}{\sqrt{3}}} = \frac{\cancel{AB}}{BC} \times \frac{BC+20}{\cancel{AB}}$$

$$\sqrt{3} \times 1.6003 = \frac{BC+20}{BC} \Rightarrow 1.732 \times 1.6003 = \frac{BC}{BC} + \frac{20}{BC}$$

$$2.7717 = 1 + \frac{20}{BC} \Rightarrow 2.7717 - 1 = \frac{20}{BC}$$

$$1.7717 = \frac{20}{BC} \Rightarrow BC = \frac{20}{1.7717}$$

$$BC = 11.28\ m \dots (3)$$

$$\therefore Width\ of\ the\ canal = 11.28\ m$$

$$(1) \Rightarrow 1.6003 = \frac{AB}{BC}$$

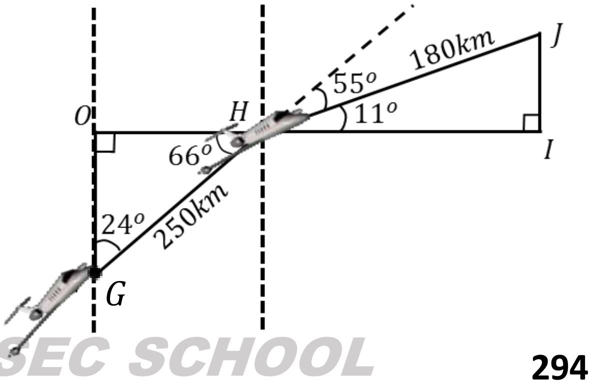
$$AB = BC \times 1.6003 = 11.28 \times 1.6003$$

$$AB = 18.05\ m$$

$$\therefore Height\ of\ the\ tower = 18.05\ m$$

Example 6.24. An aeroplane sets off from G on a bearing of 24° towards H, a point 250 km away. At H it changes course and heads towards J on a bearing of 55° and a distance of 180 km away. (i) How far is H to the North of G? (ii) How far is H to the East of G? (iii) How far is J to the North of H? (iv) How far is J to the East of H?

(i) How far is H to the North of G?



In ΔGOH,

$$cos\,\theta = \frac{adj}{hyp} \Rightarrow cos\, 24^o = \frac{OG}{GH}$$

$$0.9135 = \frac{OG}{250} \Rightarrow OG = 0.9135 \times 250$$

$$OG = 228.38$$

(ii) How far is H to the East of G?

$$In\ \Delta GOH,\ sin\,\theta = \frac{opp}{hyp}$$

$$sin\,24^o = \frac{OH}{GH} \Longrightarrow 0.4067 = \frac{OH}{250} \Longrightarrow OH = 0.4067 \times 250$$

$$OH = 101.68$$

$\therefore$ *Distance of H to the East of G* $= 101.68\ km$

(iii) How far is J to the North of H?

$$In\ \Delta HIJ,\ sin\,\theta = \frac{opp}{hyp}$$

$$sin\,11^o = \frac{IJ}{HJ} \Longrightarrow 0.1908 = \frac{IJ}{180} \Longrightarrow IJ = 0.1908 \times 180$$

$$IJ = 34.34$$

$\therefore$ *Distance of J to the North of H* $= 34.34\ km$

(iv) How far is J to the East of H?

$$In\ \Delta HIJ,\ \cos\theta = \frac{adj}{hyp}$$

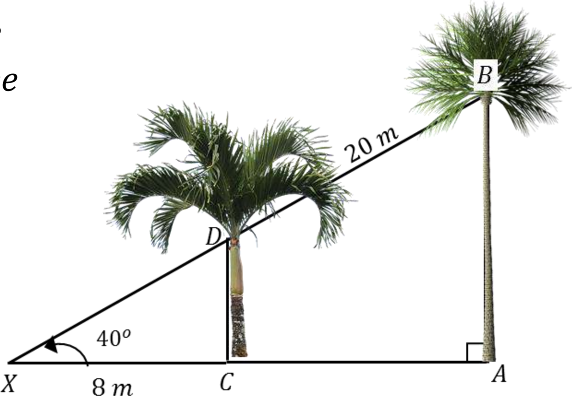
$$\cos 11^o = \frac{HI}{HJ} \Longrightarrow 0.9816 = \frac{HI}{180} \Longrightarrow HI = 0.9816 \times 180$$

$$HI = 176.69$$

$\therefore$ *Distance of J to the East of H* $= 176.69\ km$

Example 6.25. Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is 40°. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20m. calculate
(i) the distance between the point X and the top of the smaller tree
(ii) the horizontal distance between the two trees. ($cos\,40° = 0.7660$)

AB = Height of the bigger tree
CD = Height of the smaller tree



$$(i)\ In\ \Delta\,XCD\ \ \cos\theta = \frac{adj}{hyp}$$

$$\cos 45^o = \frac{CX}{XD} \Longrightarrow 0.7660 = \frac{8}{XD}$$

$$XD = \frac{8}{0.7660} \Longrightarrow XD = 10.44$$

$\therefore$ *Distance between X and the top of the smaller tree,* $XD = 10.44\ km$

(*ii*) *In Δ XAB* $\cos\theta = \frac{adj}{hyp}$

$$\cos 40^o = \frac{AX}{BX} \Rightarrow 0.7660 = \frac{AC + CX}{BD + DX}$$

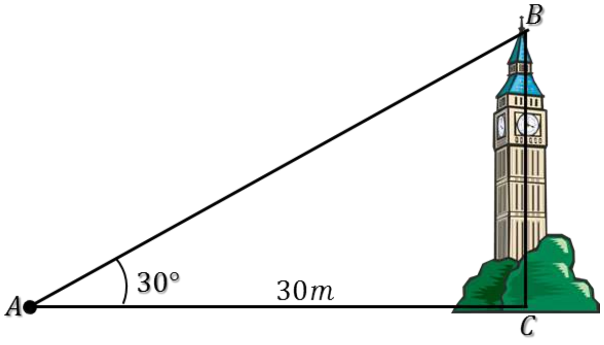
$$0.7660 = \frac{AC + 8}{20 + 10.44} \Rightarrow 0.7660 = \frac{AC + 8}{30.44}$$

$$0.7660 \times 30.44 = AC + 8 \Rightarrow \ 23.32 = AC + 8$$

$$AC = 23.32 - 8 \Rightarrow AC = 15.32$$

$\therefore$ *Horizontal distance between two trees* $= 15.32m$

1. *Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$ m.*



(*i*) *In Δ ABC*

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan\theta = \frac{\cancel{3}}{\cancel{3}\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \theta = 30^o$$

$$\therefore \angle CAB = 30^o$$

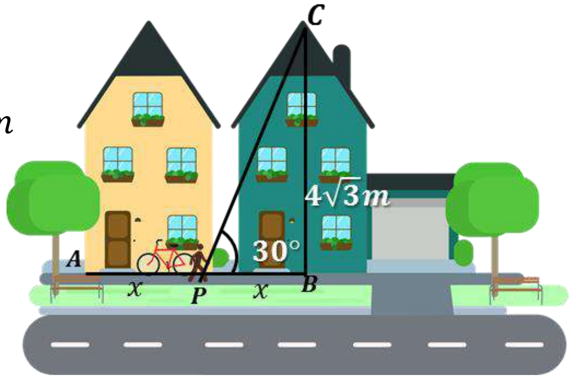
2. *A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ with no space in between them. A pedestrian is standing on the median of the road facing row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.*

Let, $AB =$ *Width of the road*

$BC =$ *Height of the row houses* $= 4\sqrt{3}m$

$P =$ *Midpoint of AB*

$PB = PA = x$ *meters*



In ΔPBC, $\tan\theta = \frac{opp}{adj} \Rightarrow \tan 30^o = \frac{BC}{PB}$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x} \Rightarrow x = 4\left(\sqrt{3} \overset{3}{\times} \sqrt{3}\right) = 4 \times 3$$

$$x = 12\,m$$

$$Width\ of\ the\ road = AB = 2x$$
$$= 2(12)$$

$$\therefore Width\ of\ the\ road = 24\,m$$

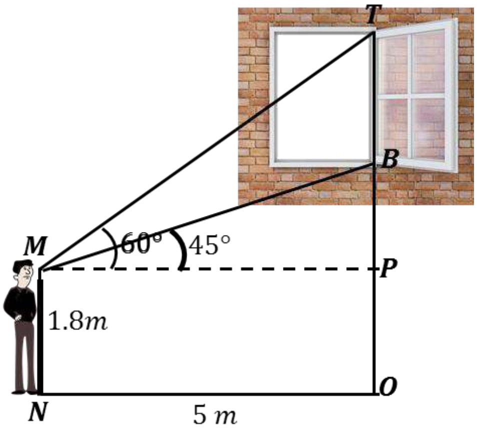
3. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ($\sqrt{3} = 1.732$)

$MN =$ height of the man $= 180\ cm = 1.8\ m$

$ON =$ distance between Man & Wall $= 5m$

$TB =$ height of window $= y$

$BP = x$



$$In\ \Delta PMB,\ \tan\theta = \frac{opp}{adj} \Longrightarrow \tan 45^o = \frac{BP}{MP}$$

$$1 = \frac{x}{5} \Longrightarrow x = 5 \ldots (1)$$

$$In\ \Delta PMT,\ \tan\theta = \frac{opp}{adj} \Longrightarrow \tan 60^o = \frac{PT}{PM}$$

$$= \frac{PB + BT}{PM} = \frac{x + y}{5}$$

$$\sqrt{3} = \frac{5 + y}{5} \Longrightarrow 5\sqrt{3} = 5 + y$$

$$y = 5\sqrt{3} - 5 = 5(\sqrt{3} - 1)$$
$$= 5(1.732 - 1) = 5(0.732)$$
$$y = 3.66$$

$$\therefore Height\ of\ the\ window = 3.66\,m$$

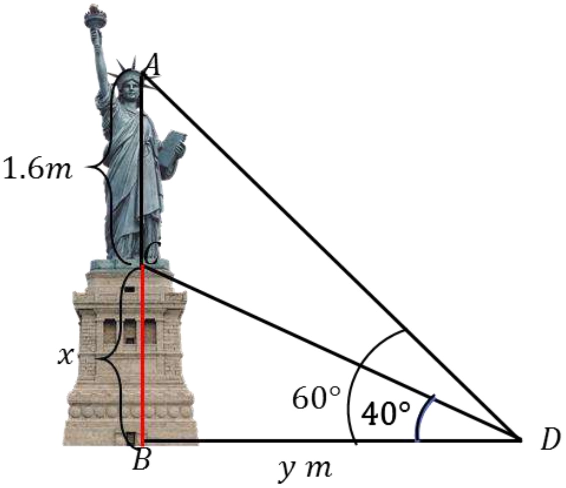
4. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal.

($\tan 40° = 0.8391, \sqrt{3} = 1.732$)

$AC =$ height of statue $= 1.6\ m$

$BC =$ height of Pedestal $= x\ m$

$BD =$ distance between the point D and pedestal $= y\ m$



In ΔBCD, $\tan\theta = \dfrac{opp}{adj}$

$$\tan 40^o = \frac{BC}{BD} \implies 0.8391 = \frac{x}{y}$$

$$y = \frac{x}{0.8391} \ldots (1)$$

In ΔBAD, $\angle BDA = 60^o$

$$\tan\theta = \frac{opp}{adj} \implies \tan 60^o = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{AC + BC}{y} \implies \sqrt{3} = \frac{x + 1.6}{y}$$

$$y = \frac{x + 1.6}{\sqrt{3}} \ldots (2)$$

From (1) & (2) $\dfrac{x}{0.8391} = \dfrac{x + 1.6}{\sqrt{3}}$

$$\sqrt{3}x = 0.8391(x + 1.6)$$

$$\sqrt{3}x = 0.8391x + (1.6 \times 0.8391) \implies \sqrt{3}x = 0.8391x + 1.343$$

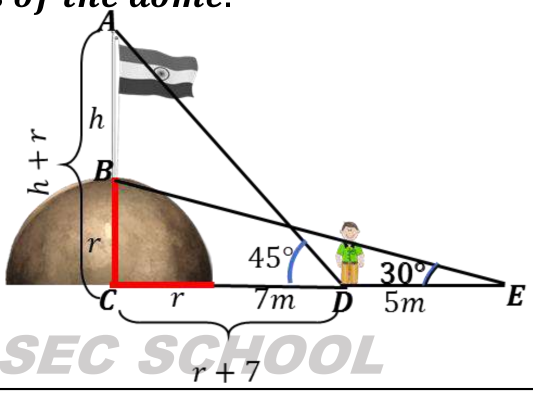
$$\sqrt{3}x - 0.8391x = 1.343 \implies \left(\sqrt{3} - 0.8391\right)x = 1.343$$

$$x = \frac{1.343}{\sqrt{3} - 0.8391} = \frac{1.343}{1.732 - 0.8391} = \frac{1.343}{0.8929}$$

$$x = 1.504m$$

$\therefore$ Height of pedestal, $x = 1.5m$

5. A flag pole 'h' metres is on the top of the hemispherical dome of radius 'r' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle 45° and moving 5 m away from the dome and seeing the bottom of the pole at an angle 30°. Find (i) the height of the pole (ii) radius of the dome.



In ΔACD, $\tan\theta = \dfrac{opp}{adj}$

$$\tan 45^o = \frac{AC}{CD} \implies 1 = \frac{h + r}{r + 7}$$

$$r + 7 = h + r \implies h = 7$$

$\therefore$ Height of the pole $= 7m$

$$In\ \Delta BCE,\ \tan\theta = \frac{opp}{adj} \Rightarrow \tan 30^o = \frac{BC}{CE}$$

$$\frac{1}{\sqrt{3}} = \frac{r}{r+7+5} \Rightarrow \frac{1}{\sqrt{3}} = \frac{r}{r+12}$$

$$\sqrt{3}\,r = r + 12 \Rightarrow \sqrt{3}\,r - r = 12 \Rightarrow r(\sqrt{3}-1) = 12$$

$$r = \frac{12}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{12(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2}$$

$$r = \frac{12(\sqrt{3}+1)}{3-1} = \frac{\overset{6}{\cancel{12}}(\sqrt{3}+1)}{\cancel{2}} = 6(\sqrt{3}+1)$$

$$= 6\,(1.732+1) = 6 \times 2.732$$

$$r = 16.39\ m$$

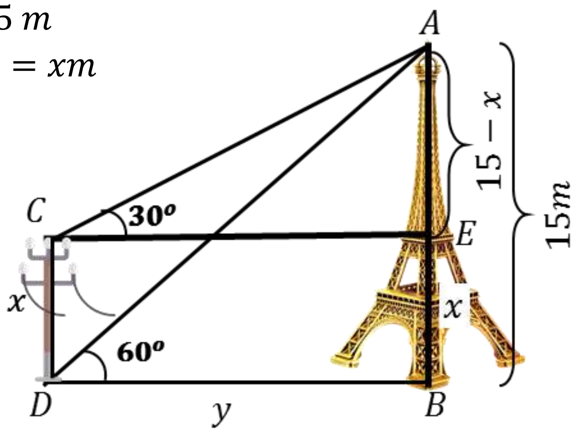
$$\therefore Radius\ of\ dome = 16.39\ m$$

6. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

Let, $AB = Height\ of\ the\ tower = 15\ m$

$CD = Height\ of\ the\ pole = BE = xm$

$AE = 15 - x$ and $BD = EC = y$



$$In\ \Delta\ ADB,\ tan\,\theta = \frac{opp}{adj}$$

$$\tan 60^o = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{15}{y}$$

$$y = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\overset{5}{\cancel{15}}\sqrt{3}}{\cancel{3}}$$

$$y = 5\sqrt{3} \quad \ldots(1)$$

$$In\ \Delta\ ACE,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 30^o = \frac{AE}{EC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{15-x}{y}$$

$$y = \sqrt{3}\,(15-x) \Rightarrow 5\sqrt{3} = \sqrt{3}\,(15-x) \qquad [\because y = 5\sqrt{3}]$$

$$5 = 15 - x \Rightarrow x = 15 - 5 \Rightarrow x = 10$$

$$\therefore Height\ of\ the\ pole = 10\ m$$

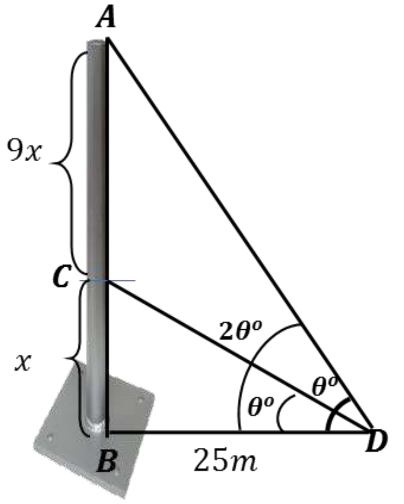
7. A vertical pole fixed to the ground is divided in the ratio 1 : 9 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a place on the ground, 25 m away from the base of the pole, what is the height of the pole

Let, $AB = height\ of\ the\ pole$

C is a point on AB which divides it in the ratio $1:9$ such that

$AC = 9x$ and $BC = x$ ($\because$ lower part is shorter than upper part)

$BD = 25\ m =$ Distance between pole of observation



In ΔBCD, $\tan\theta = \dfrac{opp}{adj}$

$$\tan\theta^o = \frac{BC}{BD} \Longrightarrow \tan\theta^o = \frac{x}{25}$$

In ΔABD, $\tan\theta = \dfrac{opp}{adj}$

$$\tan 2\theta^o = \frac{AB}{BD} \Longrightarrow \tan 2\theta^o = \frac{10x}{25}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{10x}{25} \Longrightarrow \frac{2\left(\frac{x}{25}\right)}{1-\left(\frac{x}{25}\right)^2} = \frac{2x}{5}$$

$$where\ \tan\theta^o = \frac{x}{25}$$

$$\frac{\frac{2x}{25}}{1-\frac{x^2}{25^2}} = \frac{2x}{5} \Longrightarrow \frac{\frac{2x}{\cancel{25}}}{\frac{25^2-x^2}{\cancel{25^2}\ 25}} = \frac{2x}{5}$$

$$\frac{\cancel{2x}}{\frac{25^2-x^2}{25}} = \frac{\cancel{2x}}{5} \Longrightarrow \frac{1}{\frac{25^2-x^2}{25}} = \frac{1}{5} \Longrightarrow \frac{25}{25^2-x^2} = \frac{1}{5}$$

$$25^2 - x^2 = 125 \Longrightarrow x^2 = 25^2 - 125$$

$$x^2 = 625 - 125 \Longrightarrow x^2 = 500$$

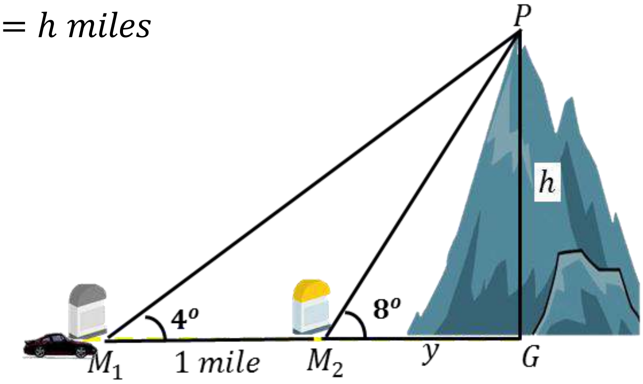
$$x^2 = 100 \times 5 \Longrightarrow x = 10\sqrt{5}$$

$\therefore$ Height of the pole $= 10x$

$$= 10 \times 10\sqrt{5} = 100\sqrt{5}m$$

8. A traveller approaches a mountain on highway. He measures the angle of elevation to the peak at each milestone. At two consecutive milestones the angles measured are 4° and 8°. What is the height of the peak if the distance between consecutive milestones is 1 mile.

($\tan 4^\circ = 0.0699, \tan 8^\circ = 0.1405$)



Let, $PG = height\ of\ the\ mountain = h\ miles$

$M_1, M_2 = Mile\ stones$

$GM_2 = y\ miles$

$Distance\ between\ M_1M_2 = 1\ mile$

$In\ \Delta PGM_2 \quad \tan\theta = \frac{opp}{adj}$

$$\tan 8^o = \frac{PG}{GM_2} \Rightarrow \tan 8^o = \frac{h}{y}$$

$$y = \frac{h}{\tan 8^o} \quad \dots (1)$$

$In\ \Delta PGM_1\ \tan\theta = \frac{opp}{adj}$

$$\tan 4^o = \frac{PK}{M_1M_2 + GM_2} \Rightarrow \tan 4^o = \frac{h}{y+1}$$

$$y + 1 = \frac{h}{\tan 4^o} \Rightarrow y = \frac{h}{\tan 4^o} - 1 \quad \dots\dots\dots (2)$$

$\therefore From\ (1)\ and\ (2)$

$$\frac{h}{\tan 8^o} = \frac{h}{\tan 4^o} - 1 \Rightarrow \frac{h}{\tan 4^o} - \frac{h}{\tan 8^o} = 1$$

$$h\left(\frac{1}{\tan 4^o} - \frac{1}{\tan 8^o}\right) = 1 \Rightarrow h\left(\frac{\tan 8^o - \tan 4^o}{\tan 8^o . \tan 4^o}\right) = 1$$

$$h = \frac{1}{\left(\frac{\tan 8^o - \tan 4^o}{\tan 8^o . \tan 4^o}\right)} \Rightarrow h = \frac{\tan 8^o \tan 4^o}{\tan 8^o - \tan 4^o}$$

$$h = \frac{0.14 \times 0.07}{0.14 - 0.07} \Rightarrow h = \frac{0.0098}{0.07}$$

$$h = 0.14\ miles$$

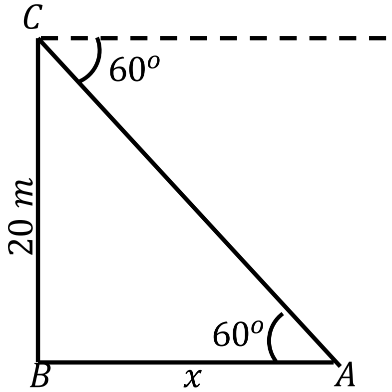
EXERCISE 6.3

Example 6.26. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball.
($\sqrt{3} = 1.732$)

Let, AB = distance between the foot of the tower and the ball = x meters

BC = height of the tower = $20\ m$

$\angle CAB = 60^o$



In $\Delta\, ABC$, $\tan\theta = \frac{opp}{adj}$

$$\tan 60^o = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\times\sqrt{3}}{3} = \frac{20\times 1.732}{3}$$

$$\boxed{x = 11.54}$$

∴ Distance between the foot of the tower and the ball = $11.54\ m$

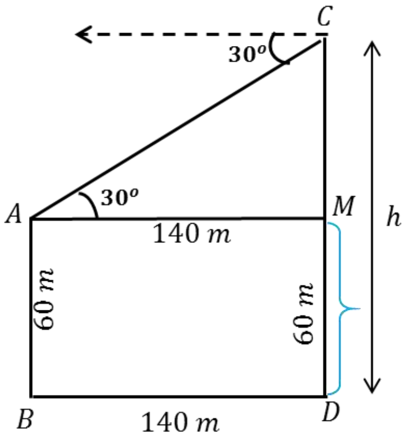
Example 6.27. The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60 m, find the height of the second building.
($\sqrt{3} = 1.732$)

Let, CD = height of the second building = h meter

AB = height of the first building = $60\ m = MD$

BD = distance between the two buidings = $140\ m = AM$

$\angle CAM = 30^o$



In $\Delta\, AMC$, $\tan\theta = \frac{opp}{adj}$

$$\tan 30^o = \frac{CM}{AM} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CM}{140}$$

$$CM = \frac{140}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow CM = \frac{140\times\sqrt{3}}{3}$$

$$CM = \frac{140\times 1.732}{3}$$

$$\boxed{CM = 80.78\ m}$$

∴ Height of the second building = CD

$$h = CM + MD = 80.78 + 60 = 140.78$$

∴ Height of the second building = $140.78\ m$

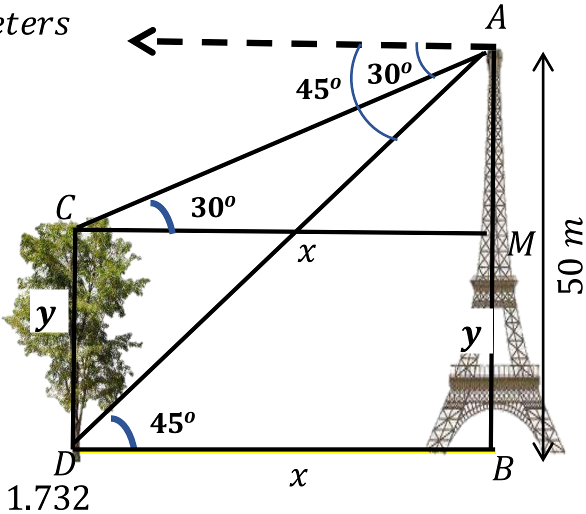
Example 6.28. From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. ($\sqrt{3} = 1.732$)

Let, CD = *Height of the tree* = y *meters*

BD = *Distance between the tree and the tower* = x *meters*

AB = *Height of the tower* = 50 *meters*

$\angle ACM = 30^o, \angle ADB = 45^o$



$$In\ \Delta ABD, \tan\theta = \frac{opp}{adj}$$

$$\tan 45^o = \frac{AB}{BD} \Rightarrow 1 = \frac{50}{x} \Rightarrow x = 50$$

$$In\ \Delta AMC,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 30^o = \frac{AM}{CM} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AM}{50}$$

$$AM = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow AM = \frac{50\sqrt{3}}{3} = \frac{50 \times 1.732}{3}$$

$$\boxed{AM = 28.85\ m}$$

$\therefore$ *Height of the tree* $= CD = MB = AB - AM$

$= 50 - 28.85$

$CD = 21.15\ m$

Example 6.29. As observed from the top of a 60 m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. ($\tan 28° = 0.5317$)

Let, AB = *Distance between the two ships*

CD = *Height of the lighthouse* = 60 *meters*

$\angle DAC = 28^o,\ \angle DBC = 45^o$

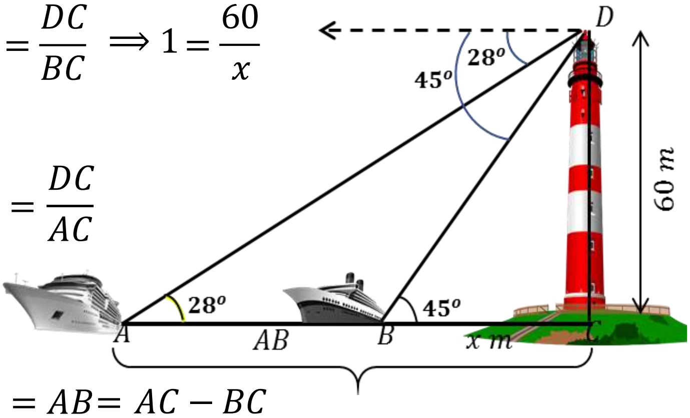
$$In\ \Delta DCB, \tan\theta = \frac{opp}{adj} \Rightarrow \tan 45^o = \frac{DC}{BC} \Rightarrow 1 = \frac{60}{x}$$

$$\boxed{x = 60\ m}$$

$$In\ \Delta DCA, \tan\theta = \frac{opp}{adj} \Rightarrow \tan 28^o = \frac{DC}{AC}$$

$$0.5137 = \frac{60}{AC} \Rightarrow AC = \frac{60}{0.5317}$$

$AC = 112.85$



$\therefore$ *Distance between the two ships* $= AB = AC - BC$

$= 112.85 - 60 = 52.85\ m$

Example 6.30. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat $\left(\text{in} \frac{km}{hr}\right)$, assuming that it is sailing in still water? ($\sqrt{3} = 1.732$)

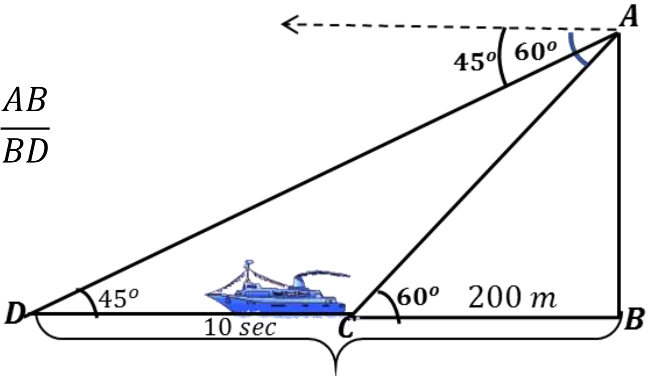
Let, $AB =$ height of the tower

$BC =$ distance between the tower and the boat $= 200m$

$\angle ACB = 60^o, \angle ADB = 45^o$

In ΔABC, $\tan\theta = \frac{opp}{adj} \Rightarrow \tan 60^o = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{200}$

$AB = 200\sqrt{3}\ \ldots(1)$



In ΔABD, $\tan\theta = \frac{opp}{adj} \Rightarrow \tan 45^o = \frac{AB}{BD}$

$1 = \frac{200\sqrt{3}}{BD} \Rightarrow BD = 200\sqrt{3}\ \ldots(2)$

$CD = BD - BC$

$= 200\sqrt{3} - 200 = 200(\sqrt{3} - 1)$

$= 200(1.732 - 1) = 200 \times 0.732$

$\boxed{CD = 146.4}$

Time taken to covered distance CD in 10 seconds

Distance of covered $CD = 146.4\ m$

$\therefore$ Speed of the boat $= \frac{Distance}{Time} = \frac{146.4}{10} = 14.64\ m/s$

$$= 14.64 \times \frac{\frac{1}{1000}}{\frac{1}{60 \times 60}} = 14.64 \times \frac{\frac{1}{1000}}{\frac{1}{3600}} = 14.64 \times \frac{3600}{1000} km/hr$$

$$= 14.64 \times \frac{360\cancel{0}}{100\cancel{0}} km/hr = 14.64 \times 3.6 km/hr$$

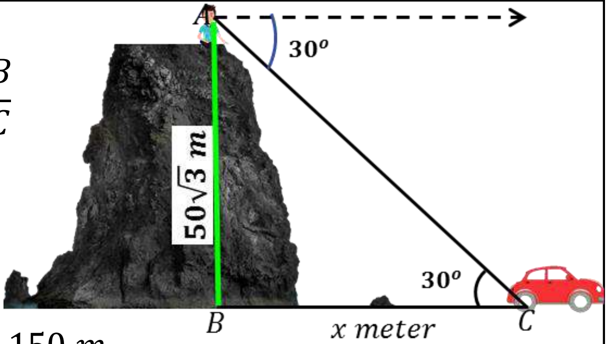
$$= 52.704\ km/hr$$

1. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

$BC =$ Distance of the car from the rock

$= x$ meters

$AB =$ Height of the rock $= 50\sqrt{3}$ meters



$$\angle ACB = 30^o$$

$$In\ \Delta\ ABC, \tan\theta = \frac{opp}{adj} \implies \tan 30^o = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x} \implies x = 50\overset{3}{(\sqrt{3}\times\sqrt{3}\ )}$$

$$x = 50 \times 3 \implies x = 150$$

$\therefore$ *Distance of the car from the rock* $= 150\ m$

2. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120m, find the height of the first building.

Let, Height of the 1^{st} *building* $= CD = EB = x$ *meter*

Height of the 2^{nd} *building* $= AB = 120$ *meter*

Distance between the two buildings $= BD = EC = 70\ m$

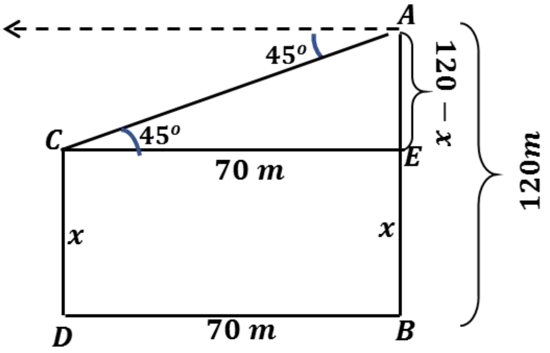
$$ACE = 45^o$$

$$In\ \Delta AEC,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 45^o = \frac{AE}{EC} = \frac{AB - EB}{EC}$$

$$1 = \frac{120 - x}{70} \implies 70 = 120 - x$$

$$x = 120 - 70 \implies x = 50$$



$\therefore$ *Height of the* 1^{st} *building* $= 50$ *meters*

3. From the top of the tower 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. $(\sqrt{3} = 1.732)$

Let, Height of the tower $= AB = 50$ *meters*

Height of the lamp post $= DC = EB = y$ *meters*

Distance of the lamp post from the tower $= BC = DE = x$ *meters*

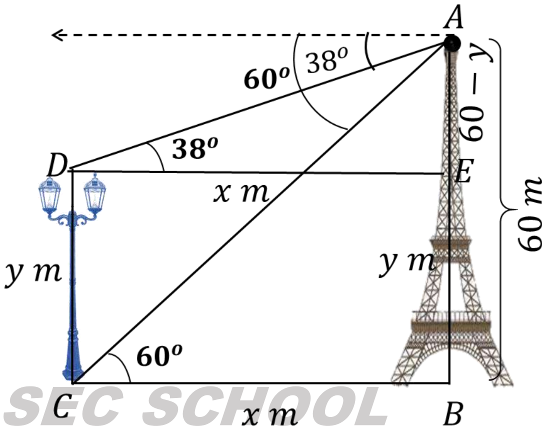
$$\angle ACB = 60^o \text{ and } \angle ADE = 38^o$$

$$In\ \Delta ACB,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 60^o = \frac{AB}{BC} \implies \sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{\overset{20}{\cancel{60}}\sqrt{3}}{\cancel{3}} \implies x = 20\sqrt{3} \ldots (1)$$



$In\ \Delta ADE,\ \tan\theta = \frac{opp}{adj}$

$$\tan 38^o = \frac{AE}{ED} \Longrightarrow \tan 38^o = \frac{60-y}{x}$$

$$0.7813 = \frac{60-y}{x} \Longrightarrow x = \frac{60-y}{0.7813} \ldots (2)$$

$From\ (1)\ \&\ (2)$

$$20\sqrt{3} = \frac{60-y}{0.7813} \Longrightarrow 20\sqrt{3} \times 0.7813 = 60 - y$$

$$20 \times 1.732 \times 0.7813 = 60 - y \Longrightarrow 34.64 \times 0.7813 = 60 - y$$

$$27.06 = 60 - y \Longrightarrow y = 60 - 27.06$$

$$y = 32.94\ m$$

$\therefore Height\ of\ the\ lamp\ post = 32.94\ m$

4. *An aeroplane at an altitude of* 1800 m *find that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are* 60° *and* 30° *respectively. Find the distance between the two boats.* **($\sqrt{3} = 1.732$)**

Let, Distance between the boat : $CD = x$

Height of the plane from the ground $= AB = 1800\ m$

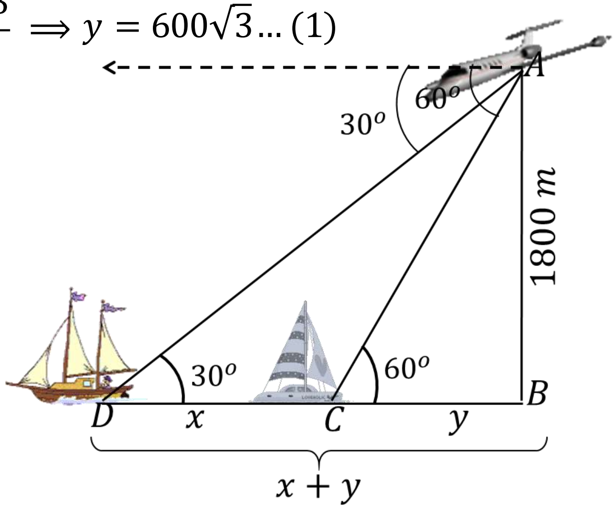
Distance between $BC = y$

$\angle ACB = 60^o\ and\ \angle ADB = 30^o$

$In\ \Delta ACB,\ \tan\theta = \frac{opp}{adj}$

$$\tan 60^o = \frac{AB}{BC} \Longrightarrow \sqrt{3} = \frac{1800}{y} \Longrightarrow y = \frac{1800}{\sqrt{3}}$$

$$y = \frac{1800}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Longrightarrow y = \frac{\overset{600}{\cancel{1800}}\sqrt{3}}{\cancel{3}} \Longrightarrow y = 600\sqrt{3} \ldots (1)$$



$In\ \Delta ADB,\ \ \tan\theta = \frac{opp}{adj}$

$$tan\ 30^o = \frac{AB}{BD} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{1800}{y+x}$$

$$y + x = 1800 \times \sqrt{3} \ldots (2)$$

$From\ (1)\ \&\ (2)$

$$600\sqrt{3} + x = 1800\sqrt{3}$$

$$x = 1800\sqrt{3} - 600\sqrt{3}$$

$$x = 1200\sqrt{3} = 1200 \times 1.732$$

$x = 2078.4$

$\therefore$ *Distance between two boats* $= 2078.4\ m$

5. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m

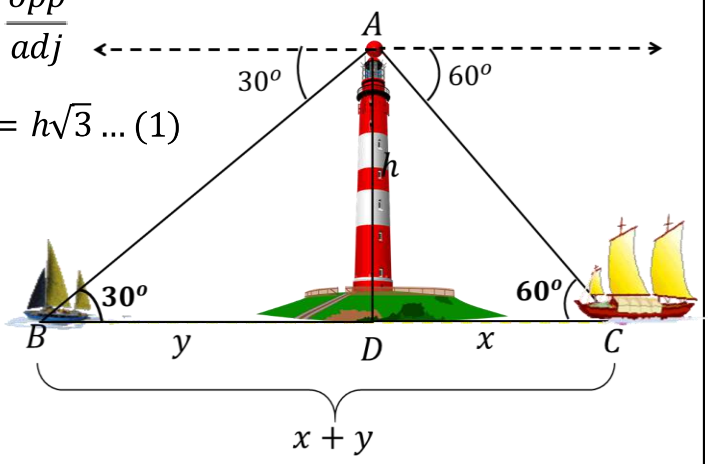
Height of the light house $= AD = h$ *meters*

Distance between B and D is $BD = y\ m$

Distance between D and C is $DC = x\ m$

To prove:

Distance between the ships is $x + y = \frac{4h}{\sqrt{3}}$



In ΔABD, $\angle ABD = 30^o$, $\tan\theta = \frac{opp}{adj}$

$$\tan 30^o = \frac{AD}{BD} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Longrightarrow x = h\sqrt{3} \ldots (1)$$

In ΔACD, $\angle ACH = 60^o$,

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60^o = \frac{AD}{CD} \Longrightarrow \sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}} \ \ldots (2)$$

Adding (1) & (2)

$$x + y = h\sqrt{3} + \frac{h}{\sqrt{3}} = \frac{h\overset{3}{(\sqrt{3}\times\sqrt{3})} + h}{\sqrt{3}}$$

$$= \frac{h\times 3 + h}{\sqrt{3}} = \frac{3h + h}{\sqrt{3}}$$

$$x + y = \frac{4h}{\sqrt{3}}$$

$\therefore$ *Distance between the two ships*, $x + y = \frac{4h}{\sqrt{3}}$

6. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.

Let, $C \rightarrow$ position of the fountain and
$P \rightarrow$ position of the lift after 2 minutes

Height of the lift $= AB = 90\ ft$
Distance between A and $P = AP = x\ ft$

Distance between the fountain & the lift is
$$BC = 30\sqrt{3} ft$$
Distance between P and B is $PB = 90 - x\ ft$
Time taken to reach from position A to $P = 2$ min.

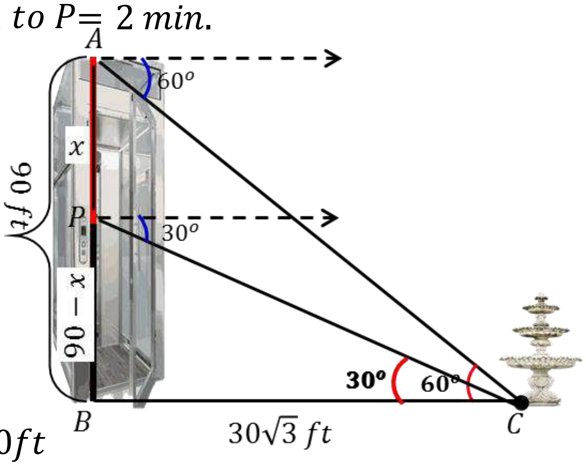
In ΔPBC, $\angle PCB = 30^o$, $\tan\theta = \dfrac{opp}{adj}$

$$\tan 30^o = \frac{PB}{BC}$$

$$\frac{1}{\cancel{\sqrt{3}}} = \frac{90 - x}{30\cancel{\sqrt{3}}} \Longrightarrow 1 = \frac{90 - x}{30}$$

$$30 = 90 - x \Longrightarrow x = 90 - 30$$

$$\boxed{x = 60}$$



$\therefore$ Distance between A and P is $AP = 60ft$

$$\therefore \text{Speed of the lift} = \frac{\text{Distance moved}}{\text{time taken}} = \frac{\cancel{60}}{\cancel{2}}$$

$\therefore$ Speed of the lift $= 30\ ft/min$

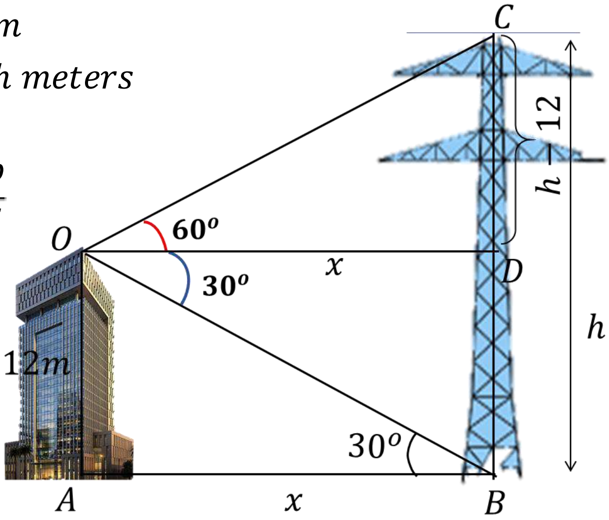
EXERCISE 6.4

Example 6.31 From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Height of the bulding is $OA = 12\ m$

$BC =$ Height of the cable tower $= h$ meters

$DC = h - 12$ and $AB = OD = x$



In ΔODC, $\angle COD = 60^o$, $\tan\theta = \frac{opp}{adj}$

$$\tan 60^o = \frac{CD}{OD} \Rightarrow \sqrt{3} = \frac{h-12}{x}$$

$$OD = \frac{h-12}{\sqrt{3}} \ \dots (1)$$

In ΔOAB, $\angle OBA = 30^o$, $\tan\theta = \frac{opp}{adj}$

$$\tan 30^o = \frac{OA}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{12}{x}$$

$$x = 12\sqrt{3} \ \dots (2)$$

From (1) and (2)

$$\frac{h-12}{\sqrt{3}} = 12\sqrt{3} \Rightarrow h - 12 = 12\sqrt{3} \times \sqrt{3}$$

$$h - 12 = 12 \times 3 \Rightarrow h - 12 = 36$$

$$h = 36 + 12 \Rightarrow h = 48$$

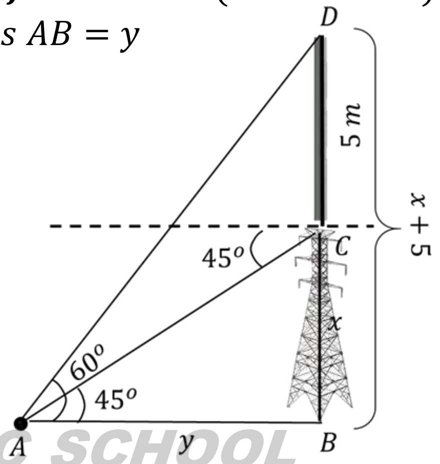
∴ Required height of the cable tower $= 48\ m$

Example 6.31 A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression to the point 'A' from the top of the tower is 45°. Find the height of the tower. $(\sqrt{3} = 1.732)$

Distance between the point and the toweris $AB = y$

Height of the tower is $BC = x$

Height of the pole is $DC = 5m$



In ΔABC, $\angle CAB = 45^o$, $\tan\theta = \frac{opp}{adj}$

$$\tan 45^o = \frac{BC}{AB} \Rightarrow 1 = \frac{x}{y}$$

$$x = y \ \dots (1)$$

In ΔABD, $\angle DAB = 60^o$, $\tan\theta = \frac{opp}{adj}$

$$tan\,60^o = \frac{BD}{AB} \Rightarrow \sqrt{3} = \frac{x+5}{y}$$

$$\sqrt{3}\,y = x + 5 \ldots (2)$$

$$sub\ y = x\ in\ (2)\ \sqrt{3}x = x + 5$$

$$\sqrt{3}x - x = 5 \Rightarrow \left(\sqrt{3}-1\right)x = 5 \Rightarrow x = \frac{5}{\sqrt{3}-1}$$

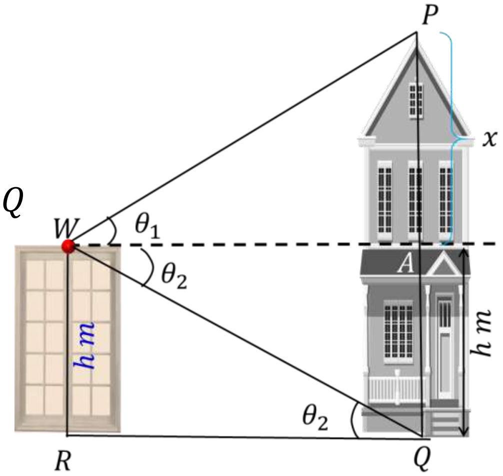
$$x = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5(\sqrt{3}+1)}{\left(\sqrt{3}\right)^2 - 1^2} = \frac{5(\sqrt{3}+1)}{3-1}$$

$$x = \frac{5(\sqrt{3}+1)}{2} = \frac{5(1.732+1)}{2} = \frac{5(\overset{1.366}{\cancel{2.732}})}{\cancel{2}}$$

$$\boxed{x = 6.83\ m}$$

$$\therefore Height\ of the\ tower = 6.83\ m$$

Example 6.32 From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ_1 and θ_2 respectively. Show that the height of the opposite house is $h\left(1 + \frac{\cot\theta_2}{\cot\theta_1}\right)$



$PQ = Height\ of\ the\ opposite\ side\ house$

$PA = x\ meters$

$WR = Height\ of\ the\ window = h\ meters = AQ$

$$In\ \Delta PAW, \angle PWA = \theta_1, \tan\theta = \frac{opp}{adj}$$

$$tan\,\theta_1 = \frac{AP}{AW} \Rightarrow tan\,\theta_1 = \frac{x}{AW}$$

$$AW = \frac{x}{tan\,\theta_1} \Rightarrow AW = x \times \frac{1}{tan\,\theta_1}$$

$$AW = x\cot\theta_1 \ldots (1)$$

$$In\ \Delta QAW, \angle WQR = \theta_2,\ \ tan\,\theta = \frac{opp}{adj}$$

$$tan\,\theta_2 = \frac{AQ}{AW} \Rightarrow tan\,\theta_2 = \frac{h}{AW} \Rightarrow AW = \frac{h}{tan\,\theta_2}$$

$$AW = h \times \frac{1}{tan\,\theta_1} \Rightarrow AW = x\,cot\,\theta_2 \ \ \ldots (2)$$

$$From\ (1)\ and\ (2)$$

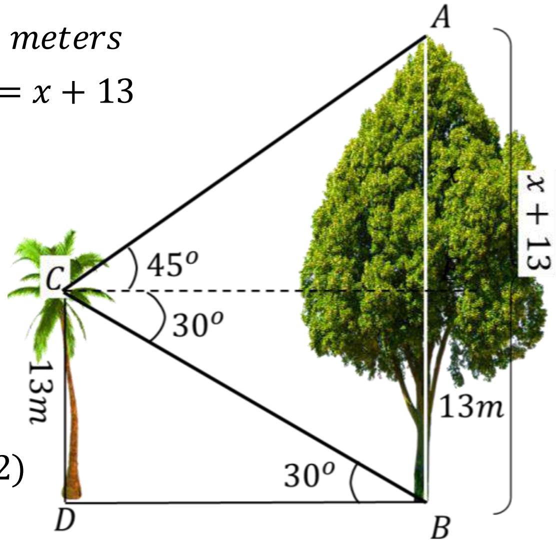
$$x\,cot\,\theta_1 = h\,cot\,\theta_2 \Rightarrow x = h\,\frac{cot\,\theta_2}{cot\,\theta_1}$$

$\therefore$ $Height\ of\ the\ opposite\ house = PQ$

$$PQ = x + h = h\,\frac{\cot\theta_2}{\cot\theta_1} + h$$

$$\therefore\ Height\ of\ the\ opposite\ house = h\left(1 + \frac{\cot\theta_2}{\cot\theta_1}\right)$$

1. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree.



$Given: CD = 13\ meters\ and\ AB = x + 13\ meters$

$Let, Height\ of\ the\ second\ tree\ is\ AE = x + 13$

$In\ \Delta CEB,\ \angle BCE = 30^o,\ \tan\theta = \dfrac{opp}{adj}$

$$\tan 30^o = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{13}{CE}$$

$CE = 13\sqrt{3} \ldots (1)$

$In\ \Delta ACE, \angle ACE = 45^o,\ \tan\theta = \dfrac{opp}{adj}$

$$\tan 45^o = \frac{AE}{CE} \Rightarrow 1 = \frac{x}{CE} \Rightarrow CE = x \ldots (2)$$

$From\ (1)\ and\ (2) \Rightarrow x = 13\sqrt{3}$

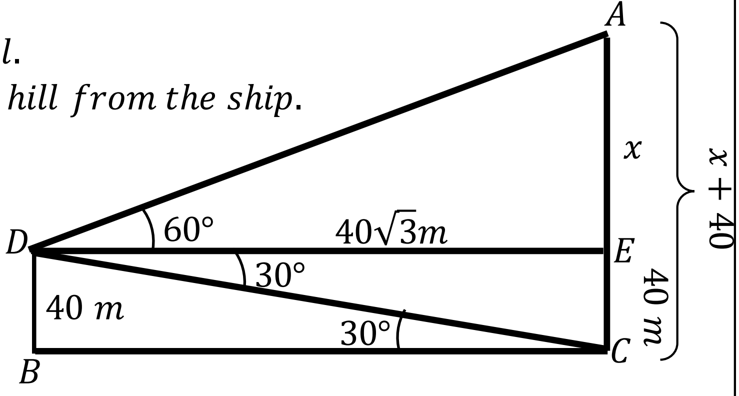
$\therefore\ Height\ of\ 2^{nd}\ tree = x + 13 = 13\sqrt{3} + 13$

$Height\ of\ 2^{nd}\ tree = 13(\sqrt{3} + 1) = 13(1.732 + 1)$

$= 13 \times 2.732 = 35.516$

$\therefore\ Height\ of\ 2^{nd}\ tree = 35.52\ meters$

2. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. $(\sqrt{3} = 1.732)$



$Let\ AC\ be\ the\ height\ of\ the\ hill.$

$Let\ DE\ be\ the\ distance\ of\ the\ hill\ from\ the\ ship.$

$In\ \Delta DEC, \angle EDC = 30°$

$$\tan\theta = \frac{opp}{adj} \Rightarrow \tan 30° = \frac{EC}{DE}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{DE} \Rightarrow DE = 40\sqrt{3}m$$

$In\ \Delta AED, \angle ADE = 60^\circ$

$$tan\,\theta = \frac{opp}{adj} \implies tan\,60^\circ = \frac{AE}{DE}$$

$$\sqrt{3} = \frac{x}{40\sqrt{3}} \implies x = 40\sqrt{3} \times \sqrt{3}m \implies x = 40 \times 3\,m$$

$$\boxed{x = 120\ m}$$

$\therefore Height\ of\ the\ hill = x + 40 = 120 + 40 = 160\ m$

$\therefore Distance\ between\ ship\ and\ hill = 40\sqrt{3}$

$$= 40 \times 1.732$$
$$= 69.28\,m$$

3. *If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\frac{h(tan\,\theta_1 + tan\,\theta_2)}{tan\,\theta_2 + tan\,\theta_1}$*

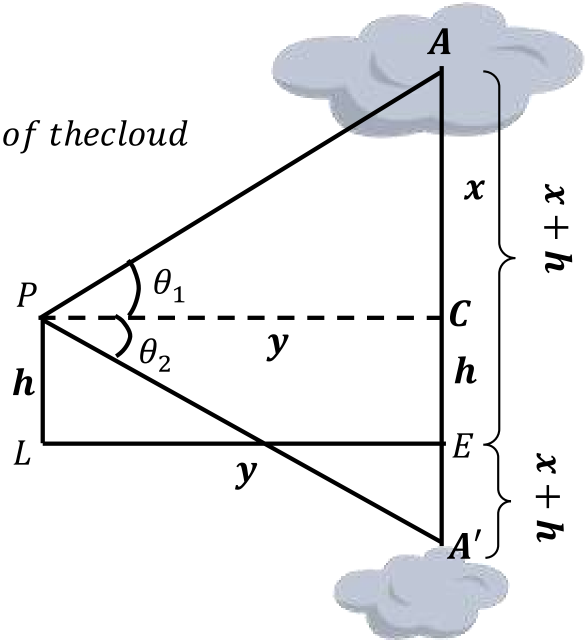
$Let,\ LE = Surface\ of\ the\ lake$

$P = Point\ of\ observation$

$A = Positions\ of\ cloud, A' = reflection\ of\ thecloud$

$AE = A'E$

$PL = CE = h\ meters$



$$To\ Prove: AE = \frac{h(tan\,\theta_1 + tan\,\theta_2)}{tan\,\theta_2 + tan\,\theta_1}$$

$$In\ \Delta APC,\ tan\,\theta = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{AC}{PC} \implies tan\,\theta_1 = \frac{x}{y}$$

$$y = \frac{x}{tan\,\theta_1} \dots (2)$$

$$In\ \Delta A'PC,\ tan\,\theta = \frac{opp}{adj}$$

$$\tan\theta_2 = \frac{CA'}{PC} = \frac{CE\ + EA'}{PC}$$

$$\tan\theta_2 = \frac{h + x + h}{y} \implies y = \frac{x + 2h}{tan\,\theta_2} \dots (2)$$

$From\ (1)\ and\ (2)$

$$\frac{x}{tan\,\theta_1} = \frac{x + 2h}{tan\,\theta_2} \implies x\,tan\,\theta_2 = (x + 2h)\,tan\,\theta_1$$

$$x\ \tan\theta_2 = x\tan\theta_1 + 2h\tan\theta_1$$

$$x\ \tan\theta_2 - x\ \tan\theta_1 = 2h\tan\theta_1 \Longrightarrow x\,(\tan\theta_2 - \tan\theta_1) = 2h\tan\theta_1$$

$$x = \frac{2h\,tan\,\theta_1}{tan\,\theta_2 - tan\,\theta_1}$$

$\therefore$ *Distance between the cloud and the ground is* $AE = h + x$

$$AE = h + \frac{2h\,tan\,\theta_1}{tan\,\theta_2 - tan\,\theta_1} = h\left[1 + \frac{2\tan\theta_1}{\tan\theta_2 - \tan\theta_1}\right]$$

$$= h\left[\frac{tan\,\theta_2 - tan\,\theta_1 + 2\,tan\,\theta_1}{tan\,\theta_2 - tan\,\theta_1}\right]$$

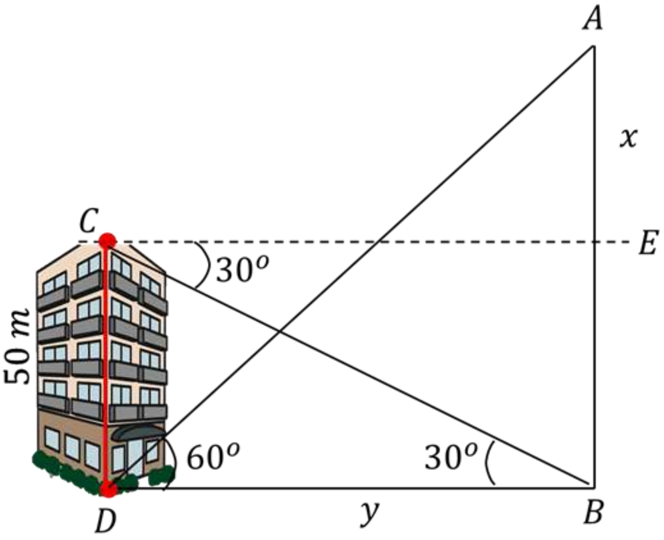
$$AE = h\left[\frac{tan\,\theta_2 + tan\,\theta_1}{tan\,\theta_2 - tan\,\theta_1}\right] \therefore \textit{Hence proved}$$

4. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.

Let, $CD = \textit{height of the apartment} = 50\ m = EB$

$AB = \textit{height of the cellphone tower} = (x + 50)\ \textit{meters}$

$BD = \textit{distance between tower and apartment} = y\ \textit{meters}\ = EC$



$$In\ \Delta CDB,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 30^o = \frac{CD}{BD} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{50}{y}$$

$$y = 50\sqrt{3} \ldots (1)$$

$$In\ \Delta ADB,\ \tan\theta = \frac{opp}{adj}$$

$$\tan 60^o = \frac{AB}{BD} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{x + 50}{y}$$

$$y = \frac{x + 50}{\sqrt{3}} \ldots (2)$$

From (1) *and* (2)

$$50\sqrt{3} = \frac{x + 50}{\sqrt{3}} \Longrightarrow 50\sqrt{3}\ \times\sqrt{3} = x + 50$$

$$50 \times 3 = x + 50 \Longrightarrow 150 = x + 50$$

$$x = 150 - 50 \Rightarrow x = 100$$

$\therefore$ *Height of the cell phone tower* $= x + 50$

$= 100 + 50$

$= 150\, m$

$\therefore 150\ m > 120\ m$

$\therefore$ *The tower does not meet the radiation norms*

8. *The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find (i) The height of the lamp post.*

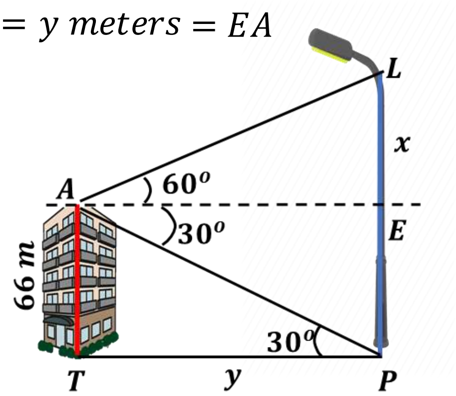
(*ii*) *The difference between height of the lamp post and the apartment.*

(*iii*) *The distance between the lamp post and the apartment.*

$AT =$ *height of the apartment* $= 66\ m = EP$

$LP =$ *height of the lamp post* $= (x + 66)$ *meters*

$PT =$ *distance between post and apartment* $= y$ *meters* $= EA$



In $\Delta APT,\ \tan\theta = \dfrac{opp}{adj}$

$$\tan 30^o = \frac{AT}{PT} \Rightarrow \frac{1}{\sqrt{3}} = \frac{66}{y}$$

$$y = 66\sqrt{3} \ldots (1)$$

In $\Delta LAE,\ \tan\theta = \dfrac{opp}{adj}$

$$\tan 60^o = \frac{LE}{EA} \Rightarrow \sqrt{3} = \frac{x}{y}$$

$$y = \frac{x}{\sqrt{3}} \ldots (2)$$

From (1) *and* (2)

$$66\sqrt{3} = \frac{x}{\sqrt{3}} \Rightarrow x = 66\overset{3}{(\sqrt{3}\times\sqrt{3})}$$

$$x = 66 \times 3 \Rightarrow x = 198$$

i) *Height of the lamp post* $= x + 66 = 198 + 66$

$= 294$ *meters*

ii) *Difference between height of the lamp post & apartment* $= 264 - 66$

$= 198$ *meters*

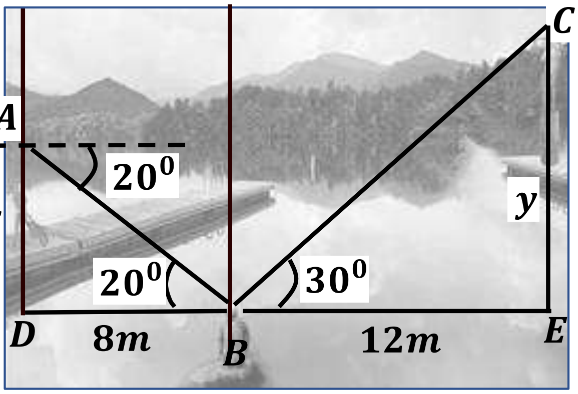
iii) *Distance between the lamp post & apartment*: $y = 66\sqrt{3} = 66(1.732)$

$= 114.312\ m$

9. $Three\ villagers\ A, B\ and\ C\ can\ see\ each\ other\ across\ a\ valley.$ $The\ horizontal\ distance\ between\ A\ and\ B\ is\ 8\ km\ and\ the\ horizontal\ distance\ between\ B\ and\ C\ is\ 12km.$ $The\ angle\ of\ depression\ of\ B\ from\ A\ is\ 20°\ and\ the\ angle\ of\ elevation\ of\ C\ from\ B\ is\ 30°.$ $Calculate:$ $(i)the\ vertical\ height\ between\ A\ and\ B$ $(ii)the\ vertical\ height\ between\ B\ and\ C.$ $given: (tan\ 20^o = 0.3640, (\sqrt{3} = 1.732)$

$A, B, C \rightarrow positions\ of\ three\ villagers$

$To\ find:\ i)AD \quad ii)CE$



$In\ \Delta ABD,\ \tan\theta = \frac{opp}{adj}$

$$\tan 20^o = \frac{AD}{DE} \Rightarrow 0.3640 = \frac{x}{8}$$

$$x = 8 \times 0.3640$$

$$x = 2.912\ km \ \dots (1)$$

$In\ \Delta CBE,\quad \tan\theta = \frac{opp}{adj}$

$$\tan 30^o = \frac{CE}{BE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{12}$$

$$y = \frac{12}{\sqrt{3}} = \frac{4\times 3}{\sqrt{3}}$$

$$= \frac{4\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$$

$$= 4 \times 1.732$$

$$y = 6.928$$

EXERCISE : 7.1

Example* 7.1 *A cylindrical drum has a height of* 20 *cm and base radius of* 14 *cm.Find its curved surface area and the total surface area.

$Given : r = 14cm \; , h = 20cm$

C.S.A of cylinder = $2\pi rh$ sq.units

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 20 = 2 \times 22 \times 2 \times 20$$

$$= 88 \times 20 = 1760 \; cm^2$$

$T.S.A \; of \; cylinder = 2\pi r(h + r)$ sq.units

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times (20 + 14)$$

$$= 2 \times 22 \times 2 \times 34 = 88 \times 34$$

$$= 2992 cm^2$$

Example* 7.2 *The curved surface area of a right circular cylinder of height* 14 *cm is* 88*cm*^{2}.*Find the diameter of the cylinder.

Given : $h = 14 \; cm$

C.S.A of cylinder = $88 \; cm^2$

$$2\pi rh = 88$$

$$2 \times \frac{22}{\cancel{7}} \times r \times \overset{2}{\cancel{14}} = 88$$

$$r = \overset{\overset{\cancel{22}}{\cancel{44}}}{\cancel{88}} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{22}} \times \frac{1}{\cancel{2}}$$

$$r = 1 \; cm$$

Diameter = $2 \times r = 2 \times 1$

Diameter = 2cm

***Example* 7.3 *A garden roller whose length is* 3 *m long and whose diameter is*2.8 *m is rolled to level a garden.How much area will it cover in* 8 *revolutions*?**

$Given : \; r = \frac{2.8}{2} m \; and \; h = 3 \; m$

$$r = 1.4 \; m$$

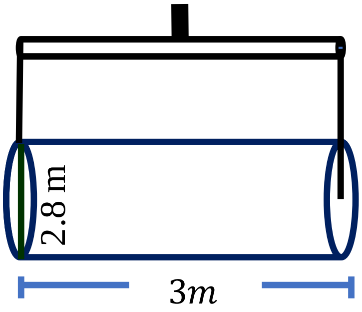
Area covered by the roller in one revolution

= Curved surface area of the garden roller

$= 2\pi rh$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{0.2}{\cancel{1.4}} \times 3 = 2 \times 22 \times 0.6 = 44 \times 0.6$$

$$= 26.4\, m^2$$



Area covered by the roller in one revolution

$$= 26.4\, m^2$$

Area covered by the roller in 8 revolutions

$$= 8 \times 26.4 = 211.2\, m^2$$

Example 7.4 If one litre of paint covers 10 $\mathbf{m^2}$, how many litres of paintis required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m, internal radius is 6 m and height is 25 m.

Given:

$r = 6\ m, \quad h = 25\ m$

$Thickness = 2$

$$\boxed{Thickness = R - r}$$

$R - r = 2$

$R - 6 = 2$

$R = 2 + 6 \implies R = 8\ m$

$$C.S.A\ of\ hollow\ cylinder = 2\pi(R + r)h\ Sq.units$$

$$= 2 \times \frac{22}{7}(8 + 6)(25)$$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 25 = 44 \times 50 = 2200m^2$$

Area covered by one litre of paint $= 10m^2$

Number of litres required to paint the tunnel $= \dfrac{2200}{10} = 220$

∴ 220 *litres of paint is needed to paint the tunnel.*

Example 7.6 If the total surface area of a cone of radius 7cm is 704 $\mathbf{cm^2}$, then find its slant height.

Given :*radius*: $r = 7\ cm$

$T.S.A\ of\ a\ cone = 704cm^2$

$\pi r(l + r) = 704$

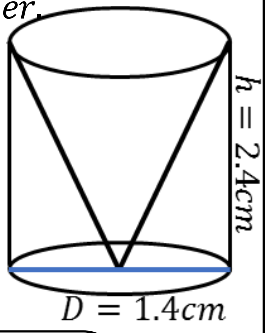
$$\frac{22}{7} \times 7 \times (l + 7) = 704 \implies l + 7 = \frac{704}{22}$$

$$l + 7 = 32 \implies l = 32 - 7$$

$$l = 25\ cm$$

∴ *slant height of the cone is* 25 *cm.*

Example 7.7 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out Find the total surface area of the remaining solid.



Let h and r be the height and radius of the cone and cylinder.

Let l be the slant height of the cone.

Given : $h = 2.4\ cm$ and $d = 1.4\ cm$

$r = 0.7\ cm$

Total surface area of the remaining solid

$= C.S.A.$ of the cylinder $+ C.S.A.$ of the cone $+$area of the bottom

$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$

$$= \frac{22}{7} \times 0.7 \times [(2 \times 2.4) + 2.5 + 0.7]$$

$$= \frac{22}{\cancel{7}} \times \overset{0.1}{\cancel{0.7}} \times [4.8 + 2.5 + 0.7] = 2.2 \times 8$$

$l = \sqrt{r^2 + h^2}$

$= \sqrt{(0.7)^2 + (2.4)^2}$

$= \sqrt{0.49 + 5.76} = \sqrt{6.25}$

$l = 2.5$ cm

$\therefore$ *Total surface area of the remaining solid is* $17.6\ cm^2$

Example 7.8 Find the diameter of a sphere whose surface area is $154m^2$

Let r be the radius of the sphere.

surface area of sphere $= 154\ m^2$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \overset{\overset{7}{\cancel{77}}}{\cancel{154}} \times \frac{1}{\underset{2}{\cancel{4}}} \times \frac{7}{\underset{2}{\cancel{22}}} \implies r^2 = \frac{49}{4}$$

$$r = \sqrt{\frac{49}{4}} \implies r = \frac{7}{2}$$

$$Diameter = 2 \times r = 2 \times \frac{7}{2}$$

$\therefore$ *diameter is* $7\ m$

Example 7.9 The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Let r_1 *and* r_2 *be the radii of the spherical balloons.*

$r_1 = 12cm$ and $r_2 = 16cm$

Ratio of C.S.A. of balloons

C.S.A of two sphere are in the ratio $= C.S.A_1 : C.S.A_2$

$$= \cancel{4\pi} r_1^2 : \cancel{4\pi} r_2^2$$

$$= r_1^2 : r_2^2$$

$$= (12)^2 : (16)^2 = \overset{3}{\cancel{12}} \times \overset{3}{\cancel{12}} : \overset{4}{\cancel{16}} \times \overset{4}{\cancel{16}}$$

$\therefore$ *Ratio of C.S.A. of balloons is* $9:16$.

Example 7.10 *If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area*?

Let r be the radius of the hemisphere.

$$Base\ area = 1386\ sq.m$$

$$\pi r^2 = 1386$$

$$T.S.A\ of\ a\ hemisphere = 3\pi r^2 sq.m$$

$$= 3 \times 1386 = 4158$$

$\therefore$ *T.S.A. of the hemispherical solid is* $4158\ m^2$.

Example 7.11 *The internal and external radii of a hollow hemispherical shellare 3 m and 5 m respectively. Find the T.S.A. and C.S.A. of the shell.*

Let the internal and external radii of the hemispherical shell be r and R

Given : $R = 5\ m, r = 3\ m$

$$C.S.A.\ of\ the\ shell = 2\pi(R^2 + r^2)\ sq.units$$

$$= 2 \times \frac{22}{7} \times (5^2 + 3^2) = 2 \times \frac{22}{7} \times (25 + 9)$$

$$= 2 \times \frac{22}{7} \times 34 = 44 \times 4.85 = 213.71 m^2$$

$$T.S.A.\ of\ the\ shell = \pi(3R^2 + r^2)\ sq.units$$

$$= \frac{22}{7} \times (3 \times 5^2 + 3^2) = \frac{22}{7} \times (3 \times 25 + 9)$$

$$= \frac{22}{7} \times (75 + 9) = \frac{22}{\cancel{7}} \times \overset{12}{\cancel{84}} = 264$$

$\therefore C.S.A = 213.71 m^2$, $T.S.A. = 264\ m^2$

Example 7.12 *A sphere, a cylinder and a cone are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.*

Required Ratio $=$ *C.S.A. of the sphere* : *C.S.A. of the cylinder* : *C.S.A. of the cone* $= 4\pi r^2 : 2\pi rh : \pi rl$

$$h = r$$

$$= 4\pi r^2 : 2\pi r \times r : \pi r \times \sqrt{2} r$$

$$= 4\cancel{\pi r^2} : 2\cancel{\pi r^2} : \sqrt{2}\cancel{\pi r^2}$$

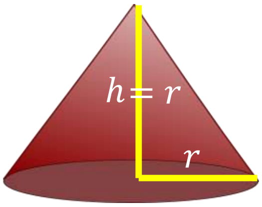
$$= 4 : 2 : \sqrt{2} \quad = 4\sqrt{2} : 2\sqrt{2} : \sqrt{2} \times \sqrt{2}$$

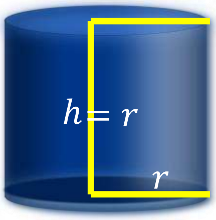
$$= \underset{\div 2}{4\sqrt{2} : 2\sqrt{2} : 2} \quad = 2\sqrt{2} : \sqrt{2} : 1$$

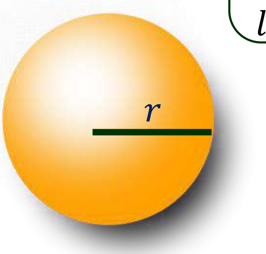
$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{r^2 + r^2} \quad = \sqrt{2r^2}$$

$$l = \sqrt{2}r \text{ units}$$

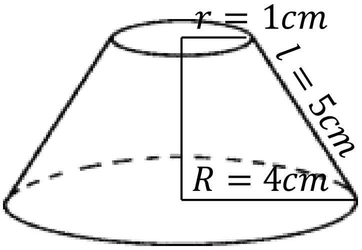






Example 7.13 The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Let l, R and r be the slant height, top radius and bottom radius of the frustum.



Given : $l = 5$ cm, $R = 4$ cm, $r = 1$ cm

C.S.A. of the frustum $= \pi(R + r)l$ sq.units

$$= \frac{22}{7} \times (4 + 1) \times 5$$

$$= \frac{22}{7} \times 5 \times 5 \ = \frac{550}{7}$$

$$= 78.57$$

$\therefore$ C.S.A. $= 78.57\ cm^2$

Example 7.14 An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

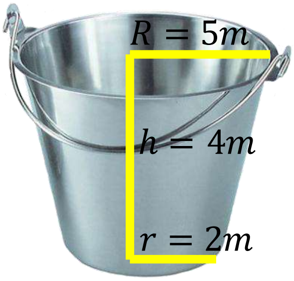
Let h, l, R and r be the height, slant height, top radius and bottom radius of the frustum.

Given : Diameter of the top $= 10$ m; $R = \frac{10}{2} = 5$ m

Diameter of the bottom $= 4$ m

$$r = \frac{4}{2} = 2\ m$$

Height $h = 4$ m



$$l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{4^2 + (5 - 2)^2} = \sqrt{16 + 3^2}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5\ m$$

C.S.A. of the frustum $= \pi(R + r)l$ sq.units $= \frac{22}{7} \times (5 + 2) \times 5$

$$= \frac{22}{7} \times 7 \times 5 = 110\ m^2$$

$$T.S.A = \pi(R + r)l + \pi R^2 + \pi r^2\ sq.units$$

$$= \pi[(R + r)l + R^2 + r^2]$$

$$= \frac{22}{7}[(5 + 2)5 + 5^2 + 2^2] = \frac{22}{7}[7 \times 5 + 25 + 4]$$

$$= \frac{22}{7}[35 + 25 + 4] = \frac{22}{7}[64]$$

$$= \frac{1408}{7} = 201.14$$

$$\therefore C.S.A. = 110\ m^2\ and\ T.S.A = 201.14\ m^2$$

1. *The radius and height of a cylinder are in the ratio* 5: 7. *If its curved surface area is* 5500 *sq.cm , find its radius and height*

Given: $r : h = 5 : 7$

Let $r = 5k,\ \ h = 7k$

C.S.A of cylinder $= 5500$

$$2\pi rh = 5500$$

$$2 \times \frac{22}{\cancel{7}} \times 5k \times \cancel{7}k = 5500$$

$$2 \times 22 \times 5 \times k^2 = 5500 \implies k^2 = \overset{\cancel{100}\ \ \cancel{50}\ \ 25}{\overset{\cancel{1100}}{\cancel{5500}}} \times \frac{1}{\cancel{2}} \times \frac{1}{\underset{\cancel{2}}{\cancel{22}}} \times \frac{1}{\cancel{5}}$$

$$k^2 = 25 \implies k = \sqrt{25}$$

$$\boxed{k = 5}$$

$$\therefore Radius : r = 5k$$

$$r = 5(5) = 25\ cm$$

$$height : h = 7k = 7(5)$$

$$\text{h} = 35\ cm$$

2. *A solid iron cylinder has total surface area of* 1848 *sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.*

T.S.A of cylinder $= 1848\ m^2$

$$\boxed{\text{C.S.A} = \frac{5}{6}(\text{T.S.A})}$$

$$\text{C.S.A} = \frac{5}{\cancel{6}}(\overset{308}{\cancel{1848}}) = 1540\ m^2$$

$$\text{C.S.A} = 1540\ m^2\ \ i.e\ 2\pi rh = 1540\ m^2$$

T.S.A of cylinder $= 1848\ m^2 \implies 2\pi r(h + r) = 1848$

$$2\pi rh + 2\pi r^2 = 1848 \implies 1540 + 2\pi r^2 = 1848$$

$$2\pi r^2 = 1848 - 1540 \implies 2\pi r^2 = 308$$

$$2 \times \frac{22}{7} \times r^2 = 308 \implies r^2 = \overset{\overset{\overset{7}{\cancel{14}}}{\cancel{154}}}{\cancel{308}} \times \frac{1}{\cancel{2}} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}}$$

$$r^2 = 7 \times 7 \implies r = \sqrt{7 \times 7}$$

$$\boxed{r = 7\ m}$$

To find height

$$Sub\ r = 7\ in\ 2\pi rh = 1540$$

$$2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times h = 1540$$

$$h = \overset{\overset{\overset{35}{\cancel{70}}}{\cancel{770}}}{\cancel{1540}} \times \frac{1}{\cancel{2}} \times \frac{1}{\underset{\cancel{2}}{\cancel{22}}} = 35\ m$$

$\therefore$ The cylinder $of\ r = 7\ m\ \ and\ h = 35\ m$

3. *The external radius and the length of a hollow wooden* log *are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.*

Given:

$$R = 16\ cm, h = 13\ cm$$

$$Thickness = 4$$

$$\boxed{\text{Thickness} = R - r}$$

$$R - r = 4$$

$$R - 4 = r$$

$$r = 16 - 4 \implies r = 12\ cm$$

$$T.S.A\ of\ hollow\ cylinder = 2\pi(R + r)(h + R - r)\ Sq, units$$

$$= 2 \times \frac{22}{7}(16 + 12)(13 + 16 - 12)$$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{4}{\cancel{28}} \times 17 = 44 \times 4 \times 17 = 176 \times 17$$

$$= 2992\ cm^2$$

4. *A right angled triangle PQR where* $\angle Q = 90°$ *is rotated about QR and PQ. If* $QR = 16\ cm$ *and* $PR = 20\ cm$*, compare the curved surface areas of the right circular cones so formed by the triangle.*

A right angled triangle PQR is rotated about QR

Pythagoras theorem

$r^2 = l^2 - h^2 \Longrightarrow \; r = \sqrt{l^2 - h^2}$

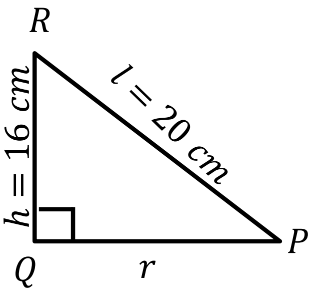
$r = \sqrt{20^2 - 16^2} \Longrightarrow \; r = \sqrt{400 - 256}$

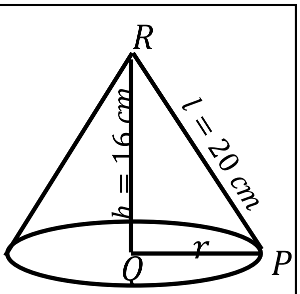
$r = \sqrt{144} \;\; \Longrightarrow \;\; r = 12\ cm$

C.S.A of a cone $= \pi r l$

$= \pi \times 12 \times 20$

$= 240\pi\ cm^2$



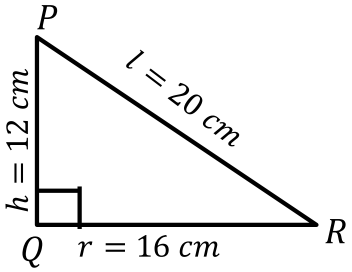


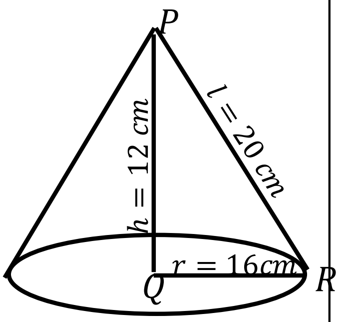
A right angled triangle PQR is rotated about PQ

C.S.A of cone $= \pi r l$

$= \pi \times 16 \times 20$

$= 320\pi\ cm^2$





C.S.A of cone rotated about PQ is greater than rotated about QR

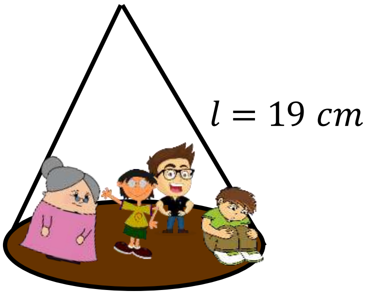
5. 4 *persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.*

Given : $l = 19\ cm$

Area required for a person $= 22\ cm^2$

Base area of a tent covered by 4 person $= 4 \times 22$

$= 88\ cm^2$



$\pi r^2 = 88 \;\; \Longrightarrow \;\; \frac{22}{7} \times r^2 = 88$

$r^2 = \overset{4}{\cancel{88}} \times \frac{7}{\cancel{22}} \;\; \Longrightarrow \;\; r^2 = 28 \;\; \Longrightarrow \boxed{r = \sqrt{28}}$

To find height of the tent

$h = \sqrt{l^2 - r^2} = \sqrt{19^2 - \left(\sqrt{28}\right)^2} = \sqrt{361 - 28}$

$h = \sqrt{333}$

$h = 18.2\ cm$

	18.2
1	333 1
28	233 224
362	900 724

6. *A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm*

Given : $r = 5\ cm, h = 12\ cm$

$$l^2 = h^2 + r^2 \implies l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{12^2 + 5^2} \implies l = \sqrt{144 + 25}$$

$$l = \sqrt{169} \implies l = 13\ cm$$

$$n \times C.S.A\ of\ cone = Area\ of\ a\ paper$$

$$n = \frac{Area\ of\ a\ paper}{C.S.A\ of\ cone}$$

$$n = \frac{5720}{\pi r l}$$

$$n = \frac{\overset{1144}{\cancel{5720}}}{\frac{22}{7} \times \cancel{5} \times 13} = \frac{1144}{\frac{22}{7} \times 13}$$

$$= \overset{\overset{\overset{4}{\cancel{52}}}{\cancel{104}}}{\cancel{1144}} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}} \times \frac{1}{\cancel{13}} = 28$$

$$no.of\ caps = 28$$

7. The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Let r_1 and r_2, h_1 and h_2 are the radius and heights of the two cones.

$$Given\ r_1 : r_2 = 1:3 \implies r_1 = k\ and\ r_2 = 3k$$

∴ Height of each cone = 3 times the radius of smaller cone

$$h_1 = 3k\ and\ h_2 = 3k$$

$$l_1 = \sqrt{h_1^2 + r_1^2} = \sqrt{(3k)^2 + k^2}$$

$$l_1 = \sqrt{9k^2 + k^2} \implies l_1 = \sqrt{10k^2}$$

$$\boxed{l_1 = \sqrt{10}k}$$

$$l_2 = \sqrt{h_2^2 + r_2^2} = \sqrt{(3k)^2 + (3k)^2}$$

$$l_1 = \sqrt{9k^2 + 9k^2} \implies l_1 = \sqrt{18k^2}$$

$$l_1 = \sqrt{2 \times 9 \times k^2} = 3\sqrt{2}k$$

$$\boxed{l_1 = 3\sqrt{2}k}$$

$$\begin{aligned}
\text{C.S.}A_1 : \text{C.S.}A_2 &= \cancel{\pi} r_1 l_1 : \cancel{\pi} r_2 l_2 \\
&= r_1 l_1 : r_2 l_2 \\
&= \cancel{k} \times \sqrt{10}\cancel{k} : 3\cancel{k} \times 3\sqrt{2}\cancel{k} \\
&= \sqrt{10} : 9\sqrt{2} \quad = \sqrt{5} \times \cancel{\sqrt{2}} : 9\cancel{\sqrt{2}} \\
&= \sqrt{5} : 9
\end{aligned}$$

8. *The radius of a sphere increases by 25%. Find the percentage increase in its surface area.*

$$Original\ Radius = r\,(100\%)$$

$$Original\ surface\ area = 4\pi r^2\ (100\%)$$

$$New\ Radius = 125\%\ of\ r\ = \frac{\overset{5}{\cancel{\overset{25}{\cancel{125}}}}}{\underset{4}{\cancel{\underset{20}{\cancel{100}}}}} \times\ r\ = \frac{5}{4} \times\ r$$

$$New\ Radius = \frac{5r}{4}$$

$$New\ surface\ area\ = 4\pi\left(\frac{5r}{4}\right)^2 = \cancel{4}\pi\left(\frac{25r^2}{\underset{4}{\cancel{16}}}\right) = \frac{25\pi r^2}{4}$$

$$New\ surface\ area = \frac{25\pi r^2}{4}$$

$$Increased\ Area = New\ Area - Original\ area$$

$$= \frac{25\pi r^2}{4} - 4\pi r^2\ = \frac{25\pi r^2 - 16\pi r^2}{4}$$

$$Increased\ Area\ = \frac{9\pi r^2}{4}$$

$$Increased\ percentage = \frac{Increased\ Area}{Original\ Area} \times 100$$

$$= \frac{\frac{9\cancel{\pi r^2}}{4}}{4\cancel{\pi r^2}} \times 100\ = \frac{\frac{9}{4}}{4} \times 100 = \frac{9}{4} \times \frac{1}{4} \times 100$$

$$= \frac{9}{16} \times 100 = 0.5625 \times 100$$

$$= 56.25\%$$

9. *The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at Rs. 0.14 per cm^2.*

$Internal\ Diameter = 20\ cm$

$$r = \frac{20}{2} = 10\ cm$$

$External\ Diameter = 28\ cm$

$$R = \frac{28}{2} = 14\ cm$$

$T.S.A\ of\ a\ hollow\ hemispherical\ vessel = \pi(3R^2 + r^2)$

$$= \frac{22}{7}(3 \times 14^2 + 10^2) = \frac{22}{7}(3 \times 196 + 100)$$

$$= \frac{22}{7}(588 + 100) = \frac{22}{7} \times 688 = \frac{15136}{7} = 2162.28\ cm^2$$

$Cost\ of\ painting = 2162.28 \times 0.14 = Rs.302.72$

10. *The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is Rs.2.*

Given $R = 12\ cm, r = 6cm, h = 8\ cm$

$$l = \sqrt{h^2 + (R-r)^2} = \sqrt{8^2 + (12-6)^2}$$

$$= \sqrt{64 + 6^2} = \sqrt{64 + 36} = \sqrt{100}$$

$$l = 10\ cm$$

C.S.A of frustum $= \pi(R + r)l$

$$= \frac{22}{7}(12 + 6)(10) = \frac{22}{7} \times 18 \times 10$$

$$= \frac{3960}{7} = 565.7cm^2$$

Area for the top $= \pi r^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7}$

$$= 113.14\ cm^2$$

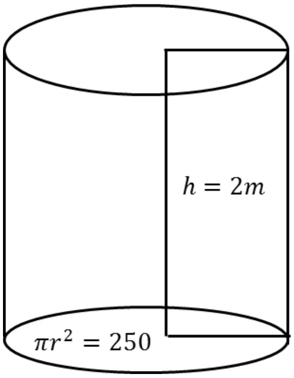
Total area $= C.S.A + Top\ area = 565.71 + 113.14 = 678.85\ cm^2$

$The\ cost\ of\ painting = 678.85 \times 2$

$$= Rs.1357.72$$

$EXERCISE: 7.2$

Example 7.15 Find the volume of a cylinder whose height is 2 m and whose base area is 250 m^2.



$Let\ r\ and\ h\ be\ the\ radius\ and\ height\ of\ the\ cylinder$

$Given : Height : h = 2\ m, Base\ area = 250\ m^2$

$$\pi r^2 = 250\ m^2$$

$$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h\ cu.units}$$

$$= \pi r^2 \times h$$

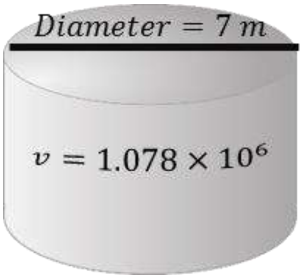
$$= 250 \times 2 = 500\ m^3$$

$\therefore Volume\ of\ a\ cylinder = 500\ m^3$

Example 7.16 The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7 m, find its height.

$Let\ r\ and\ h\ be\ the\ radius\ and\ height\ of\ the\ cylinder.$

$Given\ : Diameter = 7\ m \Rightarrow radius = \frac{7}{2}\ m$ $\boxed{1l = \frac{1}{1000}\ m^3}$



$Volume\ of\ the\ tank = 1.078 \times 10^6$

$$= 1.078 \times 10^3 \times 10^3 = 1078 \times 10^3$$

$$= 1078000\ litre = \frac{1078000}{1000}\ m^3$$

$Volume\ of\ the\ cylindrical\ tank = 1078\ m^3$

$$\pi r^2 h = 1078$$

$$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = 1078 \Rightarrow h = \overset{14}{\overset{\cancel{154}}{\cancel{1078}}} \times \frac{\cancel{7}}{\underset{\cancel{11}}{\cancel{22}}} \times \frac{\cancel{2}}{\cancel{7}} \times \frac{2}{\cancel{7}}$$

$$h = 28m$$

$\therefore height\ of\ the\ tank\ is\ 28\ m$

Example 7.17 Find the volume of the iron used to make a hollow cylinder height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

$Let\ r, R\ and\ h\ be\ the\ internal\ radius, external\ radius\ and\ height\ of\ the\ hollow\ cylinder\ respectively.$

$Given\ : r = 21cm, R = 28\ cm, h = 9\ cm$

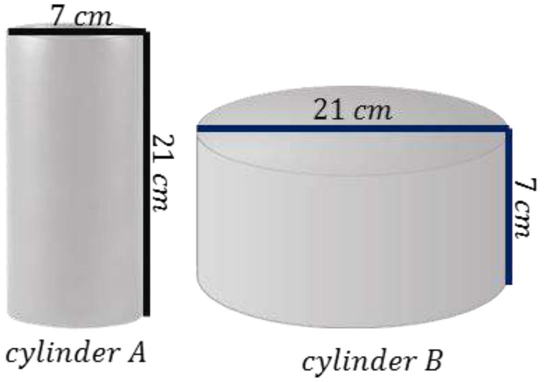
$$\boxed{Volume\ of\ hollow\ cylinder = \pi(R^2 - r^2)h\ cu.units}$$

$$= \frac{22}{7}(28^2 - 21^2) \times 9 = \frac{22}{7}(784 - 441) \times 9 = \frac{22}{\cancel{7}} \times \overset{49}{\cancel{343}} \times 9$$

$$= 22 \times 49 \times 9$$

$\therefore Volume\ of\ iron\ used = 9702\ cm^3$

Example 7.18 For the cylinders A and B (i) find out the cylinder whose volume is greater. (ii) verify whether the cylinder with greater volume has greater total surface area. (iii) find the ratios of the volumes of the cylinders A and B.



(i) *cylinder A*

$Diameter\ = 7cm \quad r\ = \frac{7}{2}cm$

$Height\ : h = 21\ cm$

$Volume\ of\ cylinder\ A = \pi r^2 h\ cu.units$

$$= \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2} \times 21$$

$Volume\ of\ cylinder\ A = 11 \times 3.5 \times 21$

$= 231 \times 3.5$

$Volume\ of\ cylinder\ A = 808.5\ cm^3$

cylinder B

$Diameter\ = 21cm \quad r\ = \frac{21}{2}cm$

$Height\ : h = 7\ cm$

$Volume\ of\ cylinder\ B = \pi r^2 h\ cu.units$

$$Volume\ of\ cylinder\ B = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{2}} \times \frac{21}{\cancel{2}} \times \overset{3.5}{\cancel{7}}$$

$= 11 \times 3 \times 21 \times 3.5 = 33 \times 21 \times 3.5 = 693 \times 3.5$

$Volume\ of\ cylinder\ B = 2425.5\ \text{cm}^3$

$\therefore$ *volume of cylinder B is greater than volume of cylinder A.*

(ii) $T.S.A\ of\ cylinder\ A = 2\pi r(h + r)\ sq.units$

$r\ = \frac{7}{2}cm,\ h = 21\ cm$

$$= \cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \left(21 + \frac{7}{2}\right) \quad = 22 \times (21 + 3.5)$$

$= 22 \times 24.5$

$T.S.A\ of\ cylinder\ A = 539\ cm^2$

$T.S.A\ of\ cylinder\ B = 2\pi r(h + r)\ sq.units$

$r\ = \frac{21}{2}cm,,h = 7cm$

$$= \cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{2}} \times \left(7 + \frac{21}{2}\right) \quad = 22 \times 3 \times (7 + 10.5)$$

$= 22 \times 3 \times 17.5 = 66 \times 17.5$

$T.S.A$ *of cylinder* $B = 1155\ cm^2$

Hence verified that cylinder B with greater volume has a greater surface area.

(iii) *Volume of cylinder A* : *Volume of cylinder B* $= 808.5 : 2425.5$

$$\frac{\textit{volume of cylinder A}}{\textit{volume of cylinder B}} = \frac{808.5}{2425.5} = \frac{\overset{\overset{1}{\cancel{1617}}}{\cancel{8085}}}{\underset{\underset{3}{\cancel{4851}}}{\cancel{24255}}} = \frac{1}{3}$$

Volume of cylinder A : *Volume of cylinder B* $= 1:3$

$\therefore$ *Ratio of the volumes of cylinders A and B is* $1 : 3$.

Example 7.19 The volume of a solid right circular cone is $\boldsymbol{11088\ cm^3}$***. If its height is 24 cm then find the radius of the cone.***

Let r and h be the radius and height of the cone.

Given : $h = 24cm$

Volume of the cone $= 11088\ cm^3$

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088 \implies r^2 = \overset{\overset{\overset{63}{\cancel{504}}}{\cancel{1008}}}{\cancel{11088}} \times \cancel{3} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}} \times \frac{1}{\underset{\cancel{8}}{\cancel{24}}}$$

$$r^2 = 63 \times 7 \implies r^2 = 9 \times 7 \times 7$$

$$r = \sqrt{9 \times 7 \times 7} \implies r = 3 \times 7$$

$\therefore$ *Radius of the cone* : $r = 21$ cm

Example 7.20 The ratio of the volumes of two cones is $\boldsymbol{2:3}$***. Find the ratio of their radii if the height of second cone is double the height of the first.***

Let r_1 *and* h_1 *be the radius and height of the cone* $- I$

let r_2 *and* h_2 *be the radius and height of the cone* $- II$.

Given: $h_2 = 2h_1$

Volume of cone $- I$: *Volume of cone* $- II = 2:3$

$$\frac{\textit{volume of the cone I}}{\textit{volume of the cone II}} = \frac{2}{3}$$

$$\frac{\cancel{\frac{1}{3}\pi} r_1^2 h_1}{\cancel{\frac{1}{3}\pi} r_2^2 h_2} = \frac{2}{3} \implies \frac{r_1^2}{r_2^2} \times \frac{h_1}{h_2} = \frac{2}{3} \implies \frac{r_1^2}{r_2^2} \times \frac{\cancel{h_1}}{2\cancel{h_1}} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} \times \frac{1}{2} = \frac{2}{3} \implies \frac{r_1^2}{r_2^2} = \frac{2}{3} \times 2 \implies \frac{r_1^2}{r_2^2} = \frac{4}{3}$$

$$\sqrt{\frac{r_1^2}{r_2^2}} = \sqrt{\frac{4}{3}} \Rightarrow \frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

$\therefore$ *Ratio of their radii* $r_1 : r_2 = 2 : \sqrt{3}$

Example 7.21 The volume of a solid hemisphere is 29106 cm^3. Another hemisphere whose volume is two – third of the above is curved out. Find the radius of the new hemisphere.

Let r be the radius of the hemisphere.

Given : Volume of hemisphere $= 29106\ cm^3$

$$\text{Volume of new hemisphere} = \frac{2}{3}\ (\text{volume of original hemisphere})$$

$$= \frac{2}{\cancel{3}} \times \overset{9702}{\cancel{29106}} = 2 \times 9702$$

volume of new hemisphere $= 19404 cm^3$

$$\frac{2}{3}\pi r^3 = 19404 \Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = 19404$$

$$r^3 = \overset{\overset{\cancel{882}\ 441}{\cancel{9702}}}{\cancel{19404}} \times \frac{3}{\cancel{2}} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}} \Rightarrow r^3 = 441 \times 21$$

$$r^3 = 441 \times 21 \Rightarrow r^3 = 21 \times 21 \times 21$$

$$r = \sqrt[3]{21 \times 21 \times 21} \Rightarrow r = 21\ cm$$

Example 7.22 Calculate the weight of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1mm, and whose density is 17.3 g/cm^3.

Let r and R be the inner and outer radii of the hollow sphere.

Given : inner diameter $d = 14\ cm \Rightarrow r = \frac{14}{2} = 7\ cm$

Thickness $= 1\ mm$

$$R - r = \frac{1}{10}\ cm \Rightarrow R - 7 = 0.1$$

$$R = 7 + 0.1 \Rightarrow R = 7.1\ cm$$

$$\boxed{\text{Volume of hollow sphere} = \frac{4}{3}\pi(R^3 - r^3)\ cu.cm}$$

$$= \frac{4}{3} \times \frac{22}{7}(7.1^3 - 7^3) = \frac{4}{3} \times \frac{22}{7}(357.91 - 343)$$

$$= \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{\overset{0.71}{\cancel{2.13}}}{\cancel{14.91}} = 4 \times 22 \times 0.71 = 62.48\ cm^3$$

Volume of hollow sphere $= 62.48\ cm^3$

$weight\ of\ brass\ in\ 1\ cm^3 = 17.3\ gm$

$$\boxed{Weight = volume \times density}$$

$Total\ weight = 62.48 \times 17.3 = 1080.90\ gm$

$\therefore Total\ weight\ is\ 1080.90\ grams.$

Example 7.23 If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

$Let\ h, r\ and\ R\ be\ the\ height, top\ and\ bottom\ radii\ of\ the\ frustum.$

$Given: h = 45cm, R = 28\ cm, r = 7\ cm$

$$\boxed{Volume\ of\ frustum = \frac{1}{3}\pi[R^2 + Rr + r^2]h\ cu.units}$$

$$= \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{7} \times [784 + 196 + 49] \times \overset{15}{\cancel{45}}$$

$$= \frac{22}{\cancel{7}} \times \overset{147}{\cancel{1029}} \times 15 = 22 \times 147 \times 15 = 48510$$

$\therefore volume\ of\ the\ frustum\ is\ 48510\ cm^3$

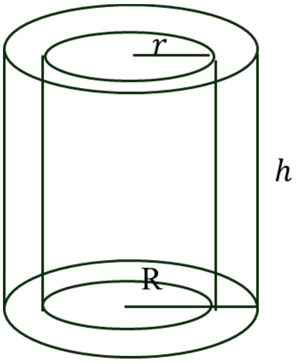
1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.

$Internal\ diameter = 10\ m$

$$r = \frac{10}{2} = 5\ m$$

$Inner\ height = 14\ m$

$$\boxed{\begin{aligned} Thickness &= R - r \\ 5 &= R - 5 \\ 5 + 5 &= R \\ R &= 10cm \end{aligned}}$$



$$\boxed{Volume\ of\ the\ cylinder = \pi r^2 h\ cubic\ units}$$

$$= \frac{22}{7} \times 5 \times 5 \times 14 = \frac{7700}{7}$$
$$= 1100\ m^3$$

$Volume\ of\ the\ hollow\ cylinder = 1100\ m^3$

$$\pi(R^2 - r^2)h = 1100$$

$$\frac{22}{7} \times (10^2 - 5^2)h = 1100 \implies \frac{22}{7} \times (100 - 25)h = 1100$$

$$\frac{22}{7} \times 75h = 1100 \implies h = 1100 \times \frac{7}{22} \times \frac{1}{75} = 4.66 = 4.67m$$

$The\ height\ of\ the \cong 4.7\ m\ embankment.$

2. *A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?*

Let r_1 *be the radius and* h_1 *be the level of water raised in the cylindrical glass*

let r_2 *and* h_2 *be the radius and height of the cylindrical metal.*

$$Given:\ r_1 = \frac{20}{2} = 10cm,\ h_1 = ?$$
$$r_2 = 5cm,\ h_2 = 4cm$$

The volume of the water raised in the cylindrical glass
= Volume of the cylindrical metal.

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$
$$\cancel{\pi} \times 10^2 \times h_1 = \cancel{\pi} \times 5^2 \times 4$$
$$100 \times h_1 = 25 \times 4$$
$$100 \times h_1 = 100 \Longrightarrow h_1 = \frac{\cancel{100}}{\cancel{100}} = 1\ cm$$

$\therefore$ *The height of the raised water in the glass* $= 1\ cm$

3. *If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.*

Given: *Circumference of the base of the cone* $= 484\ cm$, *Height* $= 105\ cm$

$$2\pi r = 484$$
$$2 \times \frac{22}{7} \times r = 484 \quad \Longrightarrow \quad r = \overset{\overset{\overset{11}{\cancel{121}}}{\cancel{242}}}{\cancel{484}} \times \frac{1}{\cancel{2}} \times \frac{7}{\underset{\cancel{11}}{\cancel{22}}} \quad \Longrightarrow r = 77\ cm$$

$$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h\ cubic\ units}$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{11}{\cancel{77}} \times 77 \times \overset{35}{\cancel{105}} = 22 \times 11 \times 77 \times 35$$
$$= 242 \times 2695 = 652190\ cm^3$$

4. *A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.*

$$\boxed{Volume\ \ of\ the\ cone = \frac{1}{3}\pi r^2 h\ \text{cu. units}}$$

$$Volume\ of\ the\ conical\ container = \frac{1}{3} \times \pi \times 10 \times 10 \times \overset{5}{\cancel{15}}$$

$$= 500\pi\ m^3 = 500 \times 3.14\ m^3 \quad = 1570\ m^3$$

$25\ m^3$ *of petrol released in* 1 *mins.*

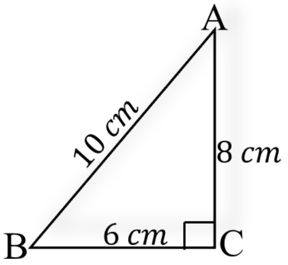
$$Time\ taken\ to\ emptied\ the\ container = \frac{\overset{314}{\cancel{1570}}}{\cancel{25}\ \cancel{5}} = 62.8\ mins.$$

$$\cong 63\ minutes.(approx.)$$

5. *A right angled triangle whose sides are* 6 *cm*, 8 *cm and* 10 *cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the solids so formed.*

When the triangle ABC is rotated about AC, the volume of the cone formed

$$\boxed{\text{Volume of cone} = \frac{1}{3}\pi r^2 h\ cu.units}$$

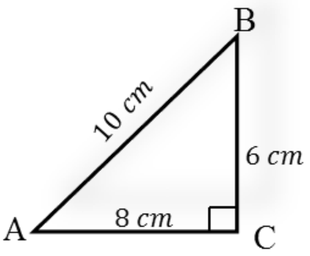


$$= \frac{1}{\cancel{3}} \times \frac{22}{7} \times \overset{2}{\cancel{6}} \times 6 \times 8$$

$$= \frac{44 \times 48}{7} = \frac{2112}{7} = 301.71\ \text{cm}^3$$

When the triangle ABC is rotated about BC, the volume of the cone formed

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h\ cu.units$$



$$= \frac{1}{\cancel{3}} \times \frac{22}{7} \times 8 \times 8 \times \cancel{6}^{\,2}$$

$$= \frac{44 \times 64}{7} = \frac{2816}{7}$$

$$= 402.28 cm^3$$

$$Difference\ between\ two\ volumes = 402.28 - 301.71$$

$$= 100.57 cm^3$$

6. *The volumes of two cones of same base radius are* 3600 cm^3 *and* 5040 cm^3. *Find the ratio of heights.*

Let r and h_1 *be the radius and height of the cone* − *I*

let r and h_2 *be the radius and height of the cone* − *II.*

$$V_1 = 3600\ cm^3,\ V_2 = 5040\ cm^3$$

$$V_1 : V_2 = 3600 : 5040$$

$$\frac{V_1}{V_2} = \frac{3600}{5040}$$

$$\frac{\cancel{\frac{1}{3}\pi r^2}h_1}{\cancel{\frac{1}{3}\pi r^2}h_2} = \frac{\overset{\cancel{360}\ 90}{360\cancel{0}}}{\underset{\cancel{504}\ 126}{504\cancel{0}}} \Longrightarrow \frac{h_1}{h_2} = \frac{\overset{\cancel{45}\ 5}{\cancel{90}}}{\underset{\cancel{63}\ 7}{\cancel{126}}}$$

$$\frac{h_1}{h_2} = \frac{5}{7} \Longrightarrow \therefore h_1 : h_2 = 5 : 7$$

7. *If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.*

Let r_1 and r_2 be the radius of the two sphere

Given: $r_1 : r_2 = 4 : 7$

$\therefore$ *Ratio of the volume of two spheres* $V_1 : V_2 = \cancel{\frac{4}{3}\pi} r_1{}^3 : \cancel{\frac{4}{3}\pi} r_2{}^3$

$$V_1 : V_2 = r_1^3 : r_2^3 \Longrightarrow V_1 : V_2 = 4^3 : 7^3$$

$$V_1 : V_2 = 64 : 343$$

8. *A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is* $3\sqrt{3} : 4$.

Let r_1 and r_2 be the radius of the sphere and hemisphere .

$$\boxed{\text{Surface area of a Sphere} = \text{Total surface area hemisphere}}$$

$$4\cancel{\pi} r_1^2 = 3\cancel{\pi} r_2^2 \Longrightarrow 4r_1^2 = 3r_2^2$$

$$\therefore \frac{r_1^2}{r_2^2} = \frac{3}{4} \Longrightarrow \sqrt{\frac{r_1^2}{r_2^2}} = \sqrt{\frac{3}{4}} \Longrightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

$$r_1 : r_2 = \sqrt{3} : 2$$

Volume of sphere : Volume of hemisphere $= \frac{4}{\cancel{3}}\cancel{\pi} r_1^3 : \frac{2}{\cancel{3}}\cancel{\pi} r_2^3$

$$= \overset{2}{\cancel{4}} \times r_1^3 : \cancel{2} \times r_2^3 = 2r_1^3 : r_2^3$$

$$= 2\left(\sqrt{3}\right)^3 : 2^3 = \cancel{2}\left(3\sqrt{3}\right) : \overset{4}{\cancel{8}}$$

The ratio of their volume $= 3\sqrt{3} : 4$

9. *The outer and the inner surface areas of a spherical copper shell are* $576\pi\ cm^2$ *and* $324\pi\ cm^2$ *respectively. Find the volume of the material required to make the shell.*

Outer surface area of a sphere : $4\pi R^2 = 576\pi\ cm^2$

Inner surface area of a sphere : $4\pi r^2 = 324\pi\ cm^2$

$$4\cancel{\pi} R^2 = 576\cancel{\pi} \Longrightarrow R^2 = \frac{576}{4}$$

$$R^2 = 144 \Longrightarrow R = \sqrt{144} \Longrightarrow R = 12\ cm$$

$$4\cancel{\pi}r^2 = 324\cancel{\pi} \implies r^2 = \frac{324}{4} \implies r^2 = 81$$

$$r = \sqrt{81} \implies r = 9\ cm$$

$$\boxed{Volume\ of\ the\ hollow\ sphere = \frac{4}{3}\pi(R^3 - r^3)\ cu.units}$$

$$= \frac{4}{3}\times\frac{22}{7}\times(12^3 - 9^3) \quad = \frac{4}{3}\times\frac{22}{7}\times(1728 - 729)$$

$$= \frac{4}{\cancel{3}}\times\frac{22}{7}\times\overset{333}{\cancel{999}} = \frac{29304}{7} = 4186.285$$

$$= 4186.29\ cu.cm$$

$$\therefore Volume\ of\ the\ material\ needed = 4186.29\ cm^3$$

10. *A container open at the top is in the form of a frustum of a cone of height 16 cm with radius of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of Rs. 40 per litre.*

Let h, r and R be the height, top and bottom radii of the frustum.

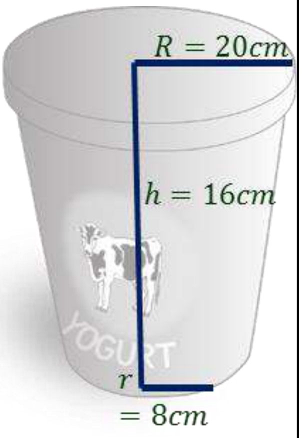
$$Given: h = 16cm, R = 20\ cm, r = 8\ cm$$

$$Volume\ of\ the\ frustum = \frac{1}{3}\pi(R^2 + Rr + r^2)h\ cu.units$$

$$= \frac{1}{3}\times\frac{22}{7}\times(20^2 + 20\times8 + 8^2)\times16$$

$$= \frac{1}{3}\times\frac{22}{7}\times(400 + 160 + 64)\times16$$

$$= \frac{1}{\cancel{3}}\times\frac{22}{7}\times\overset{208}{\cancel{624}}\times16 = \frac{73216}{7}$$



$$Volume\ of\ milk\ in\ the\ frustum = 10459.428\ cm^3$$

$$1000\ cm^3 = 1\ litre$$

$$Volume\ of\ milk\ in\ the\ frustum = \frac{10459.428\ cm^3}{1000} = 10.459428 cm^3$$

$$Volume\ of\ milk\ in\ the\ frustum = 10.459\ litres$$

$$The\ cost\ of\ milk\ @\ Rs.40\ per\ litre$$

$$cost\ of\ milk\ in\ the\ container = 10.459\times40$$

$$= 104.59\times4$$

$$cost\ of\ milk\ in\ the\ container = Rs. 418.36$$

Exercise 7.3

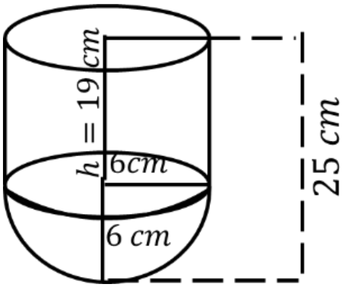
Example: 7.24 A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

Let r and h be the radius and height of the cylinder respectively.

Given : $\text{diameter} : d = 12cm$

$\text{radius} : r = 6\ cm$

$$\boxed{r = \frac{12}{2} = 6}$$



Total height of the toy $= 25\ cm$

$\therefore$ *Height of the cylinder portion* $= 25 - 6$

$= 19\ cm$

T.S.A. of the toy = *C.S.A. of the cylinder* + *C.S.A. of the hemisphere* + *Top Area of the cylinder*

$$= 2\pi rh + 2\pi r^2 + \pi r^2 = 2\pi rh + 3\pi r^2 = \pi r(2h + 3r)\ sq.units$$

$$= \frac{22}{7} \times 6 \times (2 \times 19 + 3 \times 6) = \frac{22}{7} \times 6 \times (38 + 18)$$

$$= \frac{22}{7} \times 6 \times 56 = 22 \times 48 = 1056$$

$\therefore$ **T.S.A. of the toy is $1056\ cm^2$**

Example: 7.25 A jewel box (Fig. 7.39) is in the shape of a cuboid of dimensions 30 cm × 15 cm × 10 cm surmounted by a half part of a cylinder as shown in the figure. Find the volume and T.S.A. of the box.

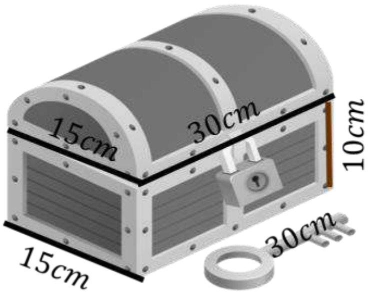
cuboid jewel box

Let l, b and h_1 be the length, breadth and height of the cuboid.

$l = 30cm,\ b = 15cm,\ h_1 = 10cm$

Half part of a cylinder

let r and h_2 be the radius and height of the cylinder.



$\text{diameter} : d = 15cm$

$$\text{radius} : r = \frac{15}{2}\ cm$$

$\therefore$ *Height of the cylinder* $= 30cm$

$$\textbf{Volume of the box} = \textbf{Volume of the cuboid} + \frac{1}{2}(\textbf{volume of cylinder})$$

$$= (l \times b \times h_1) + \frac{1}{2}(\pi r^2 h_2)\ \text{cu.units}$$

$$= (30 \times 15 \times 10) + \frac{1}{2}\left(\frac{\overset{11}{\cancel{22}}}{7} \times \frac{15}{\cancel{2}} \times \frac{15}{\cancel{2}} \times \overset{15}{\cancel{30}}\right)$$

$$\boxed{\begin{aligned} \underbrace{15 \times 15}_{} \times 15 &= 225 \times 15 \\ &= 3375 \\ 3375 \times 11 &= 37125 \end{aligned}}$$

$$= 4500 + \frac{1}{2}\left(\frac{11}{7} \times 15 \times 15 \times 15\right)$$

$$= 4500 + \frac{1}{2}\left(\frac{37125}{7}\right) = 4500 + \frac{37125}{14}$$

$$= 4500 + 2651.79 = 7151.79$$

$\therefore$ *volume of the box* $= 7151.79\ cm^3$

$$\begin{array}{r|l} & 2651.78 \\ \hline 14 & 37125 \\ & 28 \\ \hline & 91 \\ & 84 \\ \hline & 72 \\ & 70 \\ \hline & 25 \\ & 14 \\ \hline & 110 \\ & 98 \\ \hline & 120 \\ & 122 \end{array}$$

$$\boldsymbol{T.S.A.\,of\ the\ box} = \begin{array}{l}\boldsymbol{L.S.A.\,of\ the\ cuboid} \\ + \boldsymbol{\frac{1}{2}(C.S.A.\,of\ cylinder)}\end{array}$$

$$= 2(l+b)h_1 + \frac{1}{\cancel{2}}(\cancel{2}\pi r h_2)$$

$$\boxed{\begin{array}{l}\underbrace{15 \times 11}_{} \times 30 = 165 \times 30 \\ \qquad\qquad\quad = 4950\end{array}}$$

$$= 2(30+15) \times 10 + \left(\frac{\overset{11}{\cancel{22}}}{7} \times \frac{15}{\cancel{2}} \times 30\right)$$

$$= 2(450) + \left(\frac{11 \times 15 \times 30}{7}\right) = 900 + \left(\frac{4950}{7}\right)$$

$$= 900 + 707.14 = 1607.14$$

$\therefore T.S.A.$ *of the box* $= 1607.14\ cm^2$

$$\begin{array}{r|l} & 707.14 \\ \hline 7 & 4950 \\ & 49 \\ \hline & 50 \\ & 49 \\ \hline & 10 \\ & 7 \\ \hline & 30 \\ & 28 \end{array}$$

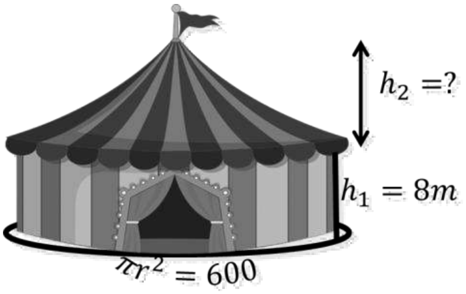
***Example*: 7.26 *Arul has to make arrangements for the accommodat – ionof 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq.m of the space on ground and 40 cu.meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m*?**

Let h_1 *and* h_2 *be the height of cylinder and cone respectively.*

Area for one person $= 4\ sq.m$

Total number of persons $= 150$

Total base area $= 150 \times 4$



$$\pi r^2 = 600 \Rightarrow \frac{22}{7} \times r^2 = 600$$

$$r^2 = \overset{300}{\cancel{600}} \times \frac{7}{\underset{11}{\cancel{22}}}$$

$$r^2 = \frac{2100}{11} \dots (1)$$

Volume of air required for 1 person $= 40\ m^3$

Total Volume of air required for 150 persons $= 150 \times 40 = 6000\ m^3$

$$\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = 6000$$

$$\pi r^2\left(h_1+\frac{1}{3}h_2\right)=6000 \Rightarrow \frac{\overset{2}{\cancel{22}}}{\cancel{7}}\times\frac{\overset{300}{\cancel{2100}}}{\cancel{11}}\left(8+\frac{1}{3}h_2\right)=6000$$

$$600\times\left(8+\frac{1}{3}h_2\right)=6000 \Rightarrow 8+\frac{1}{3}h_2=\frac{\overset{10}{\cancel{6000}}}{\cancel{600}}$$

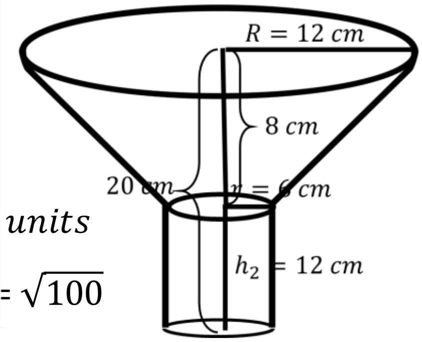
$$8+\frac{1}{3}h_2=10 \Rightarrow \frac{1}{3}h_2=10-8 \Rightarrow \frac{1}{3}h_2=2 \Rightarrow h_2=6$$

$\therefore$ *The height of the conical tent* h_2 *is* $6\,m$

Example: 7.27 A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface areaof the funnel.

Let R, r *be the top and bottom radii of the frustum.*

Let h_1, h_2 *be the heights of the frustum and cylinder*



Given : $R = 12\ cm, r = 6\ cm, h_2 = 12\ cm$

Total height of the funnel $= 20\ cm$

Heights of the frustum: $h_1 = 20 - 12 = 8\ cm$

Slant height of the frustum $l = \sqrt{(R-r)^2 + h_1^2}$ *units*

$$= \sqrt{(12-6)^2+8^2} = \sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100}$$

$$\boxed{l = 10\ cm}$$

Outer surface area $= C.S.A$ *of cylinder* $+ C.S.A$ *frustum*

$$= 2\pi r h_2 + \pi(R+r)l\ sq.units$$

$$= \pi[2rh_2 + (R+r)l]$$

$$= \pi[(2\times6\times12)+(12+6)\times10]$$

$$= \pi[144+(18\times10)]$$

$$= \pi[144+180]$$

$$= \frac{22}{7}\times324 = \frac{7128}{7} = 1018.28$$

$$\boxed{22\times324 = 7128}$$

$$\begin{array}{r|l} & 1018.28 \\ 7 & 7128 \\ & 7 \\ & 12 \\ & 7 \\ & 58 \\ & 56 \\ & 20 \\ & 14 \\ & 60 \\ & 56 \end{array}$$

$\therefore$ *outer surface area of the funnel is* $1018.28\ cm^2$

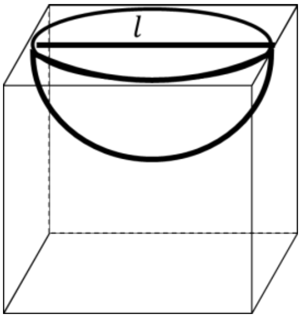
Example: 7.28 A hemispherical section is cut out from one face of a cubical block such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.

Let r be the radius of the hemisphere.

Given : Diameter of the hemisphere = side of the cube = l

$$\text{Radius of the hemisphere}: r = \frac{l}{2}$$

TSA of the remaining solid = (Surface area of the cubical part + C.S.A. of the hemispherical part − Area of the base of the hemispherical part)



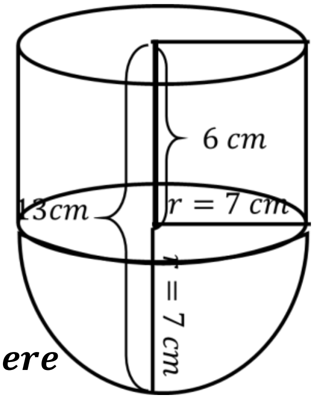
$$= 6 \times (Edge)^2 + 2\pi r^2 - \pi r^2$$

$$= 6 \times (Edge)^2 + \pi r^2 = 6 \times (l)^2 + \pi \left(\frac{l}{2}\right)^2$$

$$= 6 \times l^2 + \pi \times \frac{l^2}{4} = 6l^2 + \frac{\pi l^2}{4} = \frac{24l^2 + \pi l^2}{4}$$

$$\text{Total surface area of the remaining solid} = \frac{(24+\pi)l^2}{4}\ sq.units$$

1. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.



Given : Diameter = 14cm

$$\text{Radius}: r = \frac{14}{2} = 7cm$$

Total height = 13cm

Height of the cylinder = 13 − 7 = 6cm

Capacity of the vessel = Volume of cylinder + Volume of Hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2}{3}r\right) = \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times \left(6 + \frac{2}{3} \times 7\right)$$

$$= 22 \times 7 \times \left(6 + \frac{14}{3}\right) = 22 \times 7 \times \left(\frac{18+14}{3}\right)$$

$$= 154 \times \frac{32}{3} = 154 \times 10.67 = 1642.67 cm^3$$

```
      10.66
3 )  32
     3
    ----
     20
     18
    ----
      20
      18
    ----
       2
```

2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of themodel that Nathan made.

Given : Diameter = 3 cm

$$\text{radius} = \frac{3}{2}\ cm = 1.5cm$$

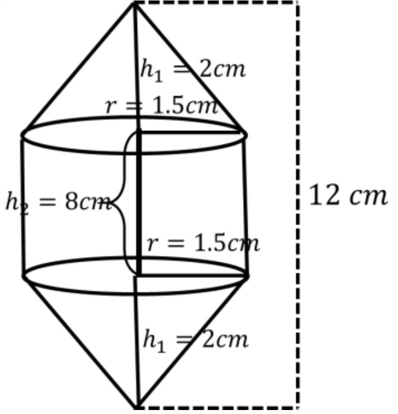
Total height = 12 cm

Let h_1 *and* h_2 *be the height of the cone and cylinder*

Cone
$h_1 = 2\ cm$

Cylinder
$h_2 = Total\ height - 2 - 2$
$= 12 - 4 = 8\ cm$
$h_2 = 8\ cm$

Volume of the model = vol. of two cone + vol. of cylinder



$$= 2 \times \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 = \frac{2}{3}\pi r^2 h_1 + \pi r^2 h_2$$

$$= \pi r^2 \left(\frac{2}{3}h_1 + h_2\right) = \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{\cancel{2}} \times \frac{3}{2} \times \left(\frac{2}{3} \times 2 + 8\right)$$

$$= \frac{11}{7} \times \frac{9}{2} \times \left(\frac{4}{3} + 8\right) = \frac{11}{7} \times \frac{9}{2} \times \left(\frac{4 + 24}{3}\right)$$

$$= \frac{11}{\cancel{7}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{2}} \times \frac{\overset{\overset{2}{\cancel{14}}}{\cancel{28}}}{\cancel{3}} = 11 \times 3 \times 2 = 66\ cm^3$$

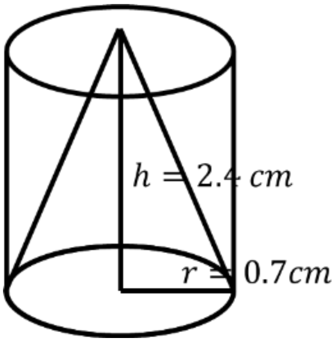
3. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.

Given : cone and cylinder has same height and radius.

$Diameter = 1.4\ cm$

$radius = \frac{1.4}{2}\ cm = 0.7\ cm$

$height\ : h = 2.4\ cm$



Volume of Remaining solid =

vol. of cylinder − vol. of cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 h\left(1 - \frac{1}{3}\right) = \frac{22}{\cancel{7}} \times \overset{0.1}{\cancel{0.7}} \times 0.7 \times \overset{0.8}{\cancel{2.4}} \times \frac{2}{\cancel{3}}$$

$$= 22 \times 0.1 \times 0.7 \times 0.8 \times 2 = 2.464\ cm^3$$

4. A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.

Cone
$r = 6$ cm
$h = 12\ cm$

Hemisphere
$r = 6\ cm$

$Volume\ of\ water\ displaced\ out\ of\ cylinder$

$$= vol. of\ cone + vol. of\ hemisphere$$

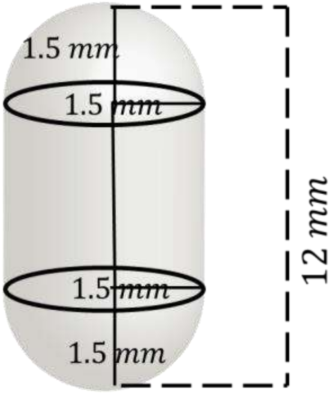
$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h+2r)$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{7} \times \overset{2}{\cancel{6}} \times 6 \times (12 + 2 \times 6)$$

$$= \frac{22}{7} \times 2 \times 6 \times (12+12) \quad = \frac{22}{7} \times 12 \times 24$$

$$= \frac{6336}{7} = 905.14\ cm^3$$

5. *A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?*



$Given: \ Diameter = 3\ mm$

$$radius: r = \frac{3}{2}\ mm = 1.5\ mm$$

$Total\ height = 12mm$

$Hemisphere$	$Cylinder$
$r = 1.5mm$	$r = 1.5mm$

$$Height\ (h) = total\ height - 2 \times 1.5$$
$$= 12 - 3 = 9\ mm$$

Volume of medicine hold in the capsule

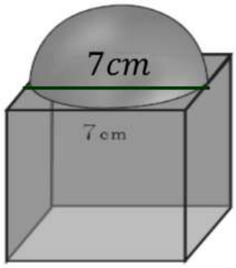
$$= \boldsymbol{vol. of\ cylinder + vol. of\ 2\ hemisphere}$$

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left(h + \frac{4}{3}r\right)$$

$$= \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{\cancel{2}} \times \frac{3}{2} \times \left(9 + \frac{\overset{2}{\cancel{4}}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}}\right) = \frac{11}{7} \times \frac{9}{2} \times 11 = \frac{121}{7} \times 4.5$$

$$= \frac{544.5}{7} = 77.78\ mm^3$$

6. *As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.*



$Cube$	$Hemisphere$	
$a = 7\ cm$	$Diameter = 7cm$	$r = \frac{7}{2}\ cm$

Surface area of the solid

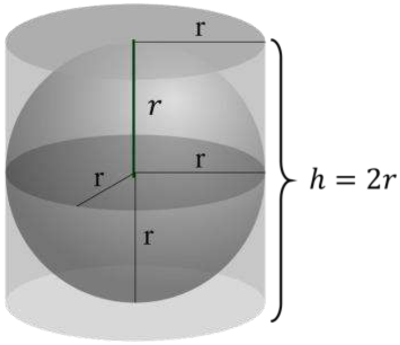
$$= \boldsymbol{T.S.A\ of\ a\ cube + \ C.S.A\ of\ Hemisphere}$$
$$\boldsymbol{- Area\ of\ the\ base\ of\ the\ Hemisphere}$$

$$= 6a^2 + 2\pi r^2 - \pi r^2 = 6a^2 + \pi r^2$$

$$= 6 \times 7 \times 7 + \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2} = 6 \times 49 + 11 \times \frac{7}{2}$$

$$= 294 + 11 \times 3.5 = 294 + 38.5 = 332.5\ cm^2$$

7. A right circular cylinder just enclose a sphere of radius r units. Calculate (i) the surface area of the sphere (ii) the curved surface area of the cylinder (iii) the ratio of the areas obtained in (i) and (ii).



Height of the cylinder $= 2r$

$$h = 2r$$

(i) *S.A of a sphere* $= 4\pi r^2$ *sq. units*

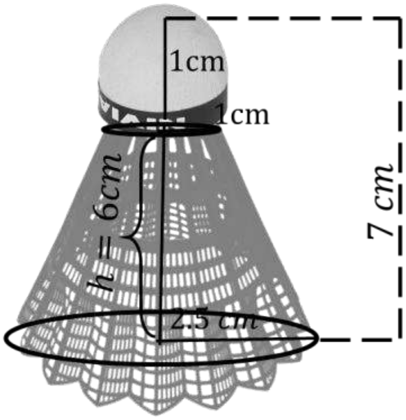
(ii) *C.S.A of cylinder* $= 2\pi rh$ *sq. units*

$$= 2\pi r \times 2r = 4\pi r^2\ sq.units$$

(iii) *S.A. of a sphere : C.S.A of cylinder*

$$= 4\cancel{\pi r^2} : 4\cancel{\pi r^2} = 1 : 1$$

8. A shuttle cock used for playing badminton has the shape of a frustum of a cone is mounted on a hemisphere. The diameters of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.



Given : Total height of the shuttle cock $= 7\ cm$

Frustum

Larger circle of diameter $= 5\ cm$

$$R = \frac{5}{2}\ cm \Rightarrow R = 2.5\ cm$$

Smaller circle of diameter $= 2\ cm$

$$r = \frac{2}{2} \Rightarrow r = 1\ cm$$

Height of the frustum

$$h = total\ height - r$$

$$h = 7 - 1 = 6\ cm \Rightarrow \boxed{h = 6cm}$$

$$l = \sqrt{(R-r)^2 + h^2} = \sqrt{(2.5-1)^2 + 6^2}$$

$$= \sqrt{(1.5)^2 + 36} = \sqrt{2.25 + 36}$$

$$l = \sqrt{38.25} \Rightarrow l = 6.18$$

$$\begin{array}{r|l} & 6\cdot 18 \\ \hline 6 & 38.25 \\ & 36 \\ \hline 121 & 225 \\ & 121 \\ \hline 1228 & 10400 \\ & 9824 \\ \hline & 576 \end{array}$$

T.S.A of shuttle cock

$$= C.S.A\ of\ frustum + C.S.A\ of\ Hemisphere$$

$$= \pi(R + r)l + 2\pi r^2 \ = \pi[(R + r)l + 2r^2]$$

$$= \frac{22}{7}[(2.5 + 1)6.18 + 2 \times 1^2] \ = \frac{22}{7}[3.5 \times 6.18 + 2]$$

$$= \frac{22}{7}[21.63 + 2] = \frac{22}{7} \times 23.63$$

$$= 3.14 \times 23.63 = 74.19\ cm^2$$

$EXERCISE: 7.4$

Example 7.29 A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

Let the number of small spheres obtained be n..

Let r be the radius of each small sphere and R be the radius of metallic sphere ... $R = 16 \text{ cm}, r = 2 \text{ cm}$

$n \times (Volume\ of\ a\ small\ sphere) = Volume\ of\ big\ metallic\ sphere$

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \implies n\left(\frac{4}{\cancel{3}}\cancel{\pi} \times 2^3\right) = \frac{4}{\cancel{3}}\cancel{\pi} \times 16^3$$

$$\cancel{8}n = \overset{2}{\cancel{16}} \times 16 \times 16 \implies n = 2 \times 16 \times 16$$

$$n = 2 \times 256$$

$$n = 512$$

$\therefore$ *There will be* 512 *small spheres.*

Example 7.30 A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Let h_1 *and* h_2 *be the heights of a cone and cylinder.*

Let r be the raius of the cone.

Radius of a cone = Radius of a cylinder

Given : Height of the cone: $h_1 = 24\ cm;$

$$Volume\ of\ cylinder = Volume\ of\ cone$$

$$\cancel{\pi r^2} h_2 = \frac{1}{3}\cancel{\pi r^2} h_1 \implies h_2 = \frac{1}{3} \times h_1 \implies h_2 = \frac{1}{\cancel{3}} \times \overset{8}{\cancel{24}} \implies \boxed{h_2 = 8}$$

$\therefore$ *Height of cylinder is 8 cm*

Example 7.31 A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemisphericalcap. Find the number of cones needed to empty the container.

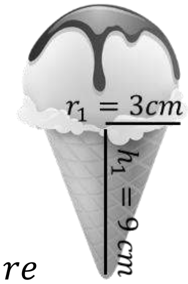
Let h and r be the height and radius of the cylinder.

Given : cylindrical container

$h = 15\ cm, r = 6\ cm$

Volume of cylindrical container $= \pi r^2 h$ *cubic units.*

$$Volume\ of\ cylindrical\ container = \frac{22}{7} \times 6 \times 6 \times 15$$



$ICE\ CREAM\ CONE$

$CONE \qquad HEMISPHERE$

$r_1 = 3\ cm \qquad r_1 = 3cm$

$h_1 = 9\ cm$

$$Vol. of\ a\ ice\ cream\ cone\ = Vol. of\ cone\ + Vol. of\ hemisphere$$

$$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3 \qquad = \frac{1}{\cancel{3}} \times \frac{22}{7} \times \cancel{3} \times 3 \times 9 + \frac{2}{\cancel{3}} \times \frac{22}{7} \times \cancel{3} \times 3 \times 3$$

$$= \frac{22}{7} \times 27 + \frac{22}{7} \times 18 = \frac{22}{7}(27 + 18)$$

$$Vol. of\ a\ ice\ cream\ cone = \frac{22}{7} \times 45$$

$$n \times (volume\ of\ a\ ice\ cream\ cone)\ = volume\ of\ the\ cylinder$$

$$Number\ of\ cones\ = \frac{volume\ of\ the\ cylinder}{volume\ of\ one\ ice\ cream\ cone}$$

$$Number\ of\ ice\ cream\ cones\ needed\ = \frac{\frac{\cancel{22}}{\cancel{7}} \times \overset{2}{\cancel{6}} \times 6 \times \cancel{15}}{\frac{\cancel{22}}{\cancel{7}} \times \cancel{45}\ \cancel{3}} = 12$$

12 $ice\ cream\ cones\ are\ required\ to\ empty\ the\ cylindrical\ container.$

1. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

$Let\ r_1\ and\ h_1\ be\ the\ radius\ and\ height\ of\ the\ cylinder$

$Let\ r_2\ be\ the\ radius\ of\ the\ sphere.$

$Cylinder \qquad Sphere$

$r_1 = 8\ cm, h_1 = ? \qquad r_2 = 12\ cm$

$$vol. of\ cylinder = vol. of\ sphere$$

$$\cancel{\pi} r_1^2 h_1 = \frac{4}{3}\cancel{\pi} r_2^3 \implies r_1^2 h_1 = \frac{4}{3} \times r_2^3$$

$$8 \times 8 \times h_1 = \frac{4}{3} \times 12 \times 12 \times 12$$

$$h_1 = \frac{\cancel{4}}{\cancel{3}} \times \overset{\cancel{4}}{\cancel{12}} \times \overset{6}{\cancel{12}} \times \overset{6}{\cancel{12}} \times \frac{1}{\underset{\cancel{2}}{\cancel{8}}} \times \frac{1}{\underset{\cancel{2}}{\cancel{8}}} \implies h_1 = 36$$

$$Height\ of\ the\ cylinder = 36\ cm$$

2. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

$Cylindrical\ pipe$

$Speed\ of\ water\ = 15\ km/hr$ $\boxed{1km = 1000m}$

$$\frac{Distance}{time} = 15\ km/hr \Longrightarrow Distance = 15\ km/hr \times Time$$

$$h = 15 \times 1000\ m/hr \times Time\ \Longrightarrow\ h = 15000\ m/hr \times Time$$

$$\boxed{\begin{array}{c} speed\ = \dfrac{Distance}{Time} \\ speed \times Time = Distance \end{array}}$$

$Diameter\ = 14\ cm$

$$r = \frac{14}{2} = 7\ cm \Longrightarrow\ r = \frac{7}{100} m$$

$Rectangular\ tank\ (Cuboid)$

$$l = 50\ m, b = 44\ m, h_1 = 21\ cm$$

$$h_1 = \frac{21}{100}\ m$$

$\boldsymbol{vol. of\ water\ flows\ through\ a\ cylindrical\ pipe}$
$\boldsymbol{= vol. of\ water\ in\ the\ cuboid}$

$$\pi r^2 h = l \times b \times h_1$$

$$\frac{22}{\cancel{7}} \times \frac{7}{10\cancel{0}} \times \frac{\cancel{7}}{1\cancel{00}} \times 15\cancel{0}\cancel{00} \times Time\ = 50 \times 44 \times \frac{21}{100}$$

$$Time\ = \cancel{50} \times \overset{2}{\cancel{44}} \times \frac{\overset{7}{\cancel{21}}}{\underset{\cancel{2}}{\cancel{100}}} \times \frac{1}{\cancel{22}} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\cancel{7}} \times \frac{1}{\underset{\cancel{3}}{\cancel{15}}}$$

$$Time = 2\ hrs$$

3. $\boldsymbol{A\ conical\ flask\ is\ full\ of\ water. The\ flask\ has\ base\ radius\ r\ units\ and\ height\ h\ units, the\ water\ poured\ into\ a\ cylindrical\ flask\ of\ base\ radius\ xr\ units. Find\ the\ height\ of\ water\ in\ the\ cylindrical\ flask.}$

$Conical\ flask$	$Cylindrical\ flask$
$radius\ = r$	$r_1\ = xr$
$height\ = h$	$h_1 = ?$

$\boldsymbol{vol. of\ water\ in\ the\ cylindrical\ flask = vol. of\ water\ in\ conical\ flask}$

$$\cancel{\pi} r_1{}^2 h_1 = \frac{1}{3} \cancel{\pi} r^2 h \Longrightarrow (xr)^2 h_1 = \frac{1}{3} r^2 h$$

$$x^2 \cancel{r}^2 h_1 = \frac{1}{3} \cancel{r}^2 h\ \Longrightarrow\ h_1 = \frac{h}{3} \times \frac{1}{x^2}$$

$$Height\ of\ the\ cylindrical\ flask\ = \frac{h}{3x^2}$$

4. $\boldsymbol{A\ solid\ right\ circular\ cone\ of\ diameter\ 14\ cm\ and\ height\ 8\ cm\ is\ melted\ to\ form\ a\ hollow\ sphere. If\ the\ external\ diameter\ of\ the\ sphere\ is\ 10\ cm, find\ the\ internal\ diameter.}$

Let R and r be the external and internal radius of the sphere

Let r_1 *and* h_1 *be the radius and height of the cone*

<u>*Hollow sphere*</u>	<u>*Cone*</u>
$External\ diameter = 10\ cm$	$Diameter = 14\ cm$
$R = \frac{10}{2} \Rightarrow R = 5\ cm$	$r_1 = \frac{14}{2} \Rightarrow r_1 = 7\ cm$
$r = ?$	$h_1 = 8\ cm$

$$\boldsymbol{vol. of\ hollow\ sphere = vol. of\ cone}$$

$$\frac{4}{\cancel{3}}\cancel{\pi}(R^3 - r^3) = \frac{1}{\cancel{3}}\cancel{\pi}r_1{}^2h_1 \Rightarrow \cancel{4}(5^3 - r^3) = 7 \times 7 \times \overset{2}{\cancel{8}}$$

$$125 - r^3 = 49 \times 2 \Rightarrow 125 - r^3 = 98$$

$$125 - 98 = r^3 \Rightarrow r^3 = 27 \Rightarrow r = \sqrt[3]{27}$$

$$r = 3\ \text{cm}$$

$$Internal\ diameter = 2 \times r$$

$$= 2 \times 3 = 6\ cm$$

$$Internal\ diameter = 6\ cm$$

5. Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.

<u>***Cuboid***</u>	<u>***Cylindrical tank***</u>
$l = 2m = 200\ cm$	$r = 60\ cm$
$b = 1.5\ m = 1.5 \times 100 = 150\ cm$	$h_1 = 105\ cm$
$h = 1\ m = 100\ cm$	

$$\boldsymbol{vol. of\ water\ left\ in\ the\ sump}$$

$$\boldsymbol{= vol. of\ water\ in\ the\ sump\ (cuboid) - vol. of\ water\ in\ the\ tank}$$

$$= l \times b \times h - \pi r^2 h_1$$

$$= 200 \times 150 \times 100 - \frac{22}{\cancel{7}} \times 60 \times 60 \times \overset{15}{\cancel{105}}$$

$$= 3000000 - 1188000$$

$$vol. of\ water\ left\ in\ the\ sump = 1812000\ cm^3$$

6. *The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, then find the height of the cylinder.*

<u>Hollow hemisphere</u>

Internal diameter $= 6\ cm$

$$r = \frac{6}{2} \Longrightarrow r = 3\ cm$$

External diameter $= 10\ cm$

$$R = \frac{10}{2} \Longrightarrow R = 5\ cm$$

<u>Solid cylinder</u>

Diameter $= 14\ cm$

$$r_1 = \frac{14}{2} \Longrightarrow r_1 = 7\ cm$$

$$h_1 = ?$$

vol. of solid cylinder = vol. of hollow hemisphere

$$\cancel{\pi} r_1{}^2 h_1 = \frac{2}{3}\cancel{\pi}(R^3 - r^3) \Longrightarrow 7 \times 7 \times h_1 = \frac{2}{3}(5^3 - 3^3)$$

$$49 \times h_1 = \frac{2}{3}(125 - 27) \Longrightarrow 49 \times h_1 = \frac{2}{3} \times 98$$

$$h_1 = \frac{2}{3} \times \overset{2}{\cancel{98}} \times \frac{1}{\cancel{49}} \Longrightarrow h_1 = \frac{4}{3} \Longrightarrow h_1 = 1.33\ cm$$

7. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

<u>Solid sphere</u>

$r_1 = 6\ cm$

<u>Hollow cylinder</u>

$R = 5\ cm, h = 32\ cm$

$r = ?$

vol. of hollow cylinder = vol. of solid sphere

$$\cancel{\pi} h(R^2 - r^2) = \frac{4}{3}\cancel{\pi} r_1{}^3 \Longrightarrow 32(5^2 - r^2) = \frac{4}{\cancel{3}} \times \overset{2}{\cancel{6}} \times 6 \times 6$$

$$5^2 - r^2 = \cancel{4} \times \cancel{2} \times \overset{3}{\cancel{6}} \times \overset{3}{\cancel{6}} \times \frac{1}{\cancel{32}\ \cancel{8}\ \cancel{4}\ \cancel{2}}$$

$$25 - r^2 = 9 \Longrightarrow 25 - 9 = r^2 \Longrightarrow r^2 = 16$$

$$r = \sqrt{16} \Longrightarrow r = 4\ cm$$

$$Thickness = R - r$$

$$= 5 - 4 = 1\ cm$$

8. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Radius of hemispherical bowl and cylinder are equal

Let h be the height of the cylinder

<u>Cylinder</u>

$h = 100\%$

$$r = 150\ \%\ of\ h \implies r = \frac{\overset{3}{\cancel{150}}}{\underset{2}{\cancel{100}}}h \implies r = \frac{3}{2}h$$

$$\boxed{h = \frac{2}{3}r}$$

$$vol.of\ juice\ in\ the\ cylinder = \pi r^2 h$$

$$= \pi \times \left(\frac{3h}{2}\right)^2 \times h = \pi \times \frac{9}{4}h^2 \times h$$

$$= \pi \times \frac{9}{4}h^3 = \pi \times \frac{9}{4} \times \left(\frac{2}{3}r\right)^3$$

$$= \pi \times \frac{\cancel{9}}{\cancel{4}} \times \frac{\cancel{8}^{\,2}}{\underset{3}{\cancel{27}}}r^3 = \frac{2}{3}\pi r^3$$

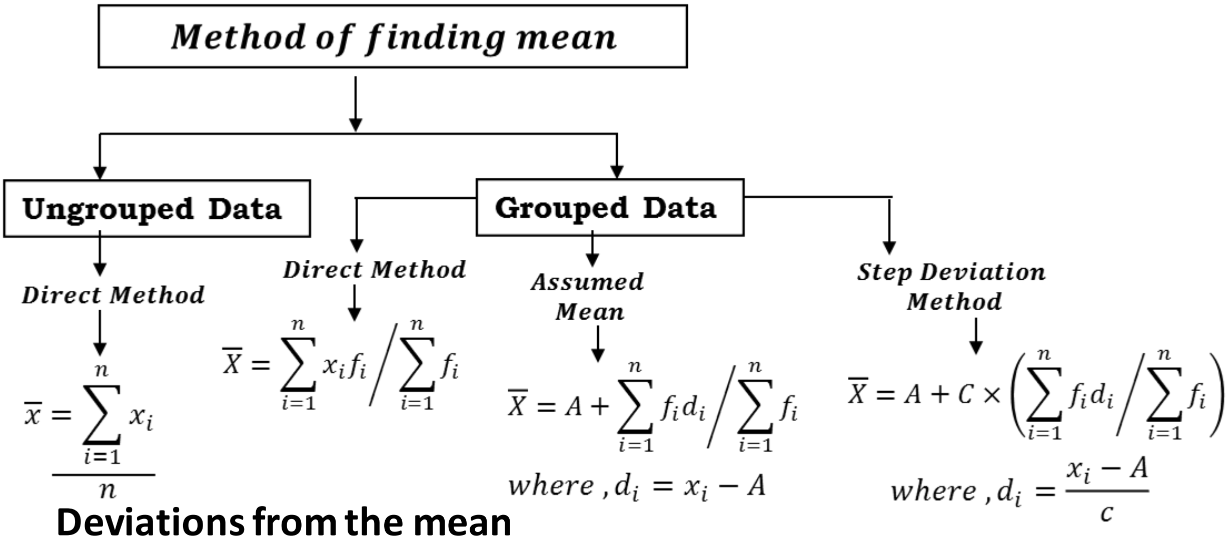
vol. of juice in the cylinder = vol. of hemispherical bowl

100 % *of juice that can be tansfered from the bowl to cylinder*

EXERCISE 8.1

Arithmetic Mean

$$\overline{x} = \frac{Sum\ of\ all\ the\ observations}{Number\ of\ observations}$$



$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\overline{X} = \sum_{i=1}^{n} x_i f_i \Big/ \sum_{i=1}^{n} f_i$$

$$\overline{X} = A + \sum_{i=1}^{n} f_i d_i \Big/ \sum_{i=1}^{n} f_i$$

$where\ , d_i = x_i - A$

$$\overline{X} = A + C \times \left(\sum_{i=1}^{n} f_i d_i \Big/ \sum_{i=1}^{n} f_i \right)$$

$where\ , d_i = \frac{x_i - A}{c}$

Deviations from the mean

$x_1 - \overline{x}, x_2 - \overline{x}, x_3 - \overline{x}, \ldots x_n - \overline{x}.$

Squares of deviations from the mean

$$\sum_{i=1}^{n} (x_i - \overline{x})^2$$

$Variance$:

The mean of the squares of the deviations from the mean is called Variance.

It is denoted by σ^2

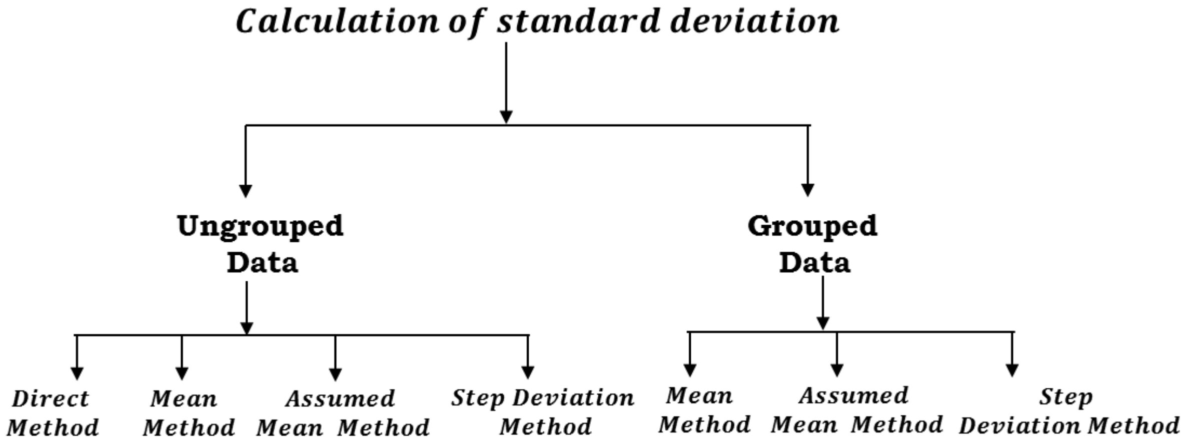
$$Variance = Mean\ of\ squares\ of\ deviations$$

$$= \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}}$$



(i) *Direct Method*

$$\boldsymbol{\sigma} = \sqrt{\frac{\sum x_i{}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

(ii) *Mean Method*

$$\sigma = \sqrt{\frac{\sum d^2}{n}} \quad where\ , \bar{x} = \frac{\sum x}{n} \quad and\ d_i = x_i - \bar{x}$$

(iii) *Assumed Mean Method*

$$\sigma = \sqrt{\frac{\sum d_i{}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} \quad where\ , d_i = x_i - A$$

(v) *Step Deviation Method*

$$\sigma = \sqrt{\frac{\sum d_i{}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} \times c \quad where\ , d_i = \frac{x_i - A}{c}$$

Measures of Dispersion are

1.Range 2. Mean deviation 3. Quartile deviation

4. Standard deviation 5. Variance 6. Coefficient of Variation

- Range

The difference between the largest value and the smallest value is called Range.

$$\boxed{\boldsymbol{R = L - S}}$$

- $\boldsymbol{Coefficient\ of\ range} = \dfrac{\boldsymbol{L - S}}{\boldsymbol{L + S}}$

Example 8.1 : Find the range and coefficient of range of the following data: 25, 67, 48, 53,18, 39, 44.

Largest value $L = 67$; *Smallest value* $S = 18$

Let as arrange the given data in the ascending order.

$\overset{S}{18}$, 25, 39, 44, 48, 53, $\overset{L}{67}$

$$Range: \; R = L - S$$

$$= 67 - 18 = 49$$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$= \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

$$\begin{array}{r|l} & 0.576 \\ \hline 85 & 49 \\ & 490 \\ & 425 \\ \hline & 0650 \\ & \;\;595 \\ \hline & \;\;550 \\ & \;\;510 \\ \hline & \;\;0400 \end{array}$$

Example 8.2 Find the range of the following distribution.

$Age\ (in\ years)$	16–18	18–20	20 – 22	22–24	24–26	26–28
$Number\ of\ students$	0	4	6	8	2	2

$$Here\ Largest\ value\ L = 28$$

$$Smallest\ value\ S = 18$$

$$Range: \; R = L - S$$

$$\text{R} = 28 - 18 = 10 \text{ Years}$$

Example 8.3 The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

$$Range: \; R = 13.67$$

$$Largest\ value: \; L = 70.08$$

$$Range: R = L - S$$

$$13.67 = 70.08 - \text{S} \Longrightarrow \text{S} = 70.08 - 13.67$$

$$\text{S} = 56.41$$

Example 8.4 The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

x_i	x_i^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 623$

$$\begin{array}{r|l} & 2{\cdot}82 \\ \hline 2 & 8 \\ & 4 \\ \hline 48 & 400 \\ & 384 \\ \hline 562 & 1600 \\ & 1124 \\ \hline & \end{array}$$

Standard deviation: $\sigma = \sqrt{\frac{\sum x_i{}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{\cancel{623}^{89}}{\cancel{7}} - \left(\frac{\cancel{63}^{9}}{\cancel{7}}\right)^2}$

$$\sigma = \sqrt{89 - (9)^2} = \sqrt{89 - 81}$$

$$\sigma = \sqrt{8} \ = 2.82$$

$$Hence, \sigma \ = 2.83$$

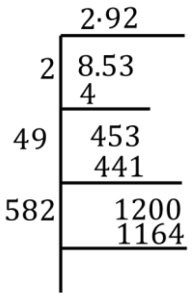
Example 8.5 The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Arranging the numbers in ascending order : 11.4, 12.5, 12.8, 16.3, 17.8, 19.2.

$$Mean = \frac{11.4 + 12.5 + 12.8 + 16.3 + 17.8 + 19.2}{6}$$

$$= \frac{\cancel{90}^{30}}{\cancel{6}_{2}} = \frac{\cancel{30}^{15}}{\cancel{2}_{1}} = 15$$

x_i	$d_i = x_i - \bar{x}$ $= x - 15$	d_i^2
11.4	$11.4 - 15 = -3.6$	12.96
12.5	$12.5 - 15 = -2.5$	6.25
12.8	$12.8 - 15 = -2.2$	4.84
16.3	$16.3 - 15 = 1.3$	1.69
17.8	$17.8 - 15 = 2.8$	7.84
19.2	$19.2 - 15 = 4.2$	17.64
		$\Sigma d_i^2 = 51.22$



Standard deviation: $\sigma = \sqrt{\frac{\sum d_i^2}{n}}$

$$= \sqrt{\frac{51.22}{6}} = \sqrt{8.53}$$

$$\boxed{\sigma = 2.9}$$

- ➢ ***Standard deviation of first 'n' natural numbers*** $\sigma = \sqrt{\frac{n^2 - 1}{12}}$
- ➢ *The value of SD will not be changed if we add (or) subtract some fixed constant to all the values.*

- *When we multiply each value of a data by a constant, the value of SD is also multiplied by the same constant.*

Example 8.6: *The marks scored by 10 students in a class test are* 25, 29, 30, 33, 35, 37, 38, 40, 44, 48.

Given data : 25, 29, 30, 33, 35, 37, 38, 40, 44, 48

x	$d_i = x - A$ $d_i = x - 35$	$d_i^{\,2}$
25	$25 - 35 = -10$	100
29	$29 - 35 = -6$ -23	36
30	$30 - 35 = -5$	25
33	$33 - 35 = -2$	4
$A = 35$	$35 - 35 = 0$	0
37	$37 - 35 = 2$	4
38	$38 - 35 = 3$ 32	9
40	$40 - 35 = 5$	25
44	$44 - 35 = 9$	81
48	$48 - 35 = 13$	169
	$\sum d_i = 9$	$\Sigma d_i^{\,2} = 453$

	$6 \cdot 67$
6	44.49
	36
1263	849
	756
1327	9300
	9289

$$\sigma = \sqrt{\frac{\Sigma d_i^{\,2}}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2} = \sqrt{45.3 - (0.9)^2}$$

$$= \sqrt{45.3 - 0.81} = \sqrt{44.49}$$

$$\boxed{\sigma \simeq 6.67}$$

Example 8.7: *The amount that the children have spent for purchasing some eatbales in one day trip of a school are* 5, 10, 15, 20, 25, 30, 35, 40. *Using step deviation method, find the standard deviation of the amount they have spent.*

5, 10, 15, 20, 25, 30, 35, 40 *class interval* : $c = 5$

x	$d_i = x - A$ $d_i = x - 20$	$d_i = \dfrac{x - A}{c} = \dfrac{x - 20}{5}$	$d_i^{\,2}$
5	$5 - 20 = -15$	$-15/5 = \cancel{-3}$	9
10	$10 - 20 = -10$	$-10/5 = \cancel{-2}$	4
15	$15 - 20 = -5$	$-5/5 = \cancel{-1}$	1
$A = 20$	$20 - 20 = 0$	$0/5 = 0$	0
25	$25 - 20 = 5$	$5/5 = \cancel{1}$	1
30	$30 - 20 = 10$	$10/5 = \cancel{2}$	4

35	$35-20=15$	$15/5=\cancel{3}$	9
40	$40-20=20$	$20/5=4$	16
		$\sum d_i = 4$	$\sum {d_i}^2 = 44$

$$\textbf{Standard deviation}: \boldsymbol{\sigma} = \sqrt{\frac{\sum \boldsymbol{d_i}^2}{\boldsymbol{n}} - \left(\frac{\sum \boldsymbol{d_i}}{\boldsymbol{n}}\right)^2} \times \boldsymbol{C}$$

$$\begin{array}{r|l} & 2\cdot 29 \\ \hline 2 & 5.25 \\ & 4 \\ \hline 42 & 125 \\ & 84 \\ \hline 449 & 4100 \\ & 4041 \\ \hline & \end{array}$$

$$= \sqrt{\frac{\cancel{44}^{\cancel{11}^{5.5}}}{\cancel{8}_{\cancel{2}}} - \left(\frac{\cancel{4}^{1}}{\cancel{8}_{2}}\right)^2} \times 5 = \sqrt{5.5 - \left(\frac{1}{2}\right)^2} \times 5 = \sqrt{5.5-(0.5)^2} \times 5$$

$$= \sqrt{5.5-0.25} \times 5 = \sqrt{5.25} \times 5 = 2.29 \times 5$$

$$\boxed{\sigma \simeq 11.455}$$

Example 8.8: ***Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.***

Data : 7, 4, 8, 10, 11

Ascending order: 4, 7, 8, 10, 11

x	$d_i = x - A$ $d_i = x - 8$	${d_i}^2$
4	$4-8=-4$	16
7	$7-8=-1$	1
$A =$ 8	$8-8=0$	0
10	$10-8=2$	4
11	$11-8=3$	9
	$\sum d_i = 0$	$\sum {d_i}^2 = 30$

$$Standard\ deviation:\ \sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$$

$$\begin{array}{r|l} & 2\cdot 44 \\ \hline 2 & 6 \\ & 4 \\ \hline 44 & 200 \\ & 176 \\ \hline 484 & 2400 \\ & 1936 \\ \hline & 464 \end{array}$$

$$= \sqrt{\frac{30}{5} - \left(\frac{0}{5}\right)^2} = \sqrt{6-0} = \sqrt{6}$$

$$\boxed{\sigma \simeq 2.45}$$

Add 3 to all the values

New values: 10, 7, 11, 13, 14

Ascending order: 7, 10, 11, 13, 14

x	$d_i = x - A$ $d_i = x - 11$	${d_i}^2$
7	$7 - 11 = -4$	16
10	$10 - 11 = -1$	1
$A = 11$	$11 - 11 = 0$	0
13	$13 - 11 = 2$	4
14	$14 - 11 = 3$	9
	$\sum d_i = 0$	$\sum {d_i}^2 = 30$

$$\sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{30}{5} - \left(\frac{0}{5}\right)^2} = \sqrt{6 - 0} = \sqrt{6}$$

$$\boxed{\sigma \simeq 2.45}$$

The standard deviation will not change when we add some fixed constant to all values.

Example 8.9: Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

Data : 2, 3, 5, 7, 8

Ascending order: 2, 3, 5, 7, 8

x	$d_i = x - A$ $d_i = x - 5$	${d_i}^2$
2	$2 - 5 = -3$	9
3	$3 - 5 = -2$	4
$A = 5$	$5 - 5 = 0$	0
7	$7 - 5 = 2$	4
8	$8 - 5 = 3$	9
	$\sum d_i = 0$	$\sum {d_i}^2 = 26$

	2·28
2	5.2 4
42	120 84
448	3600 3584

$$\sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{26}{5} - \left(\frac{0}{5}\right)^2} = \sqrt{5.2 - 0} = \sqrt{5.2} = 2.280$$

$$\boxed{\sigma \simeq 2.28}$$

Multiply each data by 4

New values: 8, 12, 20, 28, 32

x	$d_i = x - A$ $d_i = x - 20$	${d_i}^2$
8	$8 - 20 = -12$	144
12	$12 - 20 = -8$	64
$A = 20$	$20 - 20 = 0$	0
28	$28 - 20 = 8$	64
32	$32 - 20 = 12$	144
	$\sum d_i = 0$	$\sum {d_i}^2 = 416$

$$\begin{array}{r|l} & 9{\cdot}12 \\ \hline 9 & 83.2 \\ & 81 \\ \hline 182 & 220 \\ & 181 \\ \hline 1822 & 3900 \\ & 3644 \\ \hline \end{array}$$

$$S.D:\ \sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{416}{5} - \left(\frac{0}{5}\right)^2} = \sqrt{83.2 - 0} = \sqrt{83.2}$$

$$\boxed{\sigma \simeq 9.12}$$

when we multiply each data by 4, the new standard deviation gets multiplied by 4.

Example 8.10: Find the mean and variance of the first n natural numbers.

$$Mean\ \overline{x} = \frac{sum\ of\ all\ the\ observations}{number\ of\ observations} \qquad \left[\because 1 + 2 + 3 + \cdots . n = \frac{n(n+1)}{2}\right]$$

$$= \frac{\sum x_i}{n} = \frac{1 + 2 + 3 + \cdots . + n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{\cancel{n}(n+1)}{2\cancel{n}}$$

$$mean\ \overline{x} = \frac{\sum x_i}{n} = \frac{n+1}{2}$$

$$\sum {x_i}^2 = 1^2 + 2^2 + 3^2 + \cdots . n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$Variance:\ \sigma^2 = \frac{\sum {x_i}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\sigma^2 = \frac{\cancel{n}(n+1)(2n+1)}{6\cancel{n}} - \left[\frac{n+1}{2}\right]^2$$

$$= \frac{2n^2 + n + 2n + 1}{6} - \left(\frac{(n+1)^2}{2^2}\right) = \frac{2n^2 + 3n + 1}{6} - \left(\frac{n^2 + 1^2 + 2n \times 1}{4}\right)$$

$$= \frac{2n^2 + 3n + 1}{6} - \frac{n^2 + 1 + 2n}{4}$$

$$= \frac{2(2n^2 + 3n + 1) - 3(n^2 + 1 + 2n)}{12} = \frac{4n^2 + \cancel{6n} + 2 - 3n^2 - 3 - \cancel{6n}}{12}$$

$$\sigma^2 = \frac{n^2 - 1}{12} \quad \therefore variance\ \sigma^2 = \frac{n^2 - 1}{12}$$

Example 8.11: *48 students were asked to write the total number of hours per week they spent on watching television. with this information find the standard deviation of hours spent for watching television.*

x	6	7	8	9	10	12
y	3	6	9	8	5	4

$$\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

x_i	f_i	$d_i = x - A = x - 9$	${d_i}^2$	$f_i d_i$	$f_i {d_i}^2$
6	3	$6 - 9 = -3$	9	-9	27
7	6	$7 - 9 = -2$	4	-12	24
8	9	$8 - 9 = -1$	1	-9	9
9	13	$9 - 9 = 0$	0	0	0
$A =$ 10	8	$10 - 9 = 1$	1	8	8
11	5	$11 - 9 = 2$	4	10	20
12	4	$12 - 9 = 3$	9	12	36
	$N = 48$			$\sum f_i d_i = 0$	$\sum f_i {d_i}^2 = 124$

$$\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} =$$

$$\sqrt{\frac{124}{48} - \left(\frac{0}{48}\right)^2}$$

$$= \sqrt{2.8 - 0} = 1.66$$

$$\boxed{\sigma \simeq 1.6}$$

```
       1·66
  1  | 2.8
     | 1
 26  | 180
     | 156
 326 | 2400
     | 1956
```

Example 8.12: *The scored by the students in a slip test are given below:*

x	4	6	8	10	12
y	7	3	5	9	5

Find the standard deviation of their marks

Standard deviation: $\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$

x_i	f_i	$d_i = x - A = x - 8$	d_i	$f_i d_i$	$f_i {d_i}^2$
4	7	$4 - 8 = -4$	16	-28	112
6	3	$6 - 8 = -2$	4	-6	12
$A = 8$	5	$8 - 8 = 0$	0	0	0
10	9	$10 - 8 = 2$	4	18	36
12	5	$12 - 8 = 4$	16	20	80
	$N = 29$			$\sum f_i d_i = 4$	$\sum fd^2 = 240$

$$\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} = \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = \sqrt{\frac{240}{29} - \frac{16}{841}}$$

$$= \sqrt{\frac{6960 - 16}{841}} = \sqrt{\frac{6944}{841}} = \sqrt{8.25} \quad = 2.872$$

	2 · 87
2	8.25
	4
48	4 25
	384
567	4100
	3969

$\boxed{\sigma \simeq 2.87}$

Example 8.13*: *Marks of the students in a particular subject of a class are given below

Marks	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**
No. of student	**8**	**12**	**17**	**14**	**9**	**7**	**4**

Find its standard deviation.

$marks$	x_i	f_i	$d_i = x - A$ $d_i = x - 35$	$d_i = \frac{x-35}{10}$	d_i	${d_i}^2$	$f_i d_i$	$f_i {d_i}^2$
0 – 10	5	8	$5 - 35 = -30$	$= -30/10$	-3	9	-24	72
10 – 20	15	12	$15 - 35 = -20$	$= -20/10$	-2	4	-24	48
20 – 30	25	17	$25 - 35 = -10$	$= -10/10$	-1	1	-17	17
30 – 40	35 A	14	$35 - 35 = 0$	$= 0/10$	0	0	0	0
40 – 50	45	9	$45 - 35 = 10$	$= 10/10$	1	1	9	9
50 – 60	55	7	$55 - 35 = 20$	$= 20/10$	2	4	14	28
60 – 70	65	4	$65 - 35 = 30$	$= 30/10$	3	9	12	36
		$N = 71$					$\sum f_i d_i = -30$	$\sum fd^2 = 210$

$$\textbf{Standard deviation}: \boldsymbol{\sigma = c \times \sqrt{\frac{\sum f_i d_i{}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}}$$

$$= 10 \times \sqrt{\frac{210}{71} - \left(\frac{-30}{71}\right)^2} = 10 \times \sqrt{\frac{210}{71} - \frac{900}{5041}} = 10 \times \sqrt{\frac{210 \times 71 - 900}{5041}}$$

$$= 10 \times \sqrt{\frac{14910 - 900}{5041}} = 10 \times \sqrt{\frac{14010}{5041}} = 10 \times \sqrt{2.779}$$

$$= 10(1.667) = 1.667$$

$$\boxed{\sigma \simeq 1.667}$$

$$\begin{array}{r|l} & 2\cdot779 \\ \hline 5041 & 14010 \\ & 10082 \\ \hline & 39280 \\ & 35287 \\ \hline & 39930 \\ & 35287 \\ \hline & 46430 \end{array}$$

$$\begin{array}{r|l} & 1\cdot667 \\ \hline 1 & 2.779 \\ & 1 \\ \hline 26 & 177 \\ & 156 \\ \hline 326 & 2190 \\ & 1956 \\ \hline 3327 & 23400 \\ & 23289 \end{array}$$

Example 8.14: The mean and standard deviation of 1 observations are found to be 10 to 5 respectively. On rechecking it was fouund that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23?

$$mean\ \overline{x} = 10\ , standard\ deviation\ \sigma = 5\ , n = 15$$

$$\overline{x} = 10 \Rightarrow \frac{\sum x}{n} = 10 \Rightarrow \frac{\sum x}{15} = 10$$

$$\sum x = 10 \times 15 \Rightarrow \sum x = 150$$

$$wrong\ observation\ value = 8,\ \ correct\ observation\ value = 23$$

$$correct\ total \sum x = 150 - 8 + 23 = 173 - 8 = 165$$

$$corrected\ mean\ \overline{x} = \frac{\sum x}{n} = \frac{165}{15} = 11$$

$$Standard\ deviation: \sigma = \sqrt{\frac{\sum x_i{}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$In\ correct(\sigma)\ 5 = \sqrt{\frac{\sum x^2}{15} - (10)^2} \Rightarrow 5^2 = \frac{\sum x^2}{15} - 100$$

$$25 = \frac{\sum x^2 - 1500}{15} \Rightarrow 25 \times 15 = \sum x^2 - 1500 \Rightarrow 375 = \sum x^2 - 1500$$

$$375 + 1500 = \sum x^2 \Rightarrow 1875 = \sum x^2 \Rightarrow In\ correct \sum x^2 = 1875$$

$$correct \sum x^2 = 1875 - 8^2 + 23^2 \Rightarrow \sum x^2 = 1875 - 64 + 529$$

$$\boxed{\sum x^2 = 2340}$$

$$\begin{array}{r|l} & 5 \cdot 91 \\ \hline 5 & 35 \\ & 25 \\ \hline 109 & 1000 \\ & 981 \\ \hline 1181 & 1900 \\ & 1181 \\ \hline & 719 \end{array}$$

$$correct\ standard\ deviation = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{2340}{15} - (11)^2} = \sqrt{\frac{2340}{15} - 121}$$

$$= \sqrt{156 - 121} = \sqrt{35} = 5.916$$

$$\boxed{correct\ standard\ deviation \simeq 5.9}$$

1. *Find the range and coefficient of range of the following data:*

***i*) 63, 89, 98, 125, 79, 108, 117, 68**

Ascending order: 63, 68, 79, 89, 98, 108, 117, 125

$Range = L - S = 125 - 63$

$Range = 62$

$$Co-efficient\ of\ Range = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{188}$$

$$\begin{array}{r|l} & 0.3297 \\ \hline 188 & 620 \\ & 564 \\ \hline & 560 \\ & 376 \\ \hline & 1840 \\ & 1692 \\ \hline & 1480 \\ & 1316 \\ \hline & 1640 \end{array}$$

$Co-efficient\ of\ Range = 0.329$

$Co-efficient\ of\ Range = 0.33$

***ii*) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8**

Ascending order: 13.6, 18.9, 29.8, 38.4, 43.5, 61.4

$Range = L - S = 61.4 - 13.6 = 47.8$

$$Co-efficient\ of\ Range = \frac{L-S}{L+S}$$

$$= \frac{61.4-13.6}{61.4+13.6} = \frac{47.8 \times 10}{7 \times 10} = \frac{478}{750} = 0.637$$

$$\begin{array}{r|l} & 0.637 \\ \hline 750 & 4780 \\ & 4500 \\ \hline & 2800 \\ & 2250 \\ \hline & 5500 \\ & 3250 \\ \hline \end{array}$$

$Co-efficient\ of\ Range = 0.64$

2. *If the range and the smallest value of a set of data are* 36.8 *and* 13.4 *respectively, then find the largest value.*

$Range = 36.8$, $smallest\ value = 13.4$

$Range = L - S$

$36.8 = L - 13.4$

$36.8 + 13.4 = L \Rightarrow L = 50.2$

3. *Calculate the range of the following data*:

Income	400 – 450	450 – 500	500 – 550	550 – 600	600 – 650
No. of workers	8	12	30	21	6

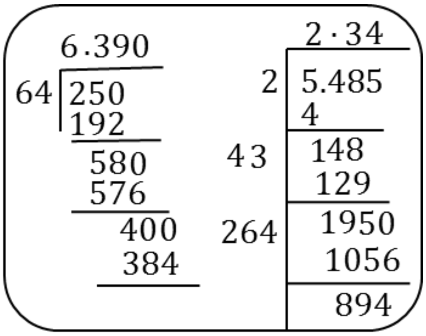
$Range = L - S = 650 - 400 = 250$

4. *A teacher asked the students to complete 60 pages of a record notebook. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be complted by them.*

complete the pages 32, 35, 37, 30, 33, 36, 35, 37

Ascending order: 30, 32, 33, 35, 35, 36, 37, 37

x_i	$d_i = x - A = x - 35$	${d_i}^2$
30	$30 - 35 = -5$	25
32	$32 - 35 = -3$	9
33	$33 - 35 = -2$	4
35 A	$35 - 35 = 0$	0
35	$35 - 35 = 0$	0
36	$36 - 35 = 1$	1
37	$37 - 35 = 2$	4
37	$37 - 35 = 2$	4
	$\sum d_i = -5$	$\sum {d_i}^2 = 47$



$$\sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \left(\frac{-5}{8}\right)^2} = \sqrt{5.875 - \left(\frac{25}{64}\right)} = \sqrt{5.875 - 6.390} = \sqrt{5.485}$$

$$\boxed{\sigma \simeq 2.34}$$

5. *Find the rariance and standard deviation of the wages of 9 workers given below*: 310, 290, 320,280, 300, 290, 320, 310, 280.

Given data: 310, 290, 320,280, 300, 290, 320, 310, 280

Ascending order: 280, 280, 290, 290, 300, 310, 310, 320, 320

$$Standard\ deviation\ \sigma = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$$

$$d_i = \frac{x - A}{c}\ \ where\ c = 10$$

x_i	$d_i = x - A = x - 300$	$d_i = \frac{x-A}{c} = \frac{x-300}{10}$	${d_i}^2$
280	$280 - 300 = -20$	$= -20/10 = -2$	4
280	$280 - 300 = -20$	$= -20/10 = -2$	4
290	$290 - 300 = -10$	$= -10/10 = -1$	1
290	$290 - 300 = -10$	$= -10/10 = -1$	1
300 A	$300 - 300 = 0$	$= 0/10 \ = 0$	0
310	$310 - 300 = 10$	$= 10/10 = 1$	1
310	$310 - 300 = 10$	$= 10/10 = 1$	1
320	$320 - 300 = 20$	$= 20/10 = 2$	4
320	$320 - 300 = 20$	$= 20/10 = 2$	4
	$\sum d_i = 0$		$\sum {d_i}^2$

$= 20$

$$\sigma = 10 \times \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = 10 \times \sqrt{\frac{20}{9} - \left(\frac{0}{9}\right)^2}$$

$$= 10 \times \sqrt{\frac{20}{9} - 0} = 10 \times \sqrt{\frac{20}{9}}$$

$$= 10\sqrt{2.222} = 10 \times 1.491$$

$$\boxed{\sigma \simeq 14.91}$$

```
         1·49                 2·222
    1 | 2.222             9 | 20
      | 1                     18
   24 | 122                   20
      |  96                   18
  289 | 2622                    20
      | 2601                    18
      |   21
```

$standard\ deviation\ \sigma = 10\sqrt{2.222}$

$variance:\ \sigma^2 = \left(10\sqrt{2.222}\right)^2 = 100 \times 2.222$

$variance\ \sigma^2 = 222.22$

6. A wall clock strikes the bell once at 10′clock, 2 times at 2 o′clock, 3 times at 3o′clock and soon .how many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

wall clock strickes the bell in 12 *hours* $1+2+3+4+\cdots.+12$

wall clock strickes the bell in 24 *hours* $2[1+2+3+4+\cdots+12]$

$$\textbf{Standard deviation } \sigma \textbf{ for n natural numbers} = \sqrt{\frac{n^2-1}{12}}$$

$$\therefore \text{The required standard deviation } \sigma = 2\left[\sqrt{\frac{n^2-1}{12}}\right]_{n=12}$$

$$= 2\sqrt{\frac{12^2-1}{12}} = 2\sqrt{\frac{144-1}{12}} = 2\sqrt{\frac{143}{12}} = 2\sqrt{11.91}$$

$$= 2 \times 3.45 = 6.86$$

$$\sigma \simeq 6.9$$

3·45
3 | 11.91
9
64 | 291
256
685 | 3500
3425
75

11.91
12 | 143
12
23
12
110
108
20
12
8

7. *Find the standard deviation of first* 21 *natural numbers*.

$$n = 21$$

$$\text{Standard deviation } \sigma \text{ for n natural numbers } = \sqrt{\frac{n^2-1}{12}}$$

$$= \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}}$$

$$= \sqrt{\frac{440}{12}} = \sqrt{36.666} = \sqrt{36.67} = 6.056$$

Standard deviation σ *for n natural numbers*

$$\sigma \simeq 6.05$$

6·05
6 | 36.67
36
120 | 67
00
1205 | 6700
6025
675

36.66
12 | 440
36
80
72
80
72
80

8. *If the standard deviation of a data is* 4. *and if each value of the data is decreased by* 5, *then find the new sandard deviation*.

Standard deviation $\sigma = 4.5$

Each data is decreased by 5

we have the standard deviation of the collection of data remains unchanged when each value is add (or) subtracted by a constant,

$\therefore$ *The new standard deviation after reducing from each data is* $\sigma = 4.5$

9. *If the standard deviation of a data is* 3.6 *and each value of the data is divided by* 3, *then find the new variance and new standard deviation*.

The standard deviation $\sigma = 3.6$

Also given that each value of the data is divided by 3.

If each value of the data is divided by a constant k, then the standard

$If\ each\ value\ of\ the\ data\ is\ divided\ by\ a\ constant\ k, then\ the\ standard$

$$\therefore The\ new\ standard\ deviation\ \sigma = \frac{3.6}{3} \Longrightarrow \sigma = 1.2$$

$$variance = (standard\ deviation)^2 = (1.2\,)^2 = 1.44$$

10. $\boldsymbol{The\ rainful\ recorded\ invarious\ places\ of\ five\ districts\ in\ a\ week\ are\ given\ below}$:

$\boldsymbol{Rainfall\ (in\ mm)}$	**45**	**50**	**55**	**60**	**65**	**70**
$\boldsymbol{Number\ of\ places}$	**5**	**13**	**4**	**9**	**5**	**4**

$\boldsymbol{Find\ its\ standard\ deviation}$:

$d_i = x - A\ \ \ where\ A\ as\ assumed\ mean$

x_i	f_i	$d_i = x - A = x - 55$	${d_i}^2$	$f_i d_i$	$f_i {d_i}^2$
45	5	$45 - 55 = -10$	100	-50	500
50	1 3	$50 - 55 = -5$	25	-65	325
55 A	4	$55 - 55 = 0$	0	0	0
60	9	$60 - 55 = 5$	25	45	225
65	5	$65 - 55 = 10$	100	50	500
70	4	$70 - 55 = 15$	225	60	900
	N= 40			$\sum f_i d_i = 40$	$\sum fd^2 = 2450$

$$Standard\ deviation\ \sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{2450}{40} - \left(\frac{40}{40}\right)^2} = \sqrt{\frac{2450}{40} - (1)^2} = \sqrt{\frac{245}{4} - 1} = \sqrt{61.25 - 1} = \sqrt{60.25}$$

$\boxed{\sigma \simeq 7.76}$

11. $\boldsymbol{In\ a\ studey\ about\ viral\ fever, the\ number\ of\ people\ affected\ in\ a\ town\ were\ noted\ as.}$

$\boldsymbol{Age\ in\ yours}$	$\boldsymbol{0 - 10}$	$\boldsymbol{10 - 20}$	$\boldsymbol{20 - 30}$	$\boldsymbol{30 - 40}$	$\boldsymbol{40 - 50}$	$\boldsymbol{50 - 60}$	$\boldsymbol{60 - 70}$
$\boldsymbol{No. of\ people\ affected}$	**3**	**5**	**16**	**18**	**12**	**7**	**4**

$\boldsymbol{Find\ its\ standard\ deviation.}$

$$d_i = \frac{x - A}{c}$$

age in years	x_i	f_i	$d_i = x - A$ $d_i = x - 35$	$d_i = \frac{x-35}{10}$	d_i	d_i^2	$f_i d_i$	$f_i {d_i}^2$
0 – 10	5	3	5 – 35 = –30	= –30/10	–3	9	–9	27
10 – 20	15	5	15 – 35 = –20	= –20/10	–2	4	–10	20
20 – 30	25	16	25 – 35 = –10	= –10/10	–1	1	–16	16
30 – 40	$\underset{A}{35}$	18	35 – 35 = 0	= 0/10	0	0	0	0
40 – 50	45	12	45 – 35 = 10	= 10/10	1	1	12	12
50 – 60	55	7	55 – 35 = 20	= 20/10	2	4	14	28
60 – 70	65	4	65 – 35 = 30	= 30/10	3	9	12	139

$N = 65$

$\sum f_i d_i = 3$ $\quad \sum f d^2 = 139$

$$\sigma = c \times \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} = \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2}$$

$$= 10 \times \sqrt{\frac{139}{65} - \frac{9}{4225}} = 10 \times \sqrt{\frac{139 \times 65 - 9}{4225}} = 10 \times \sqrt{\frac{9035 - 9}{4225}}$$

$$= 10 \times \sqrt{\frac{9026}{4225}} = 10\sqrt{2.136} = 10 \times 1.46 = 14.6$$

```
         2.136
4225 | 9026
       8450
       -----
        5760
        4225
       ------
        15350
        12675
        -----
         26750
         25350
         -----
          1400
```

$\boxed{\sigma \simeq 14.6}$

12. *The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.*

***Diamete* (cm)**	**21 – 24**	**25 – 28**	**29 – 32**	**33 – 36**	**37 – 40**	**41 – 44**
No. of plates	**15**	**18**	**20**	**16**	**8**	**7**

Find its standard deviation.

Standard deviation : $\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$

class Interval	x_i	f_i	$d_i = x - A = x - 34.5$	${d_i}^2$	$f_i d_i$	$f_i {d_i}^2$
$21 - 24$	22.5	15	$22.5 - 34.5 = -12$	144	-180	2160
$25 - 28$	26.5	18	$26.5 - 34.5 = -8$	64	-144	1152
$29 - 32$	30.5	20	$30.5 - 34.5 = -4$	16	-80	320
$33 - 36$	34.5	16	$34.5 - 34.5 = 0$	0	0	0
$37 - 40$	38.5	8	$38.5 - 34.5 = 4$	16	32	128
$41 - 44$	42.5	7	$42.5 - 34.5 = 8$	64	56	448
		$N = 84$			$\sum f_i d_i = -316$	$\sum fd^2 = 4208$

$$\sigma = \sqrt{\frac{\sum f_i {d_i}^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} = \sqrt{\frac{\cancel{4208}}{\cancel{84}} - \left(\frac{3\cancel{0}}{6\cancel{5}}\right)^2} = \sqrt{\frac{1052}{21} - \left(\frac{79}{21}\right)^2}$$

$$= \sqrt{50.095 - (3.76)^2} \quad = \sqrt{50.095 - 14.137}$$

$$= \sqrt{35.958} = \sqrt{35.96} = 5.992$$

$$\boxed{\sigma \simeq 6}$$

13. ***The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.***

Time taken (studs.)	$8.5 - 9.5$	$9.5 - 10.5$	$10.5 - 11.5$	$11.5 - 12.5$	$12.5 - 13.5$
No. of Students	6	8	17	10	9

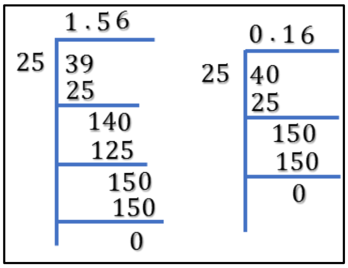
Time Taken	Mid Value xi	Number of Students fi	$di = x - A = x = 11$	d^2	$fidi$	$fidi^2$
$8.5 - 9.5$	9	6	$9 - 11 = -2$	4	-12	-12
$9.5 - 10.5$	10	8	$10 - 11 = -1$	1	-8	8
$10.5 - 11.5$	11	17	$11 - 11 = 0$	0	0	0
$11.5 - 12.5$	12	10	$12 - 11 = 1$	1	10	10
$12.5 - 13.5$	13	9	$13 - 11 = 2$	4	18	36
		$N = 50$			$\Sigma fidi = 8$	$\Sigma fidi^2 = 78$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma f_i {d_i}^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2} = \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{\frac{39}{25} - \left(\frac{4}{25}\right)^2} = \sqrt{1.56 - (0.16)^2} = \sqrt{1.56 - 0.025}$$

$$= \sqrt{1.56 - 0.03} = \sqrt{1.53} = 1.236$$

$$\sigma \simeq 1.24$$



14. $\boldsymbol{For\ a\ group\ of\ 100\ candidates\ the\ mean\ and\ standard\ deviation\ of\ their\ marks\ were\ found\ to\ be\ 60\ and\ 15\ respectively. Later\ on\ it\ was\ found\ that\ the\ scrores\ 45\ and\ 72\ were\ wrongly\ entered\ as\ 40\ and\ 27. Find\ the\ correct\ mean\ and\ standard\ deviation}$:

$Number\ of\ candidates = 100,\ n = 100,\ mean\ \overline{x} = 60,$

$Standard\ deviation\ \sigma = 15$

$$\overline{x} = \frac{\Sigma x}{n} \Longrightarrow 60 = \frac{\Sigma x}{100} \Longrightarrow 60 \times 100 = \Sigma x \Longrightarrow \Sigma x = 6000$$

$Correct\ total(\Sigma x) = 6000 + 45 + 72 - 40 - 27$

$\Sigma x = 6000 + 117 - 67 = 6000 + 50 = 6050$

$Correct: \Sigma x = 6050$

$$Correct\ mean = \frac{Correct\Sigma x}{n} = \frac{6050}{100}$$

$Correct\ mean\ \overline{x} = 60.5$

$$Standard\ deviation\ \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$15 = \sqrt{\frac{\Sigma x^2}{100} - \left(\frac{6000}{100}\right)^2} \Longrightarrow 15 = \sqrt{\frac{\Sigma x^2}{100} - (60)^2} \Longrightarrow 15 = \sqrt{\frac{\Sigma^2}{100} - 3600}$$

$$15^2 = \frac{\Sigma x^2}{100} - 3600 \Longrightarrow 225 + 3600 = \frac{\Sigma x^2}{100}$$

$$3825 = \frac{\Sigma x^2}{100} \Longrightarrow 3825 \times 100 = \Sigma x^2 \Longrightarrow 382500 = \Sigma x^2$$

$Incorrect: \Sigma x^2 = 382500$

$$\begin{aligned} Correct : \Sigma x^2 &= 382500 + 45^2 + 72^2 - 40^2 - 27^2 \\ &= 382500 + 2025 + 5184 - 1600 - 729 \\ &= 389709 - 2329 \end{aligned}$$

$Correct: \Sigma x^2 = 387380$

$$Correct\ Standard\ deviation\ \sigma = \sqrt{\frac{correct\ \Sigma x^2}{n} - \left(\frac{correct\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{387380}{100} - (60.5)^2} = \sqrt{3873.8 - 3660.25} = \sqrt{213.55} = \sqrt{213.6}$$

$$\sigma = \sqrt{2.136 \times 100} = 10 \times \sqrt{2.136} = 10 \times 1.462 = 14.62$$

$$\sigma \simeq 14.62$$

15. *The mean and variance of seven onservations are 8 and 16 respectively. If five of these are 2,4, 10, 12 and 14, then find the remaining two observations.*

$Mean\ \overline{x} = 8,\ \sigma^2 = 16,\ n = 7$

$Two\ observation = a, b$

$Five\ of\ these\ are\ 2, 4, 10, 12, 14$

$$\overline{x} = \frac{\Sigma x}{n}$$

$$8 = \frac{2 + 4 + 10 + 12 + 14 + a + b}{7} \Longrightarrow 8 = \frac{42 + a + b}{7}$$

$$8 \times 7 = 42 + a + b \Longrightarrow 56 - 42 = a + b \Longrightarrow 14 = a + b$$

$$a + b = 14$$

$The\ given\ numbers\ are\ 2, 4, a, b, 10, 12, 14.$

$$a = 6\ b = 8$$

$\therefore Two\ observation\ is\ 6\ and\ 8.$

EXERCISE 8.2

Example 8.16: The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	**Weight**
Mean	$155\ cm$	$46.50\ Kg^2$
Variance	$72.25\ cm^2$	$28.09\ Kg^2$

First data − Height:

$mean\ \overline{x_1} = 155\ cm, Variance\ {\sigma_1}^2 = 72.25\ cm^2$

$\sigma_1 = \sqrt{72.25}, \sigma_1 = 8.5$

$$\begin{array}{r|l} & 8.5 \\ \hline 8 & 72.25 \\ & 64 \\ \hline 165 & 825 \\ & 825 \\ \hline & 0 \end{array}$$

$$\begin{array}{r|l} & 5.483 \\ \hline 31 & 170 \\ & 155 \\ \hline & 150 \\ & 124 \\ \hline & 260 \\ & 248 \\ \hline & 120 \\ & 97 \\ \hline & 23 \end{array}$$

$$Coefficient\ of\ variance\ (CV_1) = \frac{\sigma_1}{\overline{x_1}} \times 100\%$$

$$= \frac{8.5}{155} \times 100\% = \frac{\overset{170}{\cancel{850}}}{\underset{31}{\cancel{155}}} = 5.483$$

$Coefficient\ of\ variance\ (CV_1) = 5.483\%$

Second data − weight

$mean\ \overline{x_2} = 46.50\ Kg^2, Variance\ {\sigma_2}^2 = 28.09\ Kg^2$

$\sigma_2 = \sqrt{28.09} \Rightarrow \sigma_2 = 5.301 \Rightarrow \sigma_2 = 5.3$

$$\begin{array}{r|l} & 5.3 \\ \hline 5 & 28.09 \\ & 25 \\ \hline 103 & 309 \\ & 309 \\ \hline & 0 \end{array}$$

$$Coefficient\ of\ Variance(CV_2) = \frac{\sigma_2}{\overline{x_2}} \times 100\% = \frac{5.3}{46.5} \times 100\%$$

$$= \frac{530}{46.5} = \frac{530 \times 10}{46.5 \times 10} = \frac{\overset{1060}{\cancel{5300}}}{\underset{93}{\cancel{465}}}$$

$$\begin{array}{r|l} & 11.397 \\ \hline 93 & 1060 \\ & 1023 \\ \hline & 370 \\ & 279 \\ \hline & 910 \\ & 837 \\ \hline & 730 \\ & 651 \\ \hline & 79 \end{array}$$

$$= 11.397 = 11.40\%$$

$\therefore CV_1 = 5.48\% \quad CV_2 = 11.40\%$

$Since\ CV_1 < CV_2$

$\therefore$ *The weight of the students is more varying then the height.*

Example 8.17: The consumption of number of guava and orange on a particular week by a family are given below.

No. of Guavas	3	5	6	4	3	5	4
No. of Oranges	1	3	7	9	2	6	2

Which fruit is consistently consumed by the family?

First we find the coefficient of variation for guavas and oranges separately.

$Number\ of\ guavas, n = 7$

$$mean\ \overline{x_1} = \frac{3+5+6+4+3+5+4}{7} = \frac{30}{7}$$

4.285
7 | 30
28
20
14
60
56
40
35
5

$$\therefore \frac{\Sigma x_1}{n} = \overline{x_1} = 4.29$$

x	x^2
3	9
5	25
6	36
4	16
3	9
5	25
4	16
$\sum x_1 = 30$	$\sum {x_1}^2 = 136$

19.428
7 | 136
7
66
63
30
28
20
14
60
36
24

1·01
1 | 1.03
1
21 | 30
21
9

$$\sigma_1 = \sqrt{\frac{\Sigma x_1^2}{n} - \left(\frac{\Sigma x_1}{n}\right)^2} = \sqrt{\frac{136}{7} - (4.29)^2} = \sqrt{19.43 - 18.40} = \sqrt{1.03}$$

$$\boxed{\sigma_1 \backsimeq 1.01}$$

$$Coefficient\ of\ variance(CV_1) = \frac{\sigma_1}{\overline{x_1}} \times 100\% = \frac{1.01}{4.29} \times 100$$

$$= \frac{101 \times 100}{4.29 \times 100} = \frac{10100}{429}$$

23.54
429 | 10100
858
1520
1287
2330
2145
1850
1696
154

$$Coefficient\ of\ variance(CV_1) = 23.54$$

$Second\ data - Oranges$

x	x^2
1	1
3	9
7	49
9	81
2	4
6	36
2	4
$\sum x_2 = 30$	$\sum {x_2}^2 = 184$

$$\sigma_2 = \sqrt{\frac{\Sigma x_2^2}{n} - \left(\frac{\Sigma x_2}{n}\right)^2} = \sqrt{\frac{184}{7} - (4.29)^2}$$

$$= \sqrt{26.29 - 18.40} = \sqrt{7.89}$$

$$\boxed{\sigma_2 \backsimeq 2.81}$$

2·808
2 | 7.89
4
48 | 389
384
560 | 500
000
5608 | 50000

$$Coefficient\ of\ variance(C.V_2) = \frac{\sigma_2}{\bar{x}_2} \times 100\%$$

$$= \frac{2.81}{4.29} \times 100 = \frac{281 \times 100}{4.29 \times 100} = \frac{28100}{429}$$

$(C.V_2) = 65.50\%$

$C.V_1 = 23.54\%, C.V_2 = 65.50\%.$ $Since, C.V_1 < C.V_2.$

We can conclude that the consumption of guvas is more than oranges.

$$\begin{array}{r|l} & 65.504 \\ 429 & 28100 \\ & \underline{2574\downarrow} \\ & 2360 \\ & \underline{2145} \\ & 2330 \\ & \underline{2145} \\ & 1850 \\ & \underline{1716} \\ & 134 \end{array}$$

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

$\sigma = 6.5,\ \ mean\bar{x} = 12.5$

$$Coefficient\ of\ variation\ = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{6.5}{12.5} \times 100 = \frac{650}{12.5} = \frac{650 \times 10}{12.5 \times 10} = \frac{6500}{125}$$

$Coefficient\ of\ variation = 52\%$

$$\begin{array}{r|l} & 52 \\ 125 & 6500 \\ & \underline{625} \\ & 250 \\ & \underline{250} \\ & 0 \end{array}$$

2. The standard deviation and coefficient of variation of a data are 12 and 25.6 respectively. Find the value of mean.

$\sigma = 1.2\ , C.V = 25.6\ , \bar{x} = ?$

$$Coefficient\ of\ variation(C.V) = \frac{\sigma}{x} \times 100$$

$$25.6 = \frac{1.2}{x} \times 100 \implies 25.6 = \frac{120}{x}$$

$$\bar{x}(25.6) = 120 \implies \bar{x} = \frac{120 \times 10}{25.6 \times 10}$$

$$\bar{x} = \frac{1200}{256} \implies \bar{x} = 4.687$$

$The\ mean\ \bar{x} = 4.69$

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

$mean\ \bar{x} = 15\ , C.V = 48\ , \sigma = ?$

$$Co-efficient\ of\ variance = \frac{\sigma}{x} \times 100\%$$

$$48 = \frac{\sigma}{15} \times 100 \implies 48 \times 15 = \sigma \times 100$$

$$\frac{\overset{12}{\cancel{48}} \times \overset{3}{\cancel{15}}}{\cancel{100}_{20\ 5}} = \sigma \implies \frac{12 \times 3}{5} = \frac{36}{5} \implies \boxed{\sigma = 7.2}$$

$$\begin{array}{r|l} & 7.2 \\ 5 & 36 \\ & \underline{35} \\ & 10 \\ & \underline{10} \\ & 0 \end{array}$$

4. $\boldsymbol{If\ n = 5, \bar{x} = 6, \Sigma x^2 = 765, then\ calculate\ the\ coefficient\ of\ variation.}$

$$n = 5\ , \bar{x} = 6 = \frac{\Sigma x}{n}\ \ , \Sigma x^2 = 765\ , \sigma = ?$$

$$Standard\ deviation\ \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{153 - 36} \quad = \sqrt{117}$$

$$= 10.817202$$

$$\sigma = 10.817$$

$$Coefficient\ of\ variance = \frac{\sigma}{x} \times 100\%$$

$$= \frac{10.817}{6} \times 100 = \frac{1081.7}{6}$$

$$C.V = 180.28\%$$

	10.81
2	117
	1
20	17
	00
208	1700
	1664
2161	3600
	2161

	180.28
6	1081.7
	6
	48
	48
	17
	12
	50
	48
	2

5. $\boldsymbol{Find\ the\ coefficient\ of\ variation\ of\ 24, 26, 33, 37, 29, 31.}$

$$Co - efficient\ of\ variance = \frac{\sigma}{x} \times 100\%$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{24 + 26 + 33 + 37 + 29 + 31}{6} = \frac{180}{6}$$

$$\boxed{\bar{x} = 30}$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\frac{\Sigma x^2}{n} = \frac{24^2 + 26^2 + 33^2 + 37^2 + 29^2 + 31^2}{6}$$

$$= \frac{576 + 676 + 1089 + 1369 + 841 + 961}{6} \quad \Longrightarrow \frac{\Sigma x^2}{n} = \frac{5512}{6}$$

$$\frac{\Sigma x^2}{n} = 918.666$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{918.67 - (30)^2} \quad = \sqrt{918.67 - 900} \quad = \sqrt{18.67}$$

$$\sigma = 4.32 \ \Longrightarrow \sigma \simeq 4.32$$

$$Coefficient\ of\ variation(C.V) = \frac{\sigma}{x} \times 100\%$$

$$= \frac{4.22}{30} \times 100 = \frac{432}{30}$$

$$(C.V) = 144\%$$

6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 48, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

The given data 38, 40, 47, 44, 46, 43, 49, 53

$$\frac{\Sigma x}{n} = \frac{38+40+47+44+46+43+49+53}{8} = \frac{360}{8}$$

$$\boxed{\frac{\Sigma x}{n} = 45}$$

$$\frac{\Sigma x^2}{n} = \frac{38^2+40^2+47^2+44^2+46^2+43^2+49^2+53^2}{8}$$

$$= \frac{1444+1600+2209+1936+2116+1849+2401+2809}{8}$$

$$\frac{\Sigma x^2}{n} = \frac{16364}{8} \Rightarrow \frac{\Sigma x^2}{n} = 2045.5$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{2045.5-(45)^2} \quad = \sqrt{2045.5-2025}$$

$$= \sqrt{20.5} \Longrightarrow \sigma \simeq 4.528 \Longrightarrow \sigma \simeq 4.53$$

$$C.V = \frac{\sigma}{x} \times 100$$

$$= \frac{4.53}{45} \times 100 = \frac{\overset{151}{\cancel{453}}}{\underset{15}{\cancel{45}}} = 10.066 \quad C.V \simeq 10.07$$

Coefficient variation $(C.V) = 10.07$

2045·5
8 | 16364
16
036
32
44
40
40
40
0

4·52
4 | 20.5
16
85 | 450
425
902 | 2500
1804

10.06
15 | 151
15
100
90
10

7. The total marks scored by two students sathya and vidhya in 5 subject are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

First data – Sathya: $\sigma = 4.6$

Total mark by Sathya $\Sigma x = 460$, $n = 5$

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \frac{\overset{92}{\cancel{460}}}{\underset{1}{\cancel{5}}}$$

$$\bar{x} = 92$$

Coefficient of Variation $(c.v_1) = \frac{\sigma}{x} \times 100 = \frac{4.6}{92} \times 100\% = \frac{460}{92}$

$$c.v_1 = 5\%$$

Second data − Vidhya

$\sigma = 2.4$

$Total\ mark\ by\ Vidhya\ \Sigma x = 480\ , n = 5$

$$\bar{x} = \frac{\Sigma x}{n} \implies \bar{x} = \frac{480}{5} \implies \bar{x} = 96$$

$$Coefficient\ of\ variance\ c.v_2 = \frac{\sigma}{x} \times 100\%$$

$$= \frac{2.4}{96} \times 100 = \frac{240}{96}$$

$$CV_2 = \frac{5}{2} \Rightarrow CV_2 = 2.5\%$$

Vidhya is more consistent. The data with lesser C.V is more consistent.

8. ***The mean standard deviation of marks obtained by 40 students of a class in three subjects mathematics science and Social science are given below***

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

which of three subject shows highest variation and which shows lowest variation in marks?

$Mathematics$: $Mean\ \bar{x} = 56\ , \sigma = 12$

$$Coefficient\ of\ variance\ (C.V) = \frac{\sigma}{x} \times 100\%$$

$$= \frac{12}{56} \times 100 = \frac{1200}{56} = \frac{150}{7}$$

21·4 28
7 | 150
14
10
7
30
28
20
14
60
58
2

$Coefficient\ of\ variance (C.V) = 21.428$

Science: $Mean\ \bar{x} = 65\ , \sigma = 14$

$$C.V = \frac{\sigma}{x} \times 100$$

$$= \frac{14}{65} \times 100 = \frac{1400}{65} = \frac{280}{13}$$

$\sigma = 21.538$

21.538
13 | 280
26
20
13
70
65
50
39
110
104
6

Social science: $\bar{x} = 60\ , \sigma = 10$

$$C.V = \frac{\sigma}{x} \times 100\%$$

$$= \frac{10}{60} \times 100 = \frac{1000}{60} = \frac{50}{3}$$

16.666
3 | 50
48
20
18
20
18
20
18
2

$C.V = 16.67$

Heighest coefficient of variation in marks is science.

Lowest coefficient of variation in marks in social Science.

9. The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	**18**	**20**	**22**	**24**	**26**
Temperature of city B (in degree Celsius)	**11**	**14**	**15**	**17**	**18**

Find which city is more consistent in temperature changes?

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d_i^2}{N} - \left(\frac{\Sigma d_i}{N}\right)^2}$$

For the city A

Let the assumed mean $A = 22$

x_i	$d_i = x_i - A$ $d_i = x_i - 22$	d_i^2
18	-4	16
20	-2	4
22	0	0
24	2	4
26	4	16
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 40$

$$\sigma = \sqrt{\frac{40}{5} - (0)^2} = \sqrt{8} = 2.828 = 2.83$$

$$\text{Mean } \bar{x} = \frac{\Sigma x_i}{N} = \frac{18 + 20 + 22 + 24 + 26}{5} = \frac{110}{5} = 22$$

$$Co - efficient\ of\ variation\ C.V. = \frac{\sigma}{x} \times 100\% = \frac{2.83}{22} \times 100\% = 12.86\%$$

For the city B *Let the assumed mean* $B = 15$

x_i	$d_i = x_i - B$ $d_i = x_i - 15$	d_i^2
11	-4	16
14	-1	1
15	0	0
17	2	4
18	3	9
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 30$

$$Standard\ deviation\ \sigma = \sqrt{\frac{30}{5} - \left(\frac{0}{5}\right)^2} = \sqrt{6-0} = \sqrt{6} = 2.449$$

$$Mean\ \bar{x} = \frac{\Sigma x_i}{N} = \frac{11+14+15+17+18}{5} = \frac{75}{5} = 15.$$

$$\therefore Co - efficient\ of\ variation\ C.V = \frac{\sigma}{x} \times 100\%$$

$$= \frac{2.4495}{15} \times 100\% = \frac{244.95}{15}\% = 16.33\%$$

$C.V.of\ temperature\ of\ city\ A < C.V\ of\ temperature\ of\ city\ B.$

$\therefore City\ A\ is\ more\ consistent\ in\ temperature\ change.$

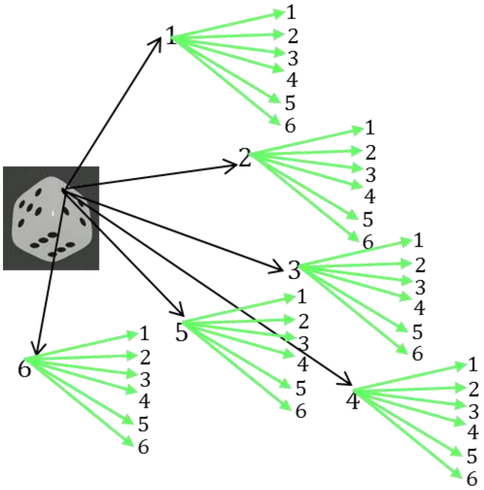
EXERCISE 8.3

Example 8.18: Express the sample space for rolling two dice using tree diagram

When we roll two dice, since each die contain 6 faces marked with 1,2,3,4,5,6 the tree diagram

Hence, the sample space can be written as

$$S = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$



Example 8.19: A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

The sample space $S = \{B_1, B_2, B_3, B_4, B_5, G_6, G_7, G_8, G_9\}$

Total number of balls $= 5 + 4 = 9$

$$n(S) = 9$$

(i) *Let A be the event of getting a Blue ball.*

$$A = \{B_1, B_2, B_3, B_4, B_5\}$$

$$n(A) = 5 \Longrightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

$$\boxed{P(A) = \frac{5}{9}}$$

ii) $\overline{A}$ *will be the event of not getting a blue ball.*

$$P(\overline{A}) = 1 - P(A)$$

$$= 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$$

$$P(\overline{A}) = \frac{4}{9}$$

Example 8.20: Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4, (ii) greater than 10, (iii) less than 13

$$\text{S} = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} \Longrightarrow n(S) = 36$$

(i) *Let A be the event of getting the sum of outcome values equal to* 4.

$$A = \{(1,3),(2,2),(3,1)\} \Longrightarrow n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} \Longrightarrow P(A) = \frac{3}{36}$$

$$\boxed{P(A) = \frac{3}{36}}$$

(ii) *Let B be the event of getting the sum of outcome is greater than* 10.

$$B = \{(5,6),(6,5),(6,6)\} \Longrightarrow n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)} \Longrightarrow P(A) = \frac{\cancel{3}^{\,1}}{\cancel{36}_{\,12}}$$

$$P(B) = \frac{1}{12}$$

(iii) *Let C be the event of getting the sum of outcomes less than* 13.

[*Since all outcomes have the sum value less than* 13.]

$$\text{C} = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} \Longrightarrow n(C) = 36$$

$$P(B) = \frac{n(C)}{n(S)} \Longrightarrow P(C) = \frac{\cancel{36}^{\,1}}{\cancel{36}} \Longrightarrow \quad P(C) = 1$$

Example 8.21: *Two coins are tossed together. What is the probability of getting different faces on the coins*?

When two coins are tossed together, the sample space is

$$S = \{HH, HT, TH, TT\} \Longrightarrow n(S) = 4$$

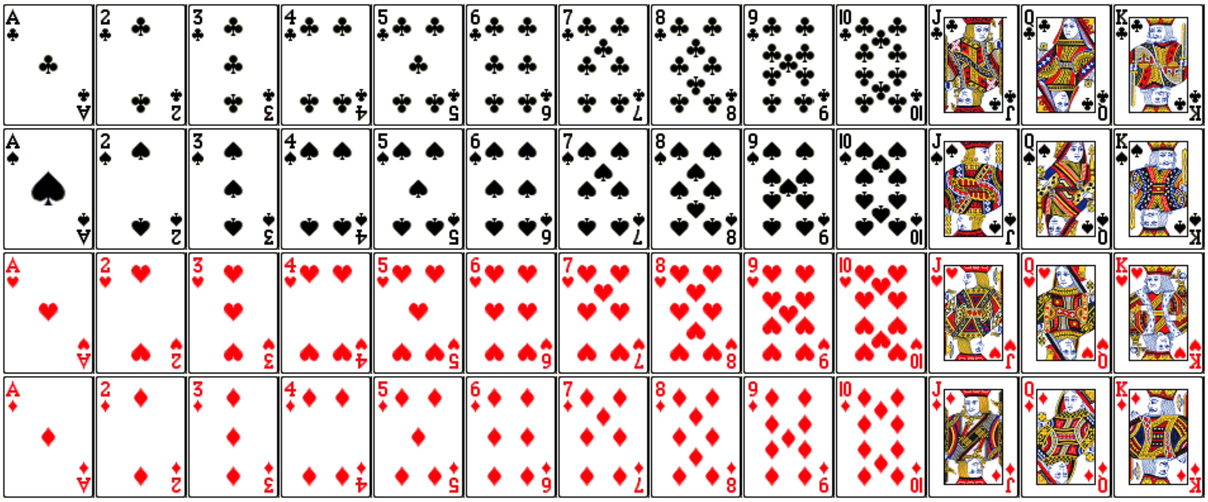
Let A be the event of getting different faces on the coins.

$$A = \{HT, TH\} \Longrightarrow n(A) = 2$$

Probability of getting different faces on the coins is

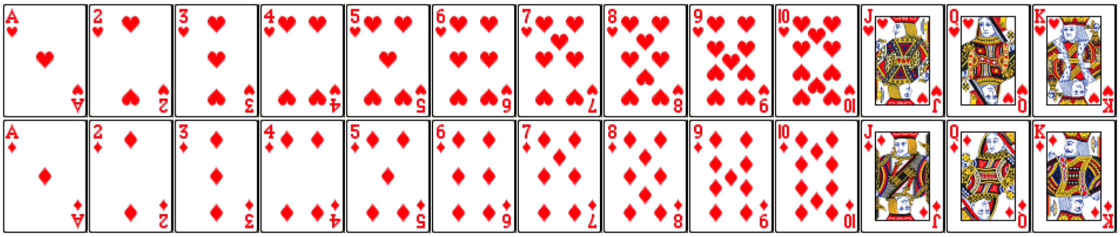
$$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{\cancel{2}\ 1}{\cancel{4}\ 2} \Rightarrow P(A) = \frac{1}{2}$$

Example 8.22: From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card



$$Total\ no.of\ cards: n(S) = 52$$

(i) Let A be the event of getting a red card.

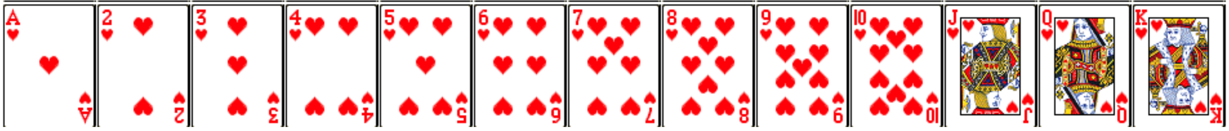


$$n(A) = 26$$

$$Probability\ of\ getting\ a\ red\ card\ is\ P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{\cancel{26}\ 1}{\cancel{52}\ 2}$$

$$P(A) = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card.



$$n\ (B) = 13$$

$$Probability\ of\ getting\ a\ heart\ card\ is\ P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{\cancel{13}\ 1}{\cancel{52}\ 4}$$

$$P(B) = \frac{1}{4}$$

(iii) Let C be the event of getting a red king card.

$$n(C) = 2$$

Probability of getting a red king card is $P(C) = \frac{n(C)}{n(S)} = \frac{\cancel{2}^{\,1}}{\cancel{52}_{\,26}}$

$$P(C) = \frac{1}{26}$$

(iv) Let D be the event of getting a face card.

The face cards are Jack (J), Queen (Q), and King (K).

$$n(D) = 12$$

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{\cancel{12}^{\,3}}{\cancel{52}_{\,13}} \Rightarrow P(D) = \frac{3}{13}$$

(v) Let E be the event of getting a number card.

The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

$$n(E) = 9$$

Probability of getting a number card is $P(E) = \frac{n(E)}{n(S)} = \frac{\cancel{36}^{\,9}}{\cancel{52}_{\,13}}$

$$\boxed{P(E) = \frac{9}{13}}$$

Example 8.23: What is the probability that a leap year selected at random will contain 53 saturdays.(Hint: 366 = 52 × 7 + 2)

Number of days in a leap year = 366 days.

366 days = 52 weeks + 2 days

52 weeks contain 52 Fridays

Remaining 2 days will have 7 possibilities

$$\begin{array}{r} 7)366\ (52 \\ \underline{35} \\ 16 \\ \underline{14} \\ 2 \end{array}$$

$S = \{(Sun - Mon, Mon - Tue, Tue - Wed, Wed - Thu, Thu - Fri, Fri - Sat, Sat - Sun)\}$

$$n(S) = 7$$

Let A be the event of getting one saturday in the remaining two days.

$$A = \{(Fri - Sat), (Sat - Sun)\} \Rightarrow n(A) = 2$$

$$p(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

The probability of getting 53 saturday in a leap year

Example 8.24: A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows anodd number and the coin shows a head.

"Sample space"

S = {(1,H),(1,T),(2,H), (2,T),(3,H),(3,T), (4,H), (4,T),(5,H),(5,T),(6,H),(6,T)}

$n\ (S) = 12$

Let A be the event of getting an odd number and a head.

A = {(1, H),(3, H), (5,H)}

$n\ (A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{3}}{\cancel{12}} \quad \frac{1}{4}$$

$$P(A) = \frac{1}{4}$$

Example 8.25: A bag contains 6 green balls, some black and red balls Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Number of green balls is $n\ (G) = 6$

Let number of red balls is $n\ (R) = x$

Number of black balls is $n\ (B)\ =\ 2x$

Total number of balls : $n(S) = 6 + x + 2x$

$$n(S) =\ 6 + 3x$$

$$P(G) = 3 \times P(R)$$

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x} \Longrightarrow 3x\ =\ 6$$

(*i*) *Number of black balls* $= 2 \times 2 = 4$

(*ii*) *Total number of balls* $= 6 + (3 \times 2) = 6 + 6 = 12$

Example 8.26: A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, … 12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

Sample space : $S\ =\ \{1,2,3,4,5,6,7,8,9,10,11,12\}$

$n(S) = 12$

(*i*) *Let A be the event of resting in 7*; $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) ***Let B be the event that the arrow will come to rest in a prime number***

$$B = \{2,3,5,7,11\}; n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

(iii) ***Let C be the event that arrow will come to rest in a composite number.***

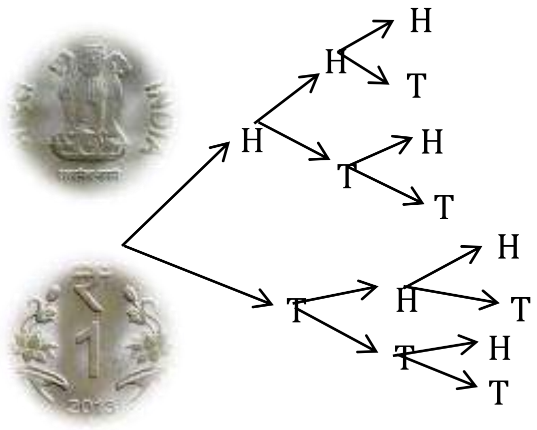
$$C = \{4,6,8,9,10,12\}; n\,(C) = 6$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{\cancel{6}^{\,1}}{\cancel{12}_{\,2}}$$

$$\boxed{P(C) = \frac{1}{2}}$$

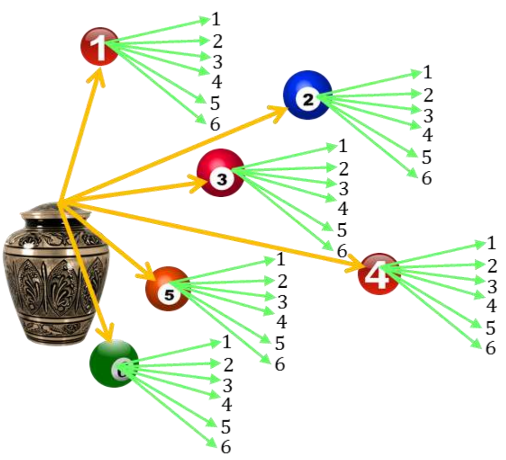
1. ***Write the sample space for tossing three coins using tree diagram.***

$$Sample\ space = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$$



2. ***Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).***

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$



3. *If A is an event of a random experiment such that*
$\boldsymbol{P(A): P(\overline{A}) = 17:15}$ ***and*** $\boldsymbol{n(S) = 640}$ ***then find*** $\boldsymbol{(i)\ P(A)\ \ (ii)\ n(A).}$

$$Given\ P(A) : P(\overline{A}) = 17:15 \implies \frac{P(A)}{P(\overline{A})} = \frac{17}{15}$$

$$\boxed{\begin{gathered} P(A) + P(\overline{A}) = 1 \\ P(\overline{A}) = 1 - P(A) \end{gathered}}$$

$$\frac{1 - P(\overline{A})}{P(\overline{A})} = \frac{17}{15}$$

$$15 - 15P(\overline{A}) = 17P(\overline{A}) \implies 32P(\overline{A}) = 15$$

$$P(\overline{A}) = \frac{15}{32} \implies P(A) = 1 - P(\overline{A})$$

$$P(A) = 1 - \frac{15}{32} \implies P(A) = \frac{32 - 15}{32} \implies P(A) = \frac{17}{32}$$

$$\frac{n(A)}{n(S)} = \frac{17}{32} \implies \frac{n(A)}{640} = \frac{17}{32} \implies n(A) = \frac{17}{32} \times 640$$

$$\boxed{n(A) = 340}$$

4. *A coin is tossed thrice. What is the probability of getting two consecutive tails?*

When a coin is tossed thrice,

$$S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(S) = 8$$

Let A bet the event of getting 2 tails continuously.

$$A = \{\text{HTT, TTH, TTT}\}$$

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} \implies P(A) = \frac{3}{8}$$

5. *At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?*

$$n(S) = 1000$$

*i) **Let A be the event of getting perfect squares between 500 and 1000***

$$A = \{23^2, 24^2, 25^2, 26^2 \ldots\ldots 31^2\}$$

$$n(A) = 9$$

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{9}{1000}$ *is the probability for the 1st player to win a prize.*

ii) When the card which was taken first is not replaced.

$$\mathrm{n}(S) = 999, \mathrm{n}(\mathrm{B}) = 8$$

$$P(B) = \frac{8}{999}$$

6. *A bag contains* 12 *blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If* 8 *more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.*

$No.\,red\ balls = x,\ No.\,white\ balls = 12$

i) Let A be the event of getting red balls

$$n(A) = x$$

$$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{x}{x+12}$$

ii) If* 8 *more red balls are added in the bag.

$$n(S) = x + 12 + 8 \Rightarrow n(S) = x + 20$$

$$No.\,red\ balls = x + 8$$

$$Now\ the\ probability = \frac{x+8}{x+20}$$

$$\frac{x+8}{x+20} = 2\left(\frac{x}{x+12}\right) \Rightarrow (x+8)(x+12) = 2x(x+20)$$

$$(x+8)(x+12) = 2x^2 + 40x \Rightarrow x^2 + 8x + 12x + 96 = 2x^2 + 40x$$

$$x^2 + 20x + 96 = 2x^2 + 40x \Rightarrow x^2 + 20x - 96 = 0$$

$$(x+24)(x-4) = 0 \Rightarrow x + 24 = 0,\ x - 4 = 0$$

$$x = -24\ and\ x = 4$$

$$\therefore x = 4$$

$$sub\ x = 4\ in\ P(A) = \frac{x}{x+12}$$

$$P(A) = \frac{4}{4+12} \Rightarrow P(A) = \frac{\cancel{4}^{\,1}}{\cancel{16}_{\,4}}$$

$$P(A) = \frac{1}{4}$$

7. Two unbiased dice are rolled once. Find the probability of getting (i) a doublet (equal numbers on both dice), (ii) the product as a prime number, (iii) the sum as a prime number, (iv) the sum as 1

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

i) Let A be the event of getting a doublet

$$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} \Longrightarrow n(A) = 6$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{6}^{\,1}}{\cancel{36}_{\,6}}$$

$$\boxed{P(A) = \frac{1}{6}}$$

ii) Let B the event of getting the product as a prime number.

$$B = \{(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)\}$$

$$n(B) = 6$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{\cancel{6}^{\,1}}{\cancel{36}_{\,6}} \Longrightarrow P(B) = \frac{1}{6}$$

iii) Let C be the event of getting the sum of numbers on the dice is prime.

$$C = \{(1,1), (1,2), (1,4), (2,1), (2,3), (2,5)\ \ (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)\}$$

$$n(C) = 14$$

$$\therefore P(C) = \frac{n(C)}{n(S)} \Longrightarrow P(C) = \frac{7}{36}$$

iv) Let D be the event of getting sum of numbers is 1.

$$n(D) = 0 \Longrightarrow P(D) = 0$$

8. Three fair coins are tossed together. Find the probability of getting (i) all heads, (ii) atleast one tail, (iii) atmost one head, (iv) atmost two tails

$$S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(S) = 8$$

i) Let A be the event of getting all heads.

$$A = \{(HHH)\} \Rightarrow n(A) = 1$$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{1}{8}$$

ii) Let B be the event of getting atleast one tail.

$$B = \{(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\} \Rightarrow n(B) = 7$$

$$\therefore P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{7}{8}$$

iii) Let C be the event of getting at most onehead.

$$C = \{(HTT), (THT), (TTH), (TTT)\} \Rightarrow n(C) = 4$$

$$\therefore P(C) = \frac{n(C)}{n(S)}$$

$$P(C) = \frac{\cancel{4}^{1}}{\cancel{8}_{2}} \Rightarrow P(C) = \frac{1}{2}$$

iv) Let D − atmost 2 tails

$$D = \{(HHH), (HHT), (HTT), (HTH), (THH), (THT), (TTH)\}$$

$$n(D) = 7$$

$$\therefore P(D) = \frac{n(D)}{n(S)} \Rightarrow P(D) = \frac{7}{8}$$

9. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

$$S = \begin{Bmatrix} (1,1), (1,1), (1,2), (1,2), (1,3), (1,3) \\ (2,1), (2,1), (2,2), (2,2), (2,3), (2,3) \\ (3,1), (3,1), (3,2), (3,2), (3,3), (3,3) \\ (4,1), (4,1), (4,2), (4,2), (4,3), (4,3) \\ (5,1), (5,1), (5,2), (5,2), (5,3), (5,3) \\ (6,1), (6,1), (6,2), (6,2), (6,3), (6,3) \end{Bmatrix} \Rightarrow n(S) = 36$$

i) Let A be the event of getting Sum of 2

$$A = \{(1, 1), (1, 1)\} \Rightarrow n(A) = 2$$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{2}{36}$$

ii) Let B be the event of geting Sum of 3

$$B = \{(1, 2), (1, 2), (2, 1), (2, 1)\} \Rightarrow n(B) = 4$$

$$\therefore P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{4}{36}$$

$\boldsymbol{iii)\ Let\ C\ be\ the\ event\ of\ getting\ \ Sum\ of\ 4}$

$$C = \{(1,3), (1,3), (2, 2), (2, 2), (3, 1), (3, 1)\} \Rightarrow n(C) = 6$$

$$\therefore P(C) = \frac{n(C)}{n(S)} \Rightarrow P(C) = \frac{6}{36}$$

$\boldsymbol{iv)\ Let\ D\ event\ of\ getting\ \ Sum\ of\ 5}$

$$n(D) = 6$$

$$D = \{(2,3), (2,3),(4, 2), (4, 2), (3,2), (3,2),(4,1), (4,1)\}$$

$$\therefore P(D) = \frac{n(D)}{n(S)} \Rightarrow P(D) = \frac{6}{36}$$

$\boldsymbol{v)\ Let\ E\ be\ the\ evnt\ of\ getting\ \ Sum\ of\ 6}$

$$n(E) = 6$$

$$E = \{(3,3), (3,3),(4, 2), (4, 2), (5,1), (5,1)\}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} \Rightarrow P(E) = \frac{6}{36}$$

$\boldsymbol{vi)\ Let\ F\ be\ the\ event\ of\ getting\ \ Sum\ of\ 7}$

$$F = \{(4,3), (4,3), (5,2), (5,2), (6,1), (6,1)\} \Rightarrow n(F) = 6$$

$$\therefore P(F) = \frac{n(F)}{n(S)} \Rightarrow P(F) = \frac{6}{36}$$

$\boldsymbol{vii)\ Let\ G\ be\ the\ event\ of\ Sum\ of\ 8}$

$$G = \{(5,3), (5,3),(6,2), (6,2)\} \Rightarrow n(G) = 4$$

$$\therefore P(G) = \frac{n(G)}{n(S)} \Rightarrow P(G) = \frac{4}{36}$$

$\boldsymbol{viii)\ Let\ H\ be\ the\ event\ of\ getting\ \ Sum\ of\ 9} \Rightarrow n(H) = 2$

$$\therefore P(H) = \frac{n(H)}{n(S)} \Rightarrow P(H) = \frac{2}{36}$$

10. ***A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black***

$No.of\ Red\ balls = 5$ $No.of\ white\ balls = 6$, $No.of\ Green\ balls = 7$

$No.of\ Black\ balls = 8$

$Total\ numbers\ of\ balls = 5 + 6 + 7 + 8$

$$n(S) = 26$$

$\boldsymbol{i)\ Let\ A\ be\ the\ event\ of\ getting\ White\ ball}$

$$n(A) = 6$$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Rightarrow \quad P(A) = \frac{\cancel{6}^{\,3}}{\cancel{26}_{\,13}}$$

$$P(A) = \frac{3}{13}$$

$\boldsymbol{ii)\ Let\ B\ be\ the\ event\ of\ getting\ Black\ (or)\ red}$

$$n(B) = 5 + 8 = 13$$

$$\therefore P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{\cancel{13}^{\,1}}{\cancel{26}_{\,2}}$$

$$P(B) = \frac{1}{2}$$

$\boldsymbol{iii)\ Let\ C\ be\ the\ event\ of\ getting\ not\ white}$

$$n(C) = 5 + 7 + 8 \Rightarrow \ n(C) = 20$$

$$\therefore P(C) = \frac{\cancel{20}^{\,10}}{\cancel{26}_{\,13}} \Rightarrow \ P(C) = \frac{10}{13}$$

$\boldsymbol{iv)\ Let\ D\ be\ the\ event\ of\ getting\ Neither\ white\ nor\ black}$

$$n(D) = 5 + 7 \Rightarrow n(D) = 12$$

$$\therefore P(D) = \frac{\cancel{12}^{\,6}}{\cancel{26}_{\,13}} \Rightarrow \ P(D) = \frac{6}{13}$$

$\boldsymbol{11.\ In\ a\ box\ there\ are\ 20\ non-defective\ and\ some\ defective\ bulbs.\ If\ the\ probability\ that\ a\ bulb\ selected\ at\ random\ from\ the\ box\ found\ to\ be\ defective\ is\ \frac{3}{8}\ then, find\ \ the\ number\ of\ defective\ bulbs.}$

$Let\ x\ be\ the\ number\ of\ defective\ bulbs.$
$and\ there\ are\ number\ of\ non\ -defective\ bulbs = 20.$

$$\therefore n(S) = x + 20$$

$Let\ A\ be\ the\ event\ of\ selecting\ defective\ balls$

$$\therefore n(A) = x$$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Rightarrow \ P(A) = \frac{x}{x + 20}$$

$Given\ that\ probability\ of\ defective\ bulb:$

$$\frac{x}{x+20} = \frac{3}{8} \Rightarrow \ 8x = 3(x + 20) \Rightarrow 8x = 3x + 60$$

$$8x - 3x = 60 \Rightarrow 5x = 60 \Rightarrow \ x = 12$$

$$\therefore \ Number\ of\ defective\ balls \ = \ 12.$$

12. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards an then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Total no. of cards: $n(S) = 52 - 2 - 2 - 2 = 46$

i) Let A be the event of selecting clover card.

$n(A) = 13$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Longrightarrow P(A) = \frac{13}{46}$$

ii) Let B be the event of getting queen of red card.

There is no red card in the pack of 46 cards

$n(B) = 0 \Longrightarrow P(B) = 0$

iii) Let C be the event of getting King of black cards

$n(C) = 1$

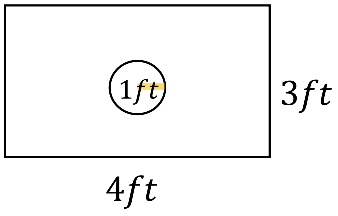
$$\therefore P(C) = \frac{n(C)}{n(S)} \Longrightarrow P(C) = \frac{1}{46}$$

13. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?

Total Area of the rectangular region = *lenght × Breath*

$= 4ft \times 3ft$

$n(S) = 12ft^2$



Area of the circular region $= \pi r^2$

$= \pi \times 1^2 = \pi ft^2$

$$\therefore \text{Probability to win the game} = \frac{\pi}{12}$$

$$= \frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$$

14. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days?

$S = \{Mon, Tues, Wed, Thurs, Fri, Sat\} \Longrightarrow n(S) = 6$

i) *Probability that both of them will visit the shop on same day* $= \frac{1}{6}$

ii) Probability that both of them will visit the shop in different day

$$= 1 - \frac{1}{6}$$

iii) Probability that both of them will visit the shop in consecutive day

$$A = \{(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat)\}$$

$$n(A) = 5$$

$$P(A) = \frac{5}{6}$$

15. *In a game, the entry fee is Rs. 150. The game consists of tossing a coin times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.*

When a coin is tossed thrice,

$$S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(S) = 8$$

i) Find the probability that she gets double entry fee

Let A be the event of getting all heads.

$$A = \{(HHH)\} \Longrightarrow n(A) = 1$$

$$\therefore P(A) = \frac{n(A)}{n(S)} \Longrightarrow P(A) = \frac{1}{8}$$

ii) Find the probability that she gets just entry fee

Let A be the event of getting one or two Head.

$$A = \{HTT, THT, TTH, HHT, HTH, THH\}$$

$$n(A) = 6 \Longrightarrow P(B) = \frac{n(B)}{n(S)} = \frac{6}{8}$$

$$P(B) = \frac{3}{4}$$

iii) Find the probability that she loss the entry fee

$$\text{C} = \{\text{TTT}\}$$

$$n(C) = 1 \Longrightarrow P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

EXERCISE : 8.4

Example 8.27: If $P(A) = 0.37, P(B) = 0.42$ and $P(A \cap B) = 0.09$ then find $P(A \cup B)$

Given : $P(A) = 0.37, P(B) = 0.42$ *and* $P(A \cap B) = 0.09$

Find $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.37 + 0.42 - 0.09$$
$$P(A \cup B) = 0.7$$

Example 8.28: What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

$$n(S) = 52$$

Let A be the event of drawing a king card. There are 4 king cards

$$n(A) = 4$$

$$\text{Probability}: P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Let B be the event of drawing a queen card. There are 4 queen cards

$$n(B) = 4$$

$$\text{Probability}: P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$\boxed{P(A \cup B) = P(A) + P(B)}$ *A and B are mutually exclusive i.e* $P(A \cap B) = 0$

$$= \frac{1}{13} + \frac{1}{13} = \frac{1+1}{13} = \frac{2}{13}$$

$\therefore$ *probability of drawing either a king or a queen* $= \frac{2}{13}$

Example 8.29: Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

$$n(S) = 36$$

$A =$ *event of getting a doublet*

$$A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$n(A) = 6 \Longrightarrow P(A) = \frac{6}{36}$$

$B =$ *event of getting face sum* 4

$$B = \{(1,3),(2,2),(3,1)\}$$

$$n(B) = 3 \Longrightarrow P(B) = \frac{3}{36}$$

$$A \cap B = \{(2,2)\}$$

$$n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{36}$$

$$\therefore \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{6+3-1}{36} = \frac{\overset{2}{\cancel{8}}}{\underset{9}{\cancel{36}}}$$

$$\therefore \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{2}{9}$$

Example 8.30: ***If A and B are two events such that*** $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ ***and*** $P(A \text{ and } B) = \frac{1}{8}$***, find (i) P(A or B) (ii) P(not A and not B)***

(i) $\mathrm{P}(A \text{ or } B) = \mathrm{P}(A \cup B)$

$$= \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(A \cap B)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$= \frac{2+4-1}{8} = \frac{5}{8}$$

$$\boxed{\therefore \mathrm{P}(A \text{ or } B) = \frac{5}{8}}$$

(ii) $\mathrm{P}(\text{not } A \text{ and not } B) = P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$$

$$\boxed{\therefore \mathrm{P}(\text{not } A \text{ and not } B) = \frac{3}{8}}$$

Example 8.31: ***A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.***

$$n(S) = 52$$

Let A be the event of drawing a king card. There are 4 king cards

$$n(A) = 4$$

$$\textit{Probability} : \mathrm{P}(\mathrm{A}) = \frac{\mathrm{n(A)}}{\mathrm{n(S)}} = \frac{4}{52}$$

Let B be the event of drawing a heart card. There are 13 heart cards

$$n(B) = 13$$

$$\textit{Probability} : \mathrm{P}(\mathrm{B}) = \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{13}{52}$$

Let C be the event of drawing a red card. There are 26 red cards

$$n(C) = 26$$

$$\textit{Probability} : \mathrm{P}(\mathrm{C}) = \frac{\mathrm{n(C)}}{\mathrm{n(S)}} = \frac{26}{52}$$

$Number\ of\ heart\ king$: $n(A \cap B) = 1$

$$\mathrm{P}(A \cap B) = \frac{n(A \cap B)}{\mathrm{n(S)}} \Rightarrow \mathrm{P}(A \cap B) = \frac{1}{52}$$

$Number\ of\ red\ heart$: $n(B \cap C) = 13$

$$\mathrm{P}(B \cap C) = \frac{n(B \cap C)}{\mathrm{n(S)}} \Rightarrow \mathrm{P}(B \cap C) = \frac{13}{52}$$

$Number\ of\ red\ king$:$n(A \cap C) = 2$

$$\mathrm{P}(A \cap C) = \frac{n(A \cap C)}{\mathrm{n(S)}} \Rightarrow \mathrm{P}(A \cap C) = \frac{2}{52}$$

$Number\ of\ heart\ ,king\ which\ is\ red$: $\mathrm{n}(A \cap B \cap C) = 1$

$$\mathrm{P}(A \cap B \cap C) = \frac{\mathrm{n}(A \cap B \cap C)}{\mathrm{n(S)}} \Rightarrow \mathrm{P}(A \cap B \cap C) = \frac{1}{52}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$-P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{4 + 13 + 26 - 1 - 13 - 2 + 1}{52}$$

$$= \frac{28}{52} \quad = \frac{7}{13}$$

Example 8.32: In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
(i) The student opted for NCC but not NSS.
(ii) The student opted for NSS but not NCC.
(iii) The student opted for exactly one of them.

$Total\ number\ of\ students$ ∴ $n(S) = 50$

$Let\ A\ and\ B\ be\ the\ events\ of\ students\ opted\ for\ NCC\ and\ NSS\ respectively.$

$\mathrm{n(A)} = 28, \mathrm{n(B)} = 30$ and $\mathrm{n}(A \cap B) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$$

$(i)\ Probability\ of\ the\ student\ opted\ for\ NCC\ but\ not\ NSS$

$$P\left(A \cap \overline{B}\right) = P(A) - P(A \cap B)$$

$$=\frac{28}{50}-\frac{18}{50}=\frac{28-18}{50}=\frac{\cancel{10}}{\cancel{50}_5}=\frac{1}{5}$$

(ii) Probability of the student opted for NSS but not NCC

$$P(\overline{A}\cap B)=P(B)-P(A\cap B)$$

$$=\frac{30}{50}-\frac{18}{50}=\frac{30-18}{50}=\frac{\overset{6}{\cancel{12}}}{\underset{25}{\cancel{50}}}=\frac{6}{25}$$

(iii) Probability of the student opted for exactly one of them

$$=\mathrm{P}(A\cup\overline{B})+\mathrm{P}(\overline{A}\cup B)$$

$$=\frac{1}{5}+\frac{6}{25}=\frac{5+6}{25}=\frac{11}{25}$$

Example 8.32: A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

$$P(A)=0.5,\ P(A\cap B)=0.3$$

$$P(A\cup B)\le 1$$

$$\mathrm{P(A)}+\mathrm{P(B)}-\mathrm{P}(A\cap B)\le 1$$

$$\mathrm{P(B)}\ +0.2\le 1$$

$$\mathrm{P(B)}\ \le 1-0.2$$

$$\mathrm{P(B)}\ \le 0.8$$

Therefore, probability of B getting selected is atmost 0.8.

1. *If* $P(A)=\frac{2}{3}$, $P(B)=\frac{2}{5}$ *and* $P(A\cup B)=\frac{1}{3}$ *then find* $P(A\cap B)$.

$$\boxed{\mathrm{P}(A\cup B)=\mathrm{P(A)}+\mathrm{P(B)}-\mathrm{P}(A\cap B)}$$

$$\mathrm{P(A\cap B)}=\mathrm{P(A)}+\mathrm{P(B)}-\mathrm{P(A\cup B)}$$

$$=\frac{2}{3}+\frac{2}{5}-\frac{1}{3}=\frac{10+6-5}{15}$$

$$\boxed{\mathrm{P}(A\cap B)=\frac{11}{15}}$$

2. *A and B are two events such that* $P(A)=0.42$, $P(B)=0.48$, $P(A\cap B)=0.16$ *Find (i) P(not A) (ii) P(not B) (iii) P(A or B)*

(i) Find P(not A)

$$P(not\ A)=\ P(\overline{A})$$

$$P(\overline{A})=1-P(A)=1-0.42$$

$$P(\overline{A})=0.58$$

(ii) *Find* $P(not\ B)$

$$P(not\ B) = P(\overline{B})$$

$$P(\overline{B}) = 1 - P(B) = 1 - 0.48$$

$$P(\overline{B}) = 0.52$$

(iii) *Find* $P(A\ or\ B)$

$$P(A\ or\ B) = P(A \cup B)$$

$$P(A \cup B) = \mathrm{P(A) + P(B) - P(A \cap} B)$$

$$= 0.42 + 0.48 - 0.16 = 0.90 - 0.16$$

$$\mathrm{P(A \cup} B) = 0.74$$

3. *If A and B are two mutually exclusive events of a random experiment and* $\boldsymbol{P(not\ A) = 0.45, P(A \cup B) = 0.65}$***, then find*** $\boldsymbol{P(B)}$**.**

Given : $P(not\ A) = 0.45$

$$P(\overline{A}) = 0.45$$

$$1 - P(A) = 0.45 \implies 1 - 0.45 = P(A)$$

$$P(A) = 0.55$$

A and B are mutually exclusive i.e $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.65 = 0.55 + P(B) - 0$$

$$0.65 - 0.55 = P(B) \implies P(B) = 0.10$$

4. *The probability that atleast one of A and B occur is* 0.6*. If A and B occur simultaneously with probability* 0.2*, then find* $\boldsymbol{P(\overline{A}) + P(\overline{B})}$**.**

The probability that atleast one of A and B occur is 0.6

$$P(A\ or\ B) = P(A \cup B) = 0.6$$

A and B occur simultaneously with probability 0.2

$$P(A \cap B) = 0.2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = P(A) + P(B) - 0.2 \implies 0.6 + 0.2 = P(A) + P(B)$$

$$P(A) + P(B) = 0.8$$

$$1 - P(\overline{A}) + 1 - P(\overline{B}) = 0.8 \implies 2 - P(\overline{A}) - P(\overline{B}) = 0.8$$

$$2 - 0.8 = P(\overline{A}) + P(\overline{B}) \implies P(\overline{A}) + P(\overline{B}) = 1.2$$

5. *The probability of happening of an event A is* 0.5 *and that of B is* 0.3*. If A and B are mutually exclusive events, then find the probability that neither A nor B happen*

Given : $P(A) = 0.5, P(B) = 0.3$

$A \text{ and } B \text{ are mutually exclusive i.e } P(A \cap B) = 0$

$\text{To find } P(\text{either } A \text{ or } B) = P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.5 + 0.3 - 0$$

$$P(A \cup B) = 0.8$$

$\text{To find } P(\text{neither } A \text{ nor } B) = P(\overline{A \cup B})$

$$P(\overline{A \cup B}) = 1 - 0.8$$

$$P(\overline{A \cup B}) = 0.2$$

6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Let S be the sample space

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ \boxed{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)} \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ \boxed{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)} \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ \boxed{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)} \end{Bmatrix} \Rightarrow n(S) = 36$$

Let A be the event of getting an even nber in the first time. um

$A = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$$n(A) = 18$$

$$P\ (A) = \frac{n(A)}{n(S)} = \frac{18}{36} \Rightarrow \ \mathrm{P(A)} = \frac{18}{36}$$

Let B be the event of getting a total 8.

$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}\ ; \ n(B) = 5$

$$\mathrm{P\ (B)} = \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{5}{36} \Rightarrow \ \mathrm{P(B)} = \frac{5}{36}$$

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),\boxed{(2,6)} \\ (3,1),(3,2),(3,3),(3,4),\boxed{(3,5)},(3,6) \\ (4,1),(4,2),(4,3),\boxed{(4,4)},(4,5),(4,6) \\ (5,1),(5,2),\boxed{(5,3)},(5,4),(5,5),(5,6) \\ (6,1),\boxed{(6,2)},(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$A \cap B = \{(2,6), (4,4), (6,2)\}\ ; \Rightarrow \ n(A \cap B) = 3$

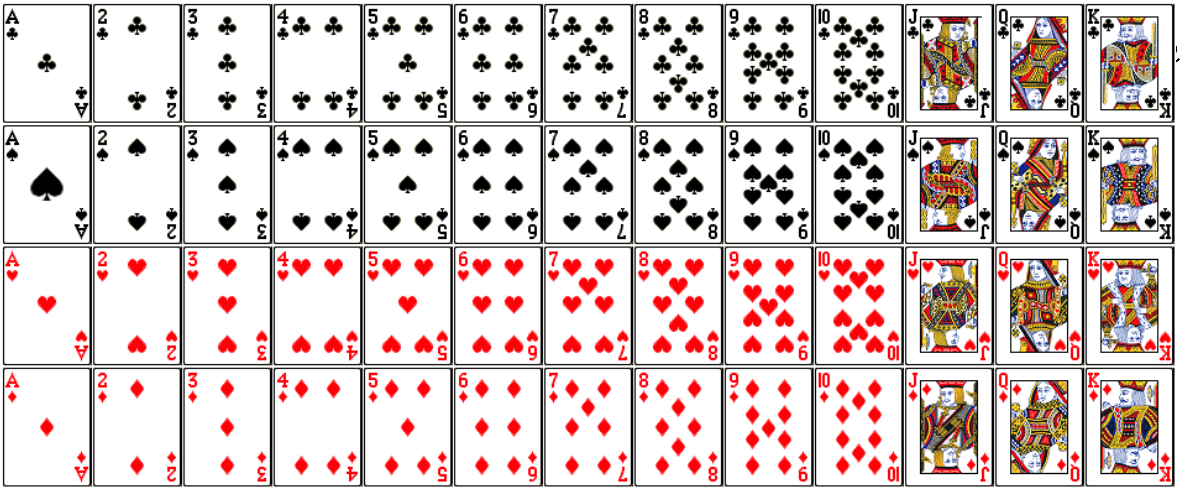
$$\mathrm{P}(A \cap B) = \frac{n(A \cap B)}{\mathrm{n(S)}} = \frac{3}{36} \Rightarrow \ \mathrm{P}(A \cap B) = \frac{3}{36}$$

$$\mathrm{P}(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P}(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{18 + 5 - 3}{36} = \frac{\cancel{20}^{\,5}}{\cancel{36}_{\,9}}$$

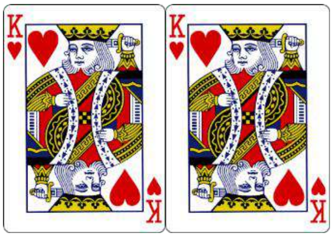
$$\mathrm{P}(A \cup B) = \frac{5}{9}$$

7. *From a well – shuffled pack of 52 cards, a card is drawn at random.*



$$Total\ no.of\ cards: n(S) = 52$$

Let A be the event of drawing a red king card, *there are 2 red king .*



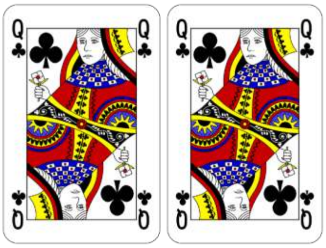
$$n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52}$$

$$P(A) = \frac{2}{52}$$

Let B be the event of drawing a black queen card,

there are 2 black queen card.



$$n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52}$$

$$P(B) = \frac{2}{52}$$

Since A and B are mutually exclusive i.e $P(A \cap B) = 0$

$$\mathrm{P}(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cup B) = \frac{2}{52} + \frac{2}{52} - 0 = \frac{\cancel{4}^{\,1}}{\cancel{52}_{\,13}}$$

$$\boxed{\mathrm{P}(A \cup B) = \frac{1}{13}}$$

8. *A box contains cards numbered* 3, 5, 7, 9, … 35, 37. *A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.*

Box in the card $= \{3, 5, 7, 9, \ldots 35, 37\}$

To find number of cards

$3, 5, 7, 9, \ldots 35, 37$ *which are in A.P*

$$a = 3, l = 37$$

$$d = t_2 - t_1$$

$$d = 5 - 3 \Rightarrow \ d = 2$$

$$\text{Number of cards : } n(S) = \left(\frac{l-a}{d}\right) + 1$$

$$n(S) = \left(\frac{37-3}{2}\right) + 1 \Rightarrow n(S) = \left(\frac{\overset{17}{\cancel{34}}}{\cancel{2}}\right) + 1$$

$$n(S) = 17 + 1 \Rightarrow n(S) = 18$$

$Let\ A\ be\ the\ event\ of\ getting\ selecting\ a\ number\ which\ is\ multiple\ of\ 7$

$$\text{A} = \{7, 21, 35\} \Rightarrow n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{18}$$

$$P(A) = \frac{3}{18}$$

$Let\ B\ be\ the\ event\ of\ choosing\ a\ prime\ number.$

$$\text{B} = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$$

$$n(B) = 11$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{18}$$

$$P(B) = \frac{11}{18}$$

$$A \cap B = \{7\} \Rightarrow \text{n}(\text{A} \cap \text{B}) = 1$$

$$\text{P}(A \cap B) = \frac{n(A \cap B)}{n(S)} \Rightarrow \text{P}(A \cap B) = \frac{1}{18}$$

$$\text{P}(\text{A} \cup \text{B}) = \text{P}(\text{A}) + \text{P}(\text{B}) - \text{P}(\text{A} \cap \text{B})$$

$$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{3 + 11 - 1}{18} = \frac{13}{18}$$

9. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

$$\text{S} = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(S) = 8$$

$Let\ A\ be\ the\ event\ of\ getting\ atmost\ 2\ tails$

$$A = \{\text{HTT, THT, TTH, HHT, HTH, THH, HHH}\}$$

$n(A) = 7$

Probability of getting atmost 2 tails is

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

Let B be the event of getting at least 2 Head.

$B = \{\text{ HHT, HTH, THH, HHH}\}$

$n(B) = 4$

Probability of getting at least 2 head is

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8}$$

$A \cap B = \{\text{ HHT,HTH,THH, HHH}\}$

$n(A \cap B) = 4$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \Longrightarrow P(A \cap B) = \frac{4}{8}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{8} + \cancel{\frac{4}{8}} - \cancel{\frac{4}{8}}$$

$$P(A \cup B) = \frac{7}{8}$$

Probability of getting atmost 2 tails or atleast 2 heads is $\frac{7}{8}$

10. *The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?*

Let A be the event of getting electrification contract

$$P(A) = \frac{3}{5}$$

Let B be the event of getting plumbing contract

$$P(\overline{B}) = \frac{5}{8}$$

$$1 - P(B) = \frac{5}{8} \Longrightarrow 1 - \frac{5}{8} = P(B) \Longrightarrow P(B) = \frac{8-5}{8}$$

$$P(B) = \frac{3}{8}$$

$The\ probability\ that\ atleast\ one\ contract\ is\ \frac{5}{7}$

$$P(A \cup B) = \frac{5}{7}$$

$$P(A) + P(B) - P(A \cap B) = \frac{5}{7} \quad where\ P(A) = \frac{3}{5}\ and\ P(B) = \frac{3}{8}$$

$$\frac{3}{5} + \frac{3}{8} - P(A \cap B) = \frac{5}{7} \implies \frac{3}{5} + \frac{3}{8} - \frac{5}{7} = P(A \cap B)$$

$$P(A \cap B) = \frac{3 \times 56 + 3 \times 35 - 5 \times 40}{280}$$

$$P(A \cap B) = \frac{168 + 105 - 200}{280} \implies P(A \cap B) = \frac{273 - 200}{280}$$

$$P(A \cap B) = \frac{73}{280}$$

11. *In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years*?

$$Number\ of\ people\ in\ a\ town = 8000$$

$$n(S) = 8000$$

$Let\ A\ be\ the\ event\ of\ selecting\ a\ female$

$Number\ of\ females\ n(A) = 3000$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3000}{8000}$$

$Let\ B\ be\ the\ event\ of\ selecting\ an\ female\ individual\ over\ 50\ years$

$Number\ of\ people\ who\ are\ over\ 50\ years = 1300$

$$n(B) = 1300$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1300}{8000}$$

$$n(A \cap B) = 30\%\ of\ 3000$$

$$n(A \cap B) = \frac{30}{\cancel{100}} \times 300\cancel{0} \implies n(A \cap B) = 900$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \implies P(A \cap B) = \frac{900}{8000}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3000}{8000} + \frac{1300}{8000} - \frac{900}{8000}$$

$$= \frac{3000 + 1300 - 900}{8000} = \frac{4300 - 900}{8000}$$

$$= \frac{340\cancel{0}}{800\cancel{0}} = \frac{34}{80} = \frac{17}{40}$$

12. *Three coins are tossed simultaneously. Find the probability of getting exactly two heads or atleast one tail or consecutively two heads.*

$$S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(S) = 8$$

Let A be the event of getting exactly two heads

$$A = \{\text{HHT, HTH, THH}\}$$

$$n(A) = 3$$

Probability of getting exactly two heads is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Let B be the event of getting at least one tail.

$$B = \{\text{HHT, HTH, THH, HTT, THT, TTH, TTT}\}$$

$$n(B) = 7$$

Probability of getting at least one tail is

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

Let C be the event of getting consecutively two heads.

$$C = \{\text{HHH, HHT, THH}\}$$

$$n(C) = 3$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$

$$A \cap B = \{\text{HHT, HTH, THH}\}$$

$$n(A \cap B) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \Longrightarrow P(A \cap B) = \frac{3}{8}$$

$$B \cap C = \{\text{HHT, THH}\}$$

$$n(A \cap B) = 2$$

$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} \Longrightarrow P(B \cap C) = \frac{2}{8}$$

$$\mathrm{A} \cap \mathrm{C} = \{\ \mathrm{HHT}, \mathrm{THH}\}$$

$$n(A \cap C) = 2$$

$$\mathrm{P}(\mathrm{A} \cap \mathrm{C}) = \frac{\mathrm{n}(\mathrm{A} \cap \mathrm{C})}{\mathrm{n}(\mathrm{S})} \Rightarrow P(A \cap C) = \frac{2}{8}$$

$$A \cap B \cap C = \{\ \mathrm{HHT}, \mathrm{THH}\}$$

$$n(A \cap B \cap C) = 2$$

$$\mathrm{P}(A \cap B \cap C) = \frac{\mathrm{n}(A \cap B \cap C)}{\mathrm{n}(\mathrm{S})} \Rightarrow \mathrm{P}(A \cap B \cap C) = \frac{2}{8}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$-P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8} = \frac{3 + 7 + 3 - 3 - 2 - 2 + 2}{8}$$

$$= \frac{8}{8} = 1$$

13. *If A,B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if* $\mathrm{P}(A \cap B) = \frac{1}{6}$, $\mathrm{P}(B \cap C) = \frac{1}{4}$, $\mathrm{P}(A \cup B \cup C) = \frac{9}{10}$ *and* $\mathrm{P}(A \cap B \cap C) = \frac{1}{15}$ *then find P(A),P(B) and P(C)?*

Given: $\mathrm{P}(A \cap B) = \frac{1}{6}, \mathrm{P}(B \cap C) = \frac{1}{4}, \mathrm{P}(A \cup B \cup C) = \frac{9}{10}$ *and* $\mathrm{P}(A \cap B \cap C) = \frac{1}{15}$

$$P(B) = 2P(A), P(C) = 3P(A)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$-P(A \cap C) + P(A \cap B \cap C)$$

$$\frac{9}{10} = P(A) + 2P(A) + 3P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15} = P(A) + 2P(A) + 3P(A)$$

$$6P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$$

$$= \frac{108 + 20 + 30 + 15 - 8}{120}$$

$$6P(A) = \frac{165}{120} \Rightarrow P(A) = \frac{\overset{\cancel{33}\ 11}{\cancel{165}}}{\underset{\cancel{24}\ 8}{\cancel{120}}} \times \frac{1}{6}$$

$$P(A) = \frac{11}{48}$$

$$P(B) = 2P(A) \Rightarrow P(B) = 2 \times \frac{11}{48}$$

$$P(B) = \cancel{2} \times \frac{11}{\underset{24}{\cancel{48}}} \Rightarrow P(B) = \frac{11}{24}$$

$$P(C) = 3P(A) \Rightarrow P(C) = \cancel{3} \times \frac{11}{\underset{16}{\cancel{48}}}$$

$$P(C) = \frac{11}{16}$$

14. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4: 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime rollnumber or a girl with composite roll number or an even roll number.

$$Number\ of\ students = 35$$

$$n(S) = 35$$

$$Boys : girls = 4:3$$

$$Number\ of\ boys = 4k\ and\ number\ of\ girls = 3k$$

$$No.of\ Boys + No.of girls = 35$$

$$4k + 3k = 35 \Rightarrow \cancel{7}k = \cancel{35}\ _5$$

$$k = 5$$

$$Number\ of\ boys = 4 \times 5 = 20$$

$$Number\ of\ girls = 3 \times 5 = 15$$

$$Boys\ numbered = \{1,2,3,4, \dots \dots \dots 20\}$$

$$Girls\ numbered = \{21,22,23,24, \dots \dots \dots 35\}$$

Let A be the event of getting a boy with prime roll number

$$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{35}$$

Let B be the event of getting a girl with composite.

$$B = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$$

$$n(B) = 12$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$$

$Let\ C\ be\ the\ event\ of\ getting\ an\ even\ roll\ number.$

$C = \{2, 4, 6, 8, 10, 12, 14, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$

$$n(C) = 17$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{17}{35}$$

$A\ and\ B\ are\ mutually\ exclusive\ i.e\ \ P(A \cap B) = 0$

$\mathrm{B} \cap \mathrm{C} = \{\ 22, 24, 26, 28, 30, 32, 34\}$

$$n(A \cap B) = 7$$

$$\mathrm{P(B \cap C)} = \frac{\mathrm{n(B \cap C)}}{\mathrm{n(S)}} \Longrightarrow P(B \cap C) = \frac{7}{35}$$

$\mathrm{A} \cap \mathrm{C} = \{\ 2\}$

$n(A \cap C) = 1$

$$\mathrm{P(A \cap C)} = \frac{\mathrm{n(A \cap C)}}{\mathrm{n(S)}} \Longrightarrow P(A \cap C) = \frac{1}{35}$$

$A \cap B \cap C = \{\ \}$

$n(A \cap B \cap C) = 0$

$$\mathrm{P}(A \cap B \cap C) = \frac{\mathrm{n}(A \cap B \cap C)}{\mathrm{n(S)}} \Longrightarrow \mathrm{P}(A \cap B \cap C) = \frac{0}{8}$$

$\mathrm{P}(A \cap B \cap C) = 0$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$-P(A \cap C) + P(A \cap B \cap C)$$
$$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0 = \frac{8 + 12 + 17 - 7 - 1}{35} = \frac{29}{35}$$