XI – PHYSICS

Name :

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BLUESTARS HIGHER SECONDARY SCHOOL

XI – PHYSICS

VOLUME I NOTES

UNIT – 1 NATURE OF PHYSICAL WORLD AND MEASUREMENT

TWO MARKS AND THREE MARKS:

1. Briefly explain the types of physical quantities.

1. Physical quantities are classified into two types. They are fundamental and derived quantities.
2. Fundamental or base quantities are quantities which cannot be expressed in terms of any other physical quantities. These are length, mass, time, electric current, temperature, luminous intensity and amount of substance.
3. Quantities that can be expressed in terms of fundamental quantities are called derived quantities. For example, area, volume, velocity, acceleration, force.

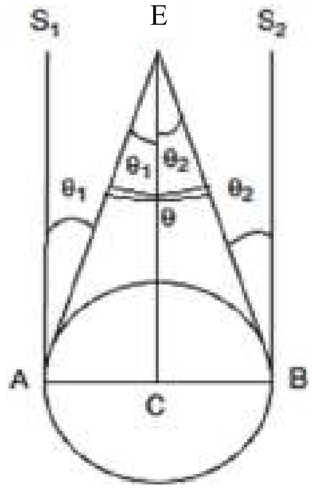
2. How will you measure the diameter of the Moon using parallax method?

i) C is the centre of the Moon. A and B are two diametrically opposite places on the surface of the Moon. From A and B, the parallaxes θ_1 and θ_2 respectively of Earth E with respect to some distant star are determined with the help of an astronomical telescope. Thus, the total parallax of the Moon subtended on Earth $\angle AEB = \theta_1 + \theta_2 = \theta$.

ii) If θ is measured in radians,

$$\theta = \frac{AB}{AE}; AE \approx EC$$

$$\theta = \frac{AB}{EC} \ or \ AB = EC.\theta;$$



Knowing the values of AB and θ, we can calculate the diameter AB of Moon from the Earth.

3. Write the rules for determining significant figures.

i) All non-zero digits are significant. Ex.1342 has **four** significant figures

ii) All zeros between two non zero digits are significant. Ex. 2008 has **four** significant figures

iii) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. Ex. 30700. has **five** significant figures

iv) The number without a decimal point, the terminal or trailing zero(s) are not significant. Ex. 30700 has **three** significant figures

All zeros are significant if they come from a measurement Ex. 30700 m has **five** significant figures

v) If the number is less than 1, the zero (s) on the right of the decimal point but to left of the first non zero digit are not significant. Ex. 0.00345 has **three** significant figures

vi) All zeros to the right of a decimal point and to the right of non-zero digit are significant. Ex. 40.00 has **four** significant figures and 0.030400 has **five** significant figures

vii) The number of significant figures does not depend on the system of units used

1.53 cm, 0.0153 m, 0.0000153 km, all have **three** significant figures

4. What are the limitations of dimensional analysis?

1. This method gives no information about the dimensionless constants in the formula like 1, 2, π, e, etc.
2. This method cannot decide whether the given quantity is a vector or a scalar.
3. This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
4. It cannot be applied to an equation involving more than three physical quantities.

5. It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation. For example using dimensional analysis , $s = ut + \frac{1}{2}at^2$ is dimensionally correct whereas the correct relation is $s = ut + 2at$

5. Define precision and accuracy. Explain with one example.

Precision: The closeness of two or more measurements to each other.

Accuracy: The closeness of a measure value to the actual value of the object being measured is called accuracy.

Ex. : The true value of a certain length is near 5.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 5.5 cm. In another experiment using a measuring instrument of greater resolution, say 0.01 cm, the length is found to be 5.38 cm. We find that the first measurement is more accurate as it is closer to the true value, but it has lesser precision. On the contrary, the second measurement is less accurate, but it is more precise.

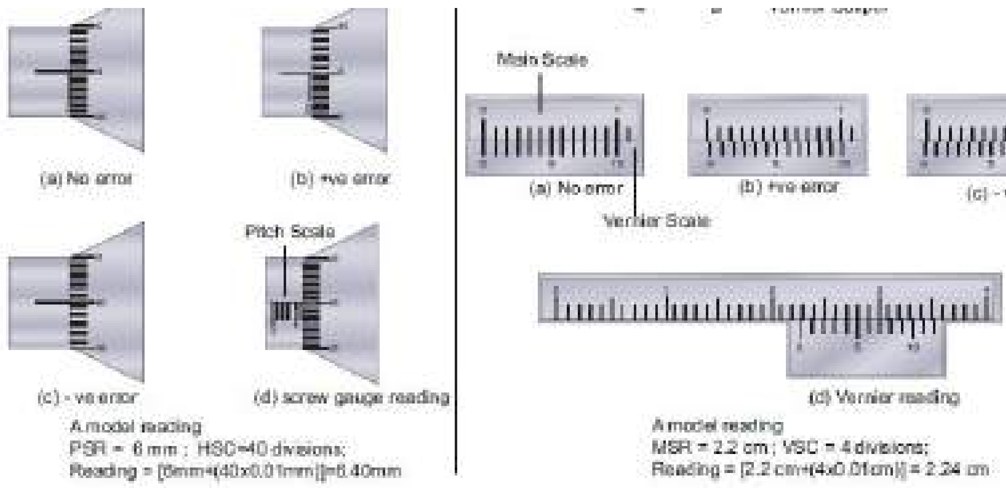
FIVE MARKS QUESTIONS

1. **Explain the use of screw gauge and vernier caliper in measuring smaller distances.**

i) Write a note on triangulation method and radar method to measure larger distances.

Measurement of small distances

The screw gauge is an instrument used for measuring accurately the dimensions of objects up to a maximum of about 50 mm. The principle of the instrument is the magnification of linear motion using the circular motion of a screw. The least count of the screw gauge is 0.01 mm *Vernier caliper:* A vernier caliper is a versatile instrument for measuring the dimensions of an object namely diameter of a hole, or a depth of a hole. The least count of the vernier caliper is 0.1 mm



Measurement of large distances

1) For measuring larger distances such as the height of a tree, distance of the Moon or a planet from the Earth, some special methods are adopted. Triangulation method, parallax method and radar method are used to determine very large distances.

Triangulation method for the height of an accessible object

i) Let AB = h be the height of the tree or tower to be measured. Let C be the point of observation at distance x from B. Place a range finder at C and measure the angle of elevation,

$\angle ACB = \theta$ as shown in Figure. From right angled triangle

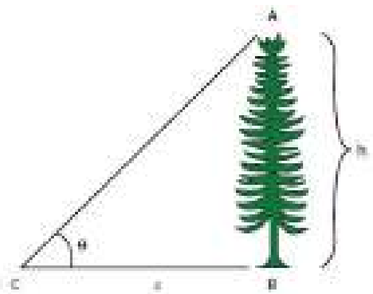
From right angled triangle ABC,

$tan\theta = \frac{AB}{BC} = \frac{h}{x}$

(or)

$\boxed{\text{Height } h = xtan\theta}$

Knowing the distance x, the height h can be determined.

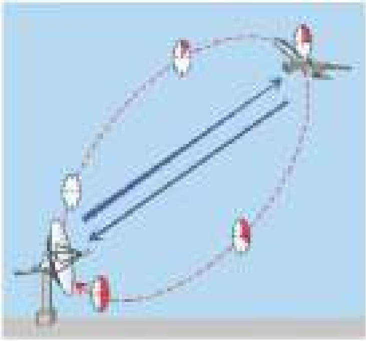


Knowing the distance x, the height h can be determined.

RADAR method

i) The word RADAR stands for radio detection and ranging. Radar can be used to measure accurately the distance of a nearby planet such as Mars. In this method, radio waves are sent from transmitters which, after reflection from the planet, are detected by the receiver.

ii) By measuring, the time interval (t) between the instants the radio waves are sent and received, the distance of the planet can be determined as $d = \frac{v \times t}{2}$



where v is the speed of the radio wave. As the time taken (t) is for the distance covered during the forward and backward path of the radio waves, it is divided by 2 to get the actual distance of the object. This method can also be used to determine the height, at which an aeroplane flies from the ground.

2. **Explain in detail the various types of errors.**

Random error, systematic error and gross error are the three possible errors

Systematic errors:

Systematic errors are reproducible inaccuracies that are consistently in the same direction.

Instrumental errors

1. When an instrument is not calibrated properly at the time of manufacture, These errors can be corrected by choosing the instrument carefully.

Imperfections in experimental technique or procedure

2. These errors arise due to the limitations in the experimental arrangement. To overcome these, necessary correction has to be applied.

Personal errors

3. These errors are due to individuals performing the experiment, may be due to incorrect initial setting up of the experiment or carelessness of the individual making the observation due to improper precautions

Errors due to external causes

4. The change in the external conditions during an experiment can cause error in measurement. For example, changes in temperature, humidity, or pressure during measurements may affect the result of the measurement.

Least count error

5. Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error.

Random errors

6. Random errors may arise due to random and unpredictable variations in experimental conditions like pressure, temperature, voltage supply etc.
7. Errors may also be due to personal errors by the observer who performs the experiment. Random errors are sometimes called **"chance error"**
8. It can be minimized by repeating the observations a large number of measurements are made and then the arithmetic mean is taken.

Gross Error

The error caused due to the shear carelessness of an observer is called gross error. These errors can be minimized only when an observer is careful and mentally alert.

3. **What do you mean by propagation of errors? Explain the propagation of errors in addition and multiplication**.

1. A number of measured quantities may be involved in the final calculation of an experiment. Different types of instruments might have been used for taking readings. Then we may have to look at the errors in measuring various quantities, collectively. The error in the final result depends on

2. The errors in the individual measurements ii) On the nature of mathematical operations performed to get the final result. So we should know the rules to combine the errors. The various possibilities of the propagation or combination of errors in different mathematical operations are discussed below:

Error in the sum of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then, Measured value of A = A ± ΔA

Measured value of B = B ± ΔB Consider the sum, Z = A + B

The error ΔZ in Z is then given by

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$= (A + B) \pm (\Delta A + \Delta B)$

$= Z \pm (\Delta A + \Delta B)$

(or) $\Delta Z = \Delta A + \Delta B$

The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

Let ΔA and ΔB be the absolute errors in the two quantities A, and B, respectively. Consider the product Z = AB

The error ΔZ in Z is given by $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= (AB) \pm (A\Delta B) \pm (B\Delta A) \pm (\Delta A.\Delta B)$

Dividing L.H.S by Z and R.H.S by AB, we get,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

As ΔA /A, ΔB / B are both small quantities, their product term ΔA/A.ΔB/B can be neglected. The maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

4. **Write short notes on the following.**

a) Unit b) Rounding – off c) Dimensionless quantities

a) Unit

1) The digits that are known reliably plus the first uncertain digit are known as Significant figures or significant digits. The units in which the fundamental quantities are measured are called fundamental or base units and the units of measurement of all other physical quantities, which can be obtained by a suitable multiplication or division of powers of fundamental units, are called derived units.

b) Rounding – off

1) The result given by a calculator has too many figures. In no case should the result have more significant figures than the figures involved in the data used for calculation. The result of calculation with numbers containing more than one uncertain digit should be rounded off.

c) Dimensionless quantities

i) Physical quantities which have no dimensions, but have variable values are called dimensionless variables. Examples are specific gravity, strain, refractive index etc

ii) Quantities which have constant values and also have no dimensions are called dimensionless constants. Examples are π, e, numbers etc.

5. **Write the rules for rounding off.**

i) If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged.

Ex. i) 7.32 is rounded off to 7.3 ii) 8.94 is rounded off to 8.9

ii) If the digit to be dropped is greater than 5, then the preceding digit should be increased by 1

Ex. i) 17.26 is rounded off to 17.3 ii) 11.89 is rounded off to 11.9

iii) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1

Ex. i) 7.352, on being rounded off to first decimal becomes 7.4

ii) 18.159 on being rounded off to first decimal, become 18.2

iv) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even

Ex. i) 3.45 is rounded off to 3.4 ii) 8.250 is rounded off to 8.2

v) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1 if it is odd

Ex. i) 3.35 is rounded off to 3.4 ii) 8.350 is rounded off to 8.4

UNIT – 2 KINEMATICS

TWO MARKS AND THREE MARKS:

1. Explain what is meant by Cartesian coordinate system?

At any given instant of time, the frame of reference with respect to which the position of the object is described in terms of position coordinates (x, y, z) (i.e., distances of the given position of an object along the x, y, and z–axes.) is called **"Cartesian coordinate system"**

2. Define a vector. Give examples

It is a quantity which is described by both magnitude and direction. Geometrically a vector is a directed line segment. **Examples** Force, velocity, displacement, position vector, acceleration, linear momentum and angular momentum

3. Define a scalar. Give examples

It is a property which can be described only by magnitude. In physics a number of quantities can be described by scalars. **Examples** Distance, mass, temperature, speed and energy

4. Write a short note on the scalar product between two vectors.

The scalar product (or dot product) of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them. $\vec{A}.\vec{B} = ABcos\theta.$ Here, A and B are magnitudes of $\vec{A}$ and $\vec{B}$

Properties

The product quantity $\vec{A}$ and $\vec{B}$ is always a scalar. The scalar product is commutative.

5. Write a short note on vector product between two vectors.

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them.

$$\vec{C} = \vec{A} \times \vec{B} = (ABsin\theta)\hat{n}.$$

6. How do you deduce that two vectors are perpendicular?

If two vector $\dot{A}$ and $\dot{\overline{B}}$ are perpendicular to each other their scalar product. $\vec{A}.\vec{B} = 0,$ because $\cos 90^0 = 0$.

7. Define displacement and distance.

Distance is the actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

Displacement the shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time**.** It is a vector quantity**.**

8. Define velocity and speed.

Velocity: The rate of change of displacement of the particle.

$Velocity\ =\ Displacement\ /\ time\ taken$. Unit: ms^{-1}. Dimensional formula: LT^{-1}

Speed: The distance travelled in unit time. It is a scalar quantity.

9. Define acceleration.

The acceleration of the particle at an instant is equal to rate of change of velocity. It is a vector quantity. SI Unit: ms^{-2}. Dimensional formula: $M^0L^1T^{-2}$.

10. What is the difference between velocity and average velocity?

Velocity is the rate at which the position changes. But the average velocity is the displacement or position change per time ratio.

11. Define a radian?

The length of the arc divided by the radius of the arc. One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

12. Define angular displacement and angular velocity.

Angular displacement: The angle described by the particle about the axis of rotation in a given time is called angular displacement. The unit of angular displacement is radian.

Angular velocity (m)

The rate of change of angular displacement is called angular velocity.

The unit of angular velocity is radian per second ($rad\ s^{-1}$).

13. What is non uniform circular motion?

If the speed of the object in circular motion is not constant, then we have non-uniform circular motion. For example, when the bob attached to a string moves in vertical circle.

14. Write down the kinematic equations for angular motion.

1. $\boldsymbol{\omega = \omega_0 + \alpha t}$ 2. $\boldsymbol{\theta = \omega_0 t + \alpha t^2}$ 3. $\boldsymbol{\omega^2 = \omega_0^2 + 2\alpha\theta}$

15. Write down the expression for angle made by resultant acceleration and radius vector in the non uniform circular motion.

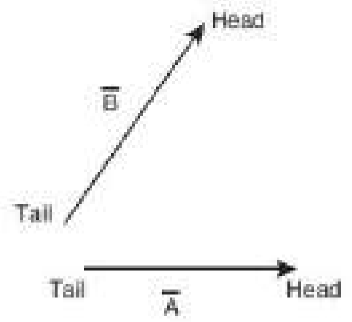
$tan\theta = \frac{a_t}{\left(\frac{v^2}{r}\right)}$ θ =Resultant acceleration angle, Centripetal acceleration $a_t = Resultant\ acceleration.$

<u>FIVE MARKS QUESTIONS</u>

01. Explain in detail the triangle law of addition.

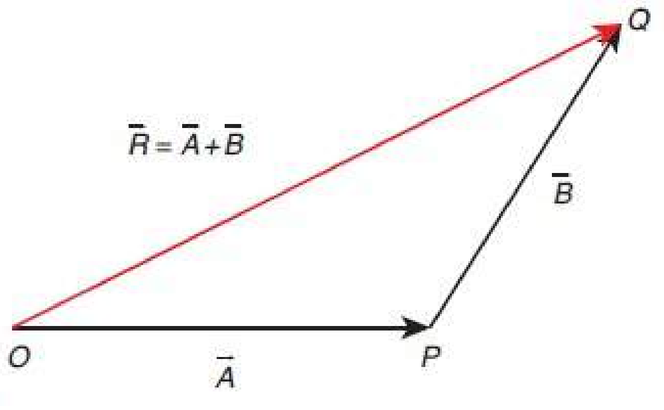
Triangular Law of addition method

Let us consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Figure.



To find the resultant of the two vectors we apply the triangular law of addition as follows:

Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle as shown in Figure.



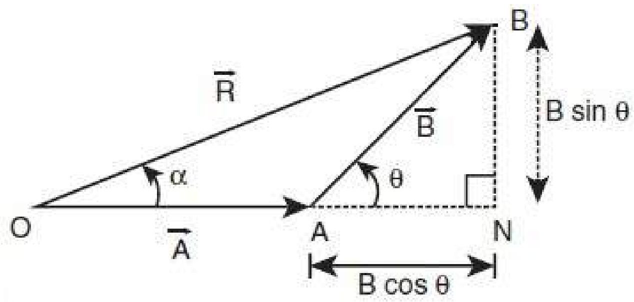
To explain further, the head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$ is the resultant vector connecting the tail of the first vector $\vec{A}$ to the head of the second vector $\vec{B}$. The magnitude of $\vec{A}$ (resultant) is given geometrically by the length of $\vec{R}$ (OQ) and the direction of the resultant vector is the angle between $\vec{R}$ and $\vec{A}$. Thus we write = $\vec{R} = \vec{A} + \vec{B}$.

$$\boxed{\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}}$$

1. Magnitude of resultant vector

The magnitude and angle of the resultant vector are determined as follows.

From Figure 2.18, consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.



From Figure

$cos\theta = \frac{AN}{B}$ $\therefore AN = Bcos\theta\ and$

$sin\theta = \frac{BN}{B} \therefore BN = Bsin\theta$ -----------(1)

For ΔOBN, we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + Bcos\theta)^2 + (Bsin\theta)^2$

$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2ABcos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2ABcos\theta$

$\Rightarrow R^2 = \sqrt{A^2 + B^2 + 2ABcos\theta}$ ------------(2)

which is the magnitude of the resultant of $\vec{A}$ and $\vec{B}$

2. Direction of resultant vectors:

If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2ABcos\theta}$ -------------(3)

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$

$tan\alpha = \frac{Bsin\theta}{A + Bcos\theta}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{Bsin\theta}{A+Bco}\right)$ ---------------(4)

1. **Discuss the properties of scalar and vector products. Properties of scalar products**

Scalar Product of Two Vectors

Definition

The scalar product (or dot product) of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having an angle θ between them, then their scalar product is defined as $\vec{A} \cdot \vec{B}$ = AB cos θ. Here, A and B are magnitudes of $\vec{A}$ and $\vec{B}$.

Properties The product quantity $\vec{A} \cdot \vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e., < 90°) and negative if the angle between them is obtuse (i.e. 90°<θ< 180°).

(i) The product quantity $\vec{\boldsymbol{A}} \cdot \vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e. $< 90°$) and negative if the angle between them is obtuse (i.e. $90° < \theta < 180°$).

(ii) The scalar product is commutative, i.e. $\vec{\boldsymbol{A}} \cdot \vec{B} = \vec{B}.\vec{\boldsymbol{A}}$.

(iii) The vectors obey distributive law i.e. $\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$

(v) The scalar product of two vectors will be maximum when $cos\theta = 1, i.e. \theta = 0°$, i.e., when the vectors are parallel;

$\left(\vec{A}.\vec{B}\right)_{max} = AB$

(vi) The scalar product of two vectors will be minimum, when $cos\theta = -1$, i.e. $\theta = 180°$.

$\left(\vec{A}.\vec{B}\right)_{min} = -AB$, when the vectors are anti-parallel.

(vii) If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product $\vec{A}.\vec{B} = 0$, because $cos90° = 0$. Then the vectors $\vec{A}$ and $\vec{B}$ are said to be mutually orthogonal.

(viii) The scalar product of a vector with itself is termed as self-dot product and is given by $\left(\vec{A}\right)^2 = \vec{A}.\vec{A} = AAcos\theta = A^2$. Here angle $\theta = 0°$.

The magnitude or norm of the vector $\vec{A}$ is $\left|\vec{A}\right| = A = \sqrt{\vec{A}.\vec{A}}$

(ix) In case of a unit vector $\hat{n}$ $\hat{n}.\hat{n} = 1 \times 1 \times cos0 = 1$. For example, $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(x) In the case of orthogonal unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$,

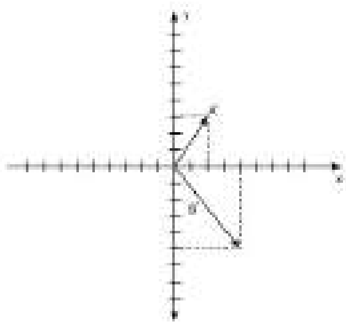
$$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 1.1cos90° = 0$$

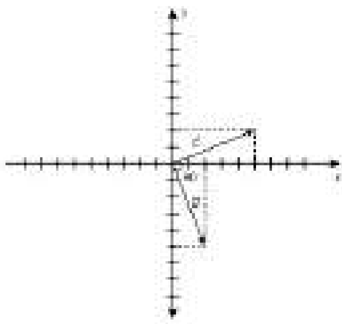
(xi) In terms of components the scalar product of $\vec{A}$ and $\vec{B}$ can be written as

$\vec{A}.\vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right).\left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) = A_xB_x + A_yB_y + A_zB_z$, with all other terms zero.

The magnitude of vectors $\left|\vec{A}\right|$ is given by

$\left|\vec{A}\right| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$





In physics, the work done by a force $\vec{F}$ to move an object through a small displacement $d\vec{r}$ is defined as,

$W = \vec{F}.d\vec{r}$

$W = Fdrcos\theta$

The work done is basically a scalar product between the force vector and the displacement vector. Apart from work done, there are other physical quantities which are also defined through scalar products.

Properties of vector (cross) product.
Definition

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. The direction of the product vector is perpendicular to the plane containing the two vectors, in accordance with the right hand screw rule or right hand thumb rule (Figure).

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product is written as $\vec{A} \times \vec{B}$ which is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A} \times \vec{B} = (ABsin\theta)\hat{n}$$

The direction $\hat{n}$ of $\vec{A} \times \vec{B}$, i.e. $\vec{C}$ is perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$ and is in the sense of advancement of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right-handed screw whose axis is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A} \times \vec{B}$ i.e. $\vec{C}$ which is illustrated in Figure.

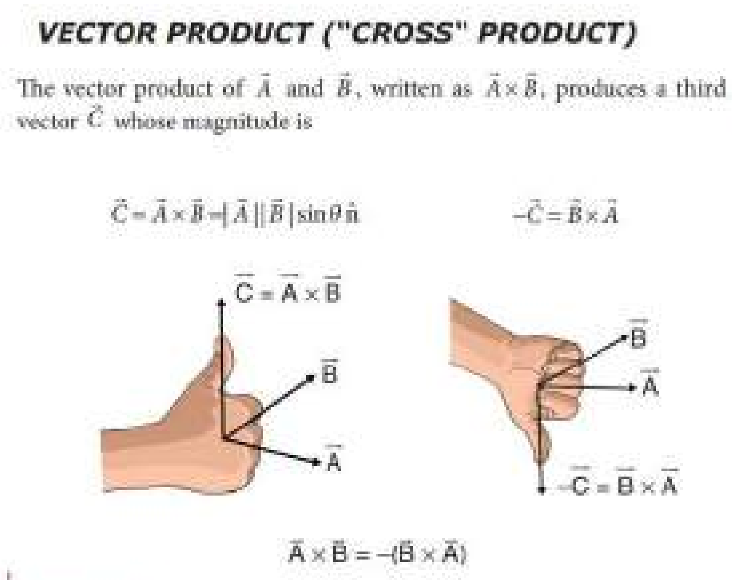


Figure 2.22 Vector product of two vectors

Properties of vector (cross) product.
A number of quantities used in Physics are defined through vector products. Particularly physical quantities representing rotational effects like torque, angular momentum, are defined through vector products.
Properties of vector (cross) product.

(i) The vector product of any two vectors is always another vector whose direction is perpendicular to the plane containing these two vectors, i.e. orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, even though the vectors $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal.

(ii) The vector product of two vectors is not commutative, i.e., $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ But,. $\vec{A} \times \ \vec{B} = -[\vec{B} \times \vec{A}]$

Here it is worthwhile to note that $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = ABsin\theta$ i.e., in the case of the product vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$, the magnitudes are equal but directions are opposite to each other.

(iii) The vector product of two vectors will have maximum magnitude when $sin\theta = 1, i.e. \theta = 90° \, i.e.,$ When the vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other.

$$(\vec{A} \times \vec{B})_{max} = AB\hat{n}$$

(iv) The vector product of two non-zero vectors will be minimum when $|sin\theta| = 0, i.e. \theta = 0°$ or $180°$.

$(\vec{A} \times \vec{B})_{max} = 0$ i.e. the vector product of two non-zero vectors vanishes, if the vectors are either parallel or antiparallel.

(v) The self-cross product, i.e., product of a vector with itself is the null vector

$$\vec{A} \times \vec{A} = AAsin0°\hat{n} = \vec{0}.$$

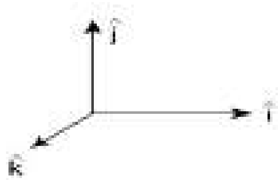
In physics the null vector $\vec{0}$ is simply denoted as zero.

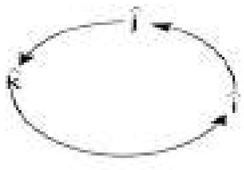
(vi) The self-vector products of unit vectors are thus zero.

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$$

(vii) In the case of orthogonal unit vectors, $\hat{\imath}, \hat{\jmath}, \hat{k}$, in accordance with the right hand screw rule:

$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{\imath} = \hat{\jmath}$





Also, since the cross product is not commutative,

$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}$ and $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

(viii) In terms of components, the vector product of two vectors $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$-\hat{\imath}(A_yB_z - A_zB_y) + \hat{\jmath}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

Note that in the $\hat{\jmath}^{th}$ component the order of multiplication is different than $\hat{\imath}^{th}$ and $\hat{k}^{th}$ components.

(ix) If two vectors $\vec{A}$ and $\vec{B}$ form adjacent sides in a parallelogram, then the magnitude of $|\vec{A} \times \vec{B}|$ will give the area of the parallelogram as represented graphically in Figure.

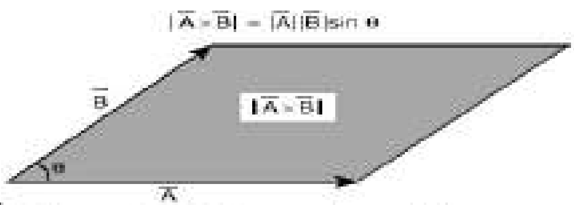


Figure 2.23 Area of parallelogram

(x) Since we can divide a parallelogram into two equal triangles as shown in the figure, the area of a triangle with $\vec{A}$ and $\vec{B}$ as sides is $\frac{1}{2}|\vec{A} \times \vec{B}|$. This is shown in the figure. (This fact will be used when we study Kepler's laws in)

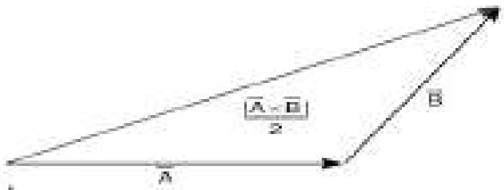


Figure 2.24 Area of triangle

(i) Torque $\vec{\tau} = \vec{r} \times \vec{F}$. Where $\vec{F}$ is force and $\vec{r}$ is position vector of a particle

(ii) Angular momentum $\vec{L} = \vec{r} \times \vec{p}$ where $\vec{p}$ is the linear momentum.

(iii) Linear Velocity $\vec{v} = \vec{\omega} \times \vec{r}$ where $\vec{\omega}$ is angular velocity.

2. Derive the kinematic equations of motion for constant acceleration.

Velocity - time relation

(i) The acceleration of the body at any instant is given by the first derivative of the velocity with respect to time,

$a = \frac{dv}{dt}$ or $dv = adt$-----------(1)

Integrating both sides with the condition that as time changes from 0 to t, the velocity changes from u to v. For the constant acceleration,

$\boldsymbol{\int_u^v dv = \int_0^t adt = a\int_0^t dt \Rightarrow [v]_u^v = a[t]_0^t}$

$\boldsymbol{v - u = at\ (or) v = u + at}$ -------------------(2)

Displacement - time relation

(ii) The velocity of the body is given by the first derivative of the displacement with respect to time.

$$v = \frac{ds}{dt}\ or\ ds = vdt$$

and since $v = u + at$,

we get $ds = (u + at)dt$ ----------------(3)

Assume that initially at time $t = 0$, the particle started from the origin. At a later time t, the particle displacement is s. Further assuming that acceleration is time-independent, we have

$\int_0^s ds = \int_0^t udt + \int_0^t atdt\ (or)\ s = ut + \frac{1}{2}at^2$ ----------------(4)

Velocity - displacement relation

(iii) The acceleration is given by the first derivative of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

[since $ds/dt = v$] where s is displacement traversed.

This is rewritten as $a = \frac{1}{2}\frac{dv^2}{ds}$

or $ds = \frac{1}{2a}d(v^2)$ ------------------(5)

Integrating the above equation, using the fact when the velocity changes from u^2 to v^2, displacement changes from 0 to s, we get

$$\int_0^s ds = \int_u^v \frac{1}{2a}d(v^2)$$

$$\therefore s = \frac{1}{2a}(v^2 - u^2)$$

$\therefore v^2 = u^2 + 2as$ ----------------(6)

We can also derive the displacement s in terms of initial velocity u and final velocity v.

From the equation (2) we can write,

$$at = v - u$$

Substitute this in equation(4), we get

$$s = ut + \frac{1}{2}(v - u)t$$

$$s = \frac{(u + v)t}{2} \ldots\ldots\ldots (7)$$

The equations (2), (4), (6) and (7) are called kinematic equations of motion, and have a wide variety of practical applications.

Kinematic equations

$$\boldsymbol{v = u + at}$$

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

$$\boldsymbol{v^2 = u^2 + 2as}$$

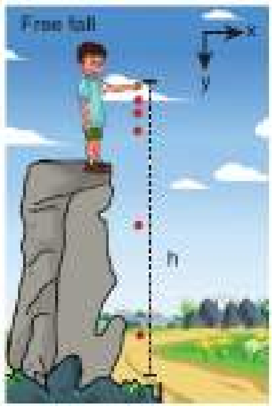
$$\boldsymbol{s = \frac{(u + v)t}{2}}$$

It is to be noted that all these kinematic equations are valid only if the motion is in a straight line with constant acceleration. For circular motion and oscillatory motion these equations are not applicable.

3. **Derive the equations of motion for a particle (a) falling vertically (b) projected vertically**

Case (1): A body falling from a height h

Consider an object of mass m falling from a height h. Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis as shown in the Figure. The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth. We can use kinematic equations to explain its motion. We have



The acceleration $\vec{a} = \hat{g}$

By comparing the components, we get

$a_x = 0, a_z = 0, a_y = g$

Let us take for simplicity, $a_y = a = g$

If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is given by

$v = u + gt$ ....(1)

$y = ut + \frac{1}{2}gt^2$ ....(2)

The square of the speed of the particle when it is at a distance y from the hill-top, is

$v^2 = u^2 + 2gy$ ----------------(3)

Suppose the particle starts from rest.

Then u = 0

Then the velocity v, the position of the particle and v^2 at any time t are given by (for a point y from the hill-top)

$v = gt$ ---------------(4)

$y = \frac{1}{2}gt^2$ ----------------(5)

$v^2 = 2gy$ -----------------(6)

The time $(t = T)$ taken by the particle to reach the ground $(for\ which\ y = h)$, is given by using equation (5),

$h = \frac{1}{2}gT^2$----------(7)

$T = \sqrt{\frac{2h}{g}}$--------------(8)

The equation (8) implies that greater the $height(h)$, particle takes more $time(T)$ to reach the ground.

For lesser height(h), it takes lesser time to reach the ground.

The speed of the particle when it reaches the ground ($y = h$) can be found using equation (2.16), we get

$v_{ground} = \sqrt{2gh}$---------------(9)

The above equation implies that the body falling from greater height(h) will have higher velocity when it reaches the ground.

The motion of a body falling towards the Earth from a small altitude (h << R), purely under the force of gravity is called free fall. (Here R is radius of the Earth

Case (ii): A body thrown vertically upwards

1) Consider an object of mass m thrown vertically upwards with an initial velocity u.
   Let us neglect the air friction.
2) In this case we choose the vertical direction as positive y axis as shown in the Figure then the acceleration $a = -g$ (neglect air friction) and g points towards the negative y axis.

The velocity and position of the object at any time t are,

$v = u - gt$ -----------(10)

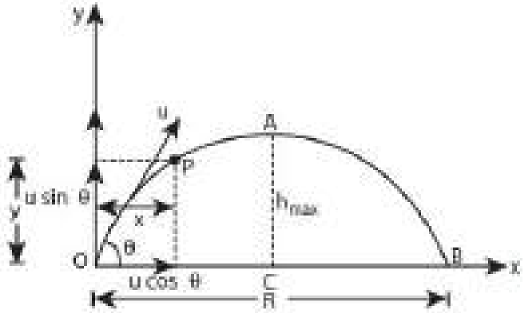
$s = ut - \frac{1}{2}gt^2$ -------------(11)

The velocity of the object at any position y (from the point where the object is thrown) is

$v^2 = u^2 - 2gy$ ----------------(12)

4. Derive the equation of motion, range and maximum height reached by the particle thrown at an oblique angle 8 **with respect to the horizontal direction.**

i) Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal.
Then



$\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}$

where $u_x = u\,cos\theta$ is the horizontal component and uy velocity.

Since the acceleration due to gravity is in the direction opposite to the direction of vertical component uy, this component will gradually reduce to zero at the maximum height of the projectile. At this maximum height, the same gravitational force will push the projectile to move downward and fall to the ground. There is no acceleration along the x direction throughout the motion. So, the horizontal component of the velocity ($ux = u\,cos\theta$) remains the same till the object reaches the ground.

Hence after the time t, the velocity along horizontal motion $vx = ux + axt = ux = u\,cos\,\theta$

The horizontal distance travelled by projectile in time t is $s_x = uxt + \frac{1}{2}\,axt^2$

Here, $sx = x, ux = u\,cos\theta, ax = 0$

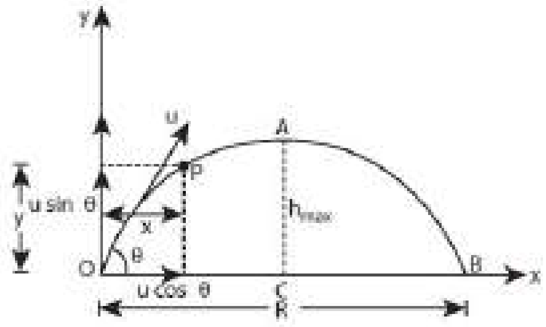


Figure 2.43. Initial velocity resolved into components

Thus, $x = ucos\theta.t$ or $t = \frac{x}{ucos\theta}$ ....(1)

Next, for the vertical motion $vy = uy + ayt$

Here $uy = u\,sin\theta, ay = -g$ (acceleration due to gravity acts opposite to the motion). Thus

Thus, $v_y = usin\theta - gt$ ....(2)

The vertical distance travelled by the projectile in the same time t is $s_y = u_yt + \frac{1}{2}\,ayt^2$

Here, $s_y = y,\ u_y = u\,sin\theta,\ a_x = -g$. Then

$y = usin\theta t - \frac{1}{2}gt^2$...........(3)

Substitute the value of t from equation (1) in equation (3), we have

$y = usin\theta\,\frac{x}{ucos\theta} - \frac{1}{2}g\,\frac{x^2}{u^2\cos^2\theta}$

$y = xtan\theta - \frac{1}{2}g\,\frac{x^2}{u^2\cos^2\theta}$ ....(4)

Thus the path followed by the projectile is an inverted parabola .

Maximum height (h_{max})

The maximum vertical distance travelled by the projectile during its journey is called maximum height. This is determined as follows:

For the vertical part of the motion,

$v_y^2 = u_y^2 + 2a_ys$

Here, $uy = u\,sin\theta$, $a = -g$, $s = h_{max}$, and at the maximum height $vy = 0$

$$\mathbf{(0)^2 = u^2 sin^2\theta = 2gh_{max}}$$

Or $\boldsymbol{h_{max}} = \boldsymbol{u^2}\,\mathbf{sin^2}\,\frac{\boldsymbol{\theta}}{\mathbf{2g}}$ ……………(5)

Time of flight (T_f)

The total time taken by the projectile from the point of projection till it hits the horizontal plane is called time of flight.

This time of flight is the time taken by the projectile to go from point O to B via point A (Figure 2.43)

We know that $S_y = u_y t + \frac{1}{2} a_y t^2$

Here, $sy = y = 0$ (net displacement in y-direction is zero), $uy = u\,sin\theta$, $ay = -g$, $t = T_f$ Then

$$\mathbf{0 = usin\theta T_f - \frac{1}{2} g T_f^2}$$

$\boldsymbol{T_f = 2u\frac{sin\theta}{g}}$ …………..(6)

Horizontal range (R)

The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R). This is found easily since the horizontal component of initial velocity remains the same. We can write

Range R = Horizontal component of velocity × time of flight = $u\,cos\theta \times Tf$

$$R = ucos\theta \times \frac{2usin\theta}{g} = \frac{2u^2 sin\theta cos\theta}{g}$$

$\therefore R = \frac{u^2 sin2}{g}$…………(7)

The horizontal range directly depends on the initial speed (u) and the sine of angle of projection (θ). It inversely depends on acceleration due to gravity ‘g’

For a given initial speed u, the maximum possible range is reached when $sin2\theta$ is maximum, $sin2\theta = 1$. This implies $2\theta = \pi/2$

or $\theta = \frac{\pi}{4}$

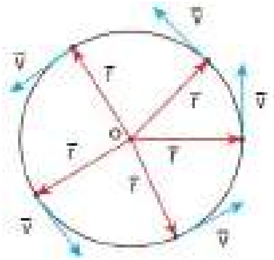
This means that if the particle is projected at 45 degrees with respect to horizontal, it attains maximum range, given by.

$R_{max} = \frac{u^2}{g}$ ……………(8)

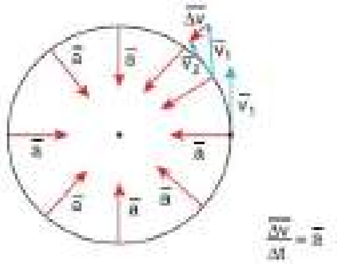
5. Derive the expression for centripetal acceleration.

Centripetal acceleration

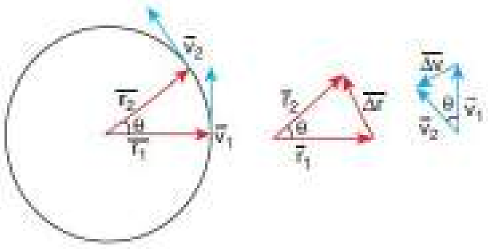
As seen already, in uniform circular motion the velocity vector turns continuously without changing its magnitude (speed), as shown in Figure 2.50.



Note that the length of the velocity vector (blue) is not changed during the motion, implying that the speed remains constant. Even though the velocity is tangential at every point in the circle, the acceleration is acting towards the center of the circle. This is called centripetal acceleration. It always points towards the center of the circle. This is shown in the Figure 2.51.



The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors **(**Figure 2.48 or Figure 2.52).



Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt, as shown in Figure. For uniform circulation motion, $r = |\vec{r}_1| = |\vec{r}_2|$ and $v = |\vec{v}_1| = |\vec{v}_2|$. If the particle moves from the position vector $\vec{r}_1$ to $\vec{r}_2$, the displacement is given by $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ and the change in velocity from $\vec{v}_1$ to $\vec{v}_2$ is given by $\Delta\vec{V} = \vec{v}_2 - \vec{v}_1$. The magnitudes of the displacement Δr and of Δv satisfy the following relation

$$\frac{\Delta r}{r} = -\frac{\Delta V}{v} = \theta$$

Here the negative sign implies that Δv points radially inward, towards the center of the circle.

$$\Delta v = -v\left(\frac{\Delta r}{r}\right)$$

Then, $a = \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right) = -\frac{v^2}{r}$

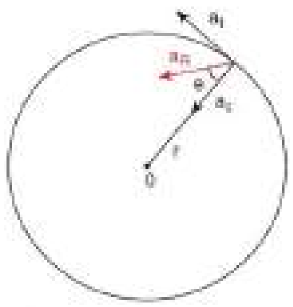
For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle about the center. Then the centripetal acceleration can be written as

$$a = -\omega^2 r$$

6. Derive the expression for acceleration in the non uniform circular motion.

Non uniform circular motion

If the speed of the object in circular motion is not constant, then we have non-uniform circular motion. For example, when the bob attached to a string moves in vertical circle, the speed of the bob is not the same at all time. Whenever the speed is not same in circular motion, the particle will have both centripetal and tangential acceleration as shown in the Figure.



The resultant acceleration is obtained by vector sum of centripetal and tangential acceleration.

Since centripetal acceleration is $\frac{v^2}{r}$, the magnitude of this resultant acceleration is given by

$$a_R = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$$

This resultant acceleration makes an angle θ with the radius vector as shown in Figure.

This angle is given by $tan\theta = \frac{a_t}{\left(\frac{v^2}{r}\right)}$.

UNIT – 3 LAWS OF MOTION

TWO MARKS AND THREE MARKS:

01. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Examples:

Inertia of Rest:

i) Passengers experience a backward push in a sudden start of bus.
ii) Tightening of seat belts in a car when it stops quickly.

Inertia of Motion:

i) Passengers experience a forward push during a sudden brake in bus.
ii) Ripe fruits fall from the trees in the direction of wind.

Inertia of Direction:

i) A stone moves tangential to Circle.
ii) When a car moves towards left, we turn to the right.

02. State Newton's second law.

The force acting on an object is equal to the rate of change of its momentum. $\vec{F} = \frac{d\vec{p}}{dt}$

03. Define One Newton

One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

04. Show that impulse is the change of momentum.

f a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

If a force (F) acts on the object in a very short interval of time (Δt), from Newton's second law in magnitude form

$Fdt = dp$ ……….(1)

Integrating over time from an initial time t_i to a final time t_f , we get

$$\int_t^f dp = \int_{t_i}^{t_f} Fdt$$

$$P_f - P_i = \int_{t_i}^{t_f} Fdt$$

P_i = initial momentum of the object at time t_i

P_f= final momentum of the object at time t_f.

$P_f - P_i = \Delta p =$Change in momentum of the object during the time interval $t_f - t_i = \Delta t$.

The integral $\int_{t_i}^{t_f} Fdt = J$ is called the impulse and it is equal to change in momentum of the object.

If the force is constant over the time interval, then

$\int_{t_i}^{t_f} Fdt = F\int_{t_i}^{t_f} dt = F(t_f - t_i) = F\Delta t$ -----(2)

$F\Delta t = \Delta p$ ……………..(3)

Equation (1) is called the 'impulse-momentum equation'.

05. Using free body diagram, show that it is easy to pull an object than to push it.

When a body is pushed at an arbitrary angle θ ($0\ to\ \pi/2$) , the applied force F can be resolved into two components as $F\ sin\theta$ parallel to the surface and F cosθ perpendicular to the surface as shown in Figure. The total downward force acting on the body is $mg\ +\ Fcos\theta$. It implies that the normal force acting on the body increases. Since there is no acceleration along the vertical direction the normal force N is equal to

$N_{push} = mg + Fcos\theta$

As a result the maximal static friction also increases and is equal to

$f_s^{max} = \mu_s N_{pu} \quad = \mu_s(mg + Fcos\theta)$ ---------(1)

Equation (1) shows that a greater force needs to be applied to push the object into motion.

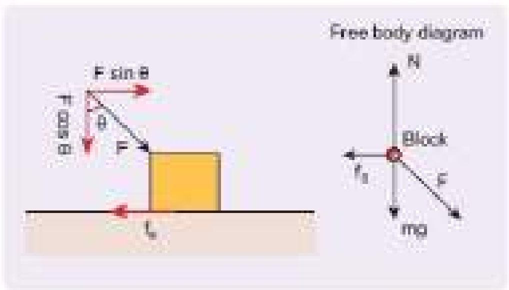


Figure 3.26 An object is pushed at an angle θ

When an object is pulled at an angle θ, the applied force is resolved into two components as shown in Figure. The total downward force acting on the object is

$N_{pull} = mg - Fcos\theta$

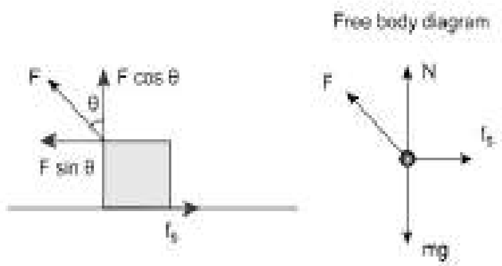


Figure 3.27 An object is pulled at an angle θ

06. **Explain various types of friction. Suggest a few methods to reduce friction.**

Static Friction ($\vec{F}_s$):

01. Static friction is the force which opposes the initiation of motion of an object on the surface.
02. When the object is at rest on the surface, only two forces act on it. They are the downward

gravitational force and upward normal force.

03. The resultant of these two forces on the object is zero.

Kinetic Friction($\vec{F}_K$):

01. When an object slides, the surface exerts a frictional force called **kinetic friction ($\vec{F}_k$)**

02. If the external force acting on the object is greater than maximum static friction, the objects begin to slide.

03. The kinetic friction does not depend on velocity.

Rolling Friction:

The force of friction that comes into act when a wheel rolls over a surface.

Methods to reduce friction:

01. By using Lubricants friction

02. By using ball bearings.

07. What is the meaning by 'pseudo force'?

Centrifugal force is called as a 'pseudo force'. A pseudo force has no origin. A pseudo force is an apparent force that acts on all masses whose motion is described using non inertial frame of reference such as a rotating reference frame.

08. State the empirical laws of static and kinetic friction.

i) The magnitude of static frictional force f_s satisfies the following empirical relation. $0 \leq f_s \leq \mu_s N_s$ where μ_s is the coefficient of static friction.

ii) The force of static friction can take any value from zero to $\mu_s N$.

iii) If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).

iv) If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.

v) When object begins to slide, the static friction (f_s) acting on the object attains maximum.

vi) The static and kinetic frictions depend on the normal force acting on the object.

vii) The static friction does not depend upon the area of contact.

09. State Newton's third law.

For every action there is an equal and opposite reaction.

10. What are inertial frames?

01. If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames.

02. Thus, there exists some special set of frames in which if an object experiences no force It moves with constant velocity or remains at rest.

11. Under what condition will a car skid on a leveled circular road

If $\frac{mv^2}{r} > \mu_s mg, or\ \mu_s < \frac{v^2}{rg} (skid)$

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid

<u>FIVE MARKS QUESTIONS:</u>

01. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.

When two particles interact with each other, they exert equal and opposite forces on each other. The particle 1 exerts force $\vec{F}_{21}$ on particle 2 and particle 2 exerts an exactly equal and opposite force $\vec{F}_{12}$ on particle 1, according to Newton's third law.

$\vec{F}_{21} = -\vec{F}_{12}$ ………………(1)

In terms of momentum of particles, the force on each particle (Newton's second law) can be written as

$\vec{F}_{12} = \frac{d\vec{p}_1}{dt}$ and $\vec{F}_{21} = \frac{d\vec{p}_2}{dt}$ ----(2)

Here $\vec{p}_1$ is the momentum of particle 1 which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. This changes due to $\vec{F}_{21}$ exerted by particle 1.

Substitute equation (1) in equation (2)

$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$ ....(3)

$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$ ....(4)

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

It implies that $\vec{P}_1 + \vec{P}_2$ = constant vector (always).

$\vec{P}_1 + \vec{P}_2$ is the total linear momentum of the two particles ($\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system. From this result, the law of conservation of linear momentum can be stated as follows.

If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{P}_{tot}$) is always a constant vector. In otherwords, the total linear momentum of the system is conserved in time. Here the word 'conserve' means that $\vec{P}_1$ and $\vec{P}_2$ can vary,in such a way that $\vec{P}_1 + \vec{P}_2$ is a constant vector.

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are called the internal forces of the system, because they act only between the two particles. There is no external force acting on the two particles from outside. In such a case the total linear momentum of the system is a constant vector or is conserved.

02. What are concurrent forces? State Lami's theorem.

Collection of forces is said to be concurrent, if the lines of forces act at a common point. Figure illustrates concurrent forces.

Concurrent forces need not be in the same plane. If they are in the same plane, they are concurrent as well as coplanar forces.

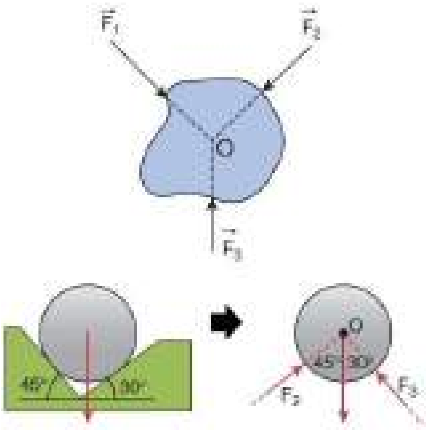


Figure 3.19 Concurrent forces

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

Let us consider three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ which act at a common point O as shown in Figure. If the point is at equilibrium, then according to Lami's theorem

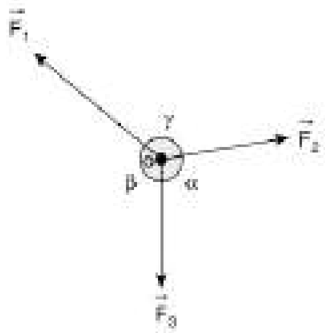


Figure 3.20 Three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting at O

$|\vec{F}_1| \propto sin\alpha$

$|\vec{F}_2| \propto sin\beta$

$|\vec{F}_3| \propto sin\gamma$

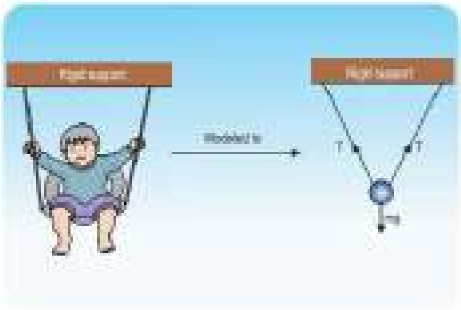
Therefore,

$$\frac{|\vec{F}_1|}{sin\alpha} = \frac{|\vec{F}_2|}{sin\beta} = \frac{|\vec{F}_3|}{sin\gamma}$$

Lami's theorem is useful to analyse the forces acting on objects which are in static equilibrium.

Example 3.14

A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.



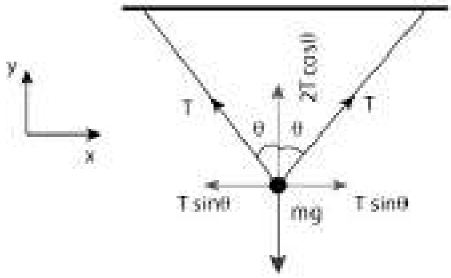
Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

i. Downward gravitational force along negative y direction (mg)

ii. Tension (T) along the two strings

These three forces are coplanar as well as concurrent as shown in the following figure.



By using Lami's theorem

$$\frac{T}{\sin(180-\theta)} = \frac{T}{\sin(180-\theta)} = \frac{mg}{\sin(2\theta)}$$

Since $\sin(180-\theta) = sin\theta$ and $\sin(2\theta) = 2sin\theta cos\theta$

We get $\frac{T}{\sin(\theta)} = \frac{mg}{2sin\theta cos\theta}$

From this, the tension on each string is $T = mg / 2cos\theta$

03. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.

Motion of Connected Bodies

When objects are connected by strings and a force F is applied either vertically or horizontally or along an inclined plane, it produces a tension T in the string, which affects the acceleration to an extent. Let us discuss various cases for the same.

Case 1: Vertical motion

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley as shown in Figure.

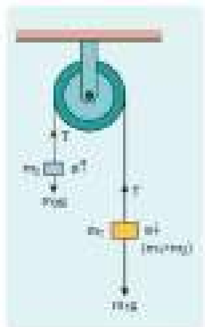


Figure 3.15 Two blocks connected by a string over a pulley

Let the tension in the string be T and acceleration a. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration a. The gravitational force m_1g on mass m_1 is used in lifting the mass m_2.

The upward direction is chosen as y direction. The free body diagrams of both masses are shown in Figure.

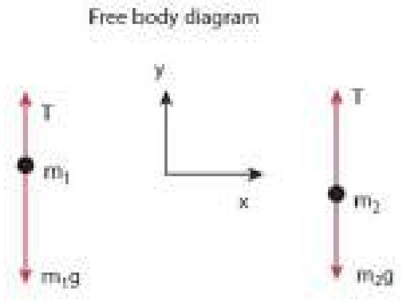


Figure 3.16 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for mass m_2

$T\vec{j} - m_2g\vec{j} = m_2a\vec{j}$ ………………(1)

The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get
$T - m^2g = m_2a$ ………………..(2)
Similarly, applying Newton's second law for mass m_1
$T\vec{j} - m_1g\vec{j} = -m_1a\hat{j}$ ……………….(3)
As mass m_1 moves downward $(-\hat{j})$, its acceleration is along $(-\hat{j})$.
By comparing the components on both sides, we get
$T - m_1g = -m_1a$
$m_1g - T = m_1a$ ………………..(4)
Adding equations (2) and (4), we get
$m_1g - m_2g = m_1a + m_2a$

$(m_1 - m_2)g = (m_1 + m_2)a$ ………..(5)

From equation (5), the acceleration of both the masses is

$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$ ………..(6)

If both the masses are equal $(m_1 = m_2)$, from equation (6)

$a = 0.$ …………………(7)

This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest.

To find the tension acting on the string, substitute the acceleration from the equation (6) into the equation (2).

$$T - m_2 g = m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g + m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \text{ …………(8)}$$

By taking $m_2 g$ common in the RHS of equation (8)

$$T = m_2 g \left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g \left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

Equation (3.12) gives only magnitude of acceleration.

For mass m_1, the acceleration vector is given by $\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right) \hat{j}$

For mass m_2, the acceleration vector is given by $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \hat{j}$

Case 2: Horizontal motion

In this case, mass m2 is kept on a horizontal table and mass m1 is hanging through a small pulley as shown in Figure. Assume that there is no friction on the surface.

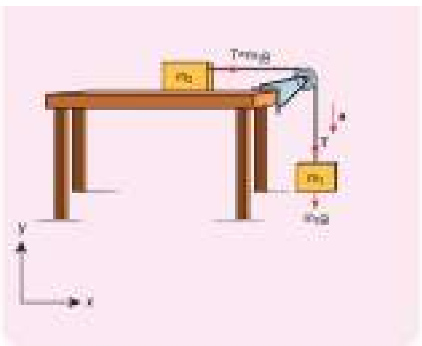


Figure 3.17 Blocks in horizontal motion

As both the blocks are connected to the unstretchable string, if m1 moves with an acceleration a downward then m2 also moves with the same acceleration a horizontally.
The forces acting on mass m2 are

i. Downward gravitational force (m2g)
ii. Upward normal force (N) exerted by the surface
iii. Horizontal tension (T) exerted by the string

The forces acting on mass m1 are

i. Downward gravitational force (m1g)
ii. Tension (T) acting upwards

The free body diagrams for both the masses is shown in Figure 3.18.

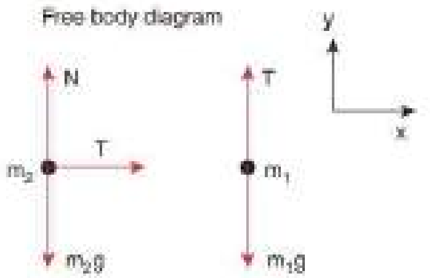


Figure 3.18 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for m_1
$T\vec{j} - m_1 g\hat{j} = -m_1 a\hat{j}$ $(along\ y\ direction)$ …………….(1)
By comparing the components on both sides of above equation,
$T - m_1 g = -m_1 a$ …………..(2)
Applying Newton's second law for m_2
$T\hat{\imath} = m_2 a\hat{\imath}$ $(along\ x\ direction)$ ……………(3)
By comparing the components on both sides of above equation,
$T = m_2 a$ ………(4)
There is no acceleration along y direction for m_2.
$N\hat{j} - m_2 g\hat{j} = 0$
By comparing the components on both sides of the above equation
$N - m_2 g = 0$
$N = m_2 g$ ……..(5)
By substituting equation (4) in equation (3), we can find the tension T

$m_2 a - m_1 g = -m_1 a$

$m_2 a + m_1 a = m_1 g$

$a = \frac{m_1}{m_1 + m_2} g$ ………(6)

Tension in the string can be obtained by substituting equation (6) in equation (4)

$T = \frac{m_1 m_2}{m_1 + m_2} g$………..(7)
Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.
This result has an important application in industries. The ropes used in conveyor belts (horizontal motion) work for longer duration than those of cranes and lifts (vertical motion).

04. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.

i) If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move.

ii) It is because of the opposing force exerted by the surface on the object which resists its motion.

iii) This force is called the frictional force which always opposes the relative motion between an object and the surface where it is placed.

iv) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ. For small angles of θ, the object may not slide down.

v) As θ is increased, for a particular value of θ, the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

vi) Consider the various forces in action here. The gravitational force mg is resolved into components parallel (mgsinθ) and perpendicular (mgcosθ) to the inclined plane.

vii) The component of force parallel to the inclined plane (mgsinθ) tries to move the object down. The component of force perpendicular to the inclined plane (mgcosθ) is balanced by the Normal force (N).

N = mg cosθ --(1)

the object just begins to move, the static friction attains its maximum value, $f_s = f_s^{max} = \mu_c$ N. This friction also satisfies the relation

$f_s^{max} = \mu_c$ mgsinθ -- (2)

Equating the right hand side of equations (1) and (2), we get

$(f^{max}) / N = \sin\theta/\cos\theta$

From the definition of angle of friction, we also know that tan0 = μ_s

in which 0 is the angle of friction.

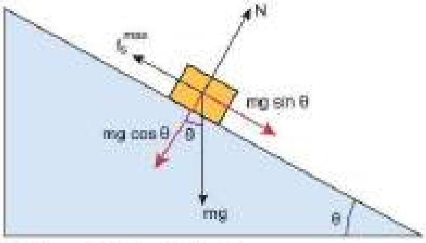


Figure 3.29 Angle of repose

05. State Newton's three laws and discuss their significance.

Newton's First Law

i) Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.

ii) This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Newton's Second Law

This law states that

The force acting on an object is equal to the rate of change of its momentum

$$\vec{F} = \frac{d\vec{p}}{dt}$$

In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{P} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$
$$\vec{F} = m\vec{a}.$$

Newton's third law

i) Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature.

ii) Newton's third law states that for every action there is an equal and opposite reaction.

iii) Here, action and reaction pair of forces do not act on the same body but on two different bodies.

iv) Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.

v) These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

06. Explain the similarities and differences of centripetal and centrifugal forces.

Centripetal force	**Centrifugal force**
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or center of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the center of the circular motion
Real force and has real effects.	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects	Origin of centrifugal force is inertia. It does not arise from interaction.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame. In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

07. Briefly explain 'centrifugal force' with suitable examples.

i) Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity m in the inertial frame (at rest).

ii) If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity m then, the stone appears to be at rest.

iii) This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$.

iv) So the total force acting on the stone in a rotating frame is equal to zero $(-m\omega^2 r + m\omega^2 r = 0)$.

v) This outward force $+m\omega^2 r$ is called the centrifugal force.

08. Briefly explain 'Rolling Friction'.

i) One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage.

ii) When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion.

iii) In rolling motion when a wheel moves on a surface, **the point of contact with surface is always at rest.**

iv) **Since the point of contact is at rest**, there is no relative motion between the wheel and surface. Hence the **frictional force is very less**. At the same time if an object moves **without a wheel**, there is a relative motion between the object and the surface.

v) As a result **frictional force is larger**. This makes it **difficult to move the object**.

vi) Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so.

vii) Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface.

viii) **Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'**. In fact, 'rolling friction' is much smaller than kinetic friction.

09. Describe the method of measuring Angle of Repose.

i)Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ. For small angles of θ , the object may not slide down.

ii) As θ is increased, for a particular value of θ, the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

iii) Consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg\ sin\theta$) and perpendicular ($mgcos\theta$) to the inclined plane.

iv) The component of force parallel to the inclined plane ($mgsin\theta$) tries to move the object down. The component of force perpendicular to the inclined plane ($mgcos\theta$) is balanced by the Normal force (N).

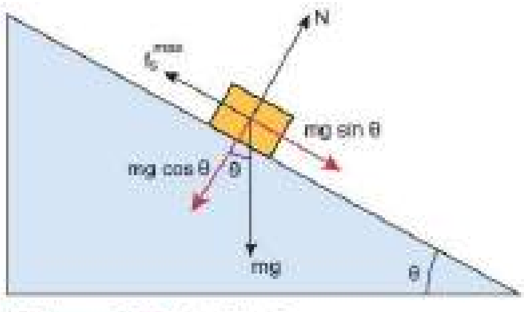


Figure 3.29 Angle of repose

Let us consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg\ sin\theta$) and perpendicular ($mg\ cos\theta$) to the inclined plane.

The component of force parallel to the inclined plane ($mg\ sin\theta$) tries to move the object down.

The component of force perpendicular to the inclined plane ($mgcos\theta$) is balanced by the Normal force (N).

$N = mgcos\theta$

When the object just begins to move, the static friction attains its maximum value

$f_s = f_s^{max} = \mu_s N$

This friction also satisfies the relation

$f_s^{max} = \mu_s mgsin\theta$

Equating the right hand side of equations (3.35) and (3.36), we get

$$(f_s^{max})/N = sin\theta/cos\theta$$

From the definition of angle of friction, we also know that

$$tan\theta = \mu_s$$

in which θ is the angle of friction.

Thus the angle of repose is the same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

10. Explain the need for banking of tracks.

In a leveled circular road, skidding mainly depends on the coefficient of static friction μs The coefficient of static friction depends on the nature of the surface which has a maximum limiting value. To avoid this problem, usually the outer edge of the road is slightly raised compared to inner edge as shown in the Figure. This is called banking of roads or tracks. This introduces an inclination, and the angle is called banking angle.

Figure 3.44 Outer edge of the road is slightly raised to avoid skidding

Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical. When the car takes a turn, there are two forces acting on the car:

a. Gravitational force mg (downwards)

b. Normal force N (perpendicular to surface)

We can resolve the normal force into two components. N cosθ and N sinθ as shown in Figure. The component $Ncos\theta$ balances the downward gravitational force 'mg' and component $Nsin\theta$ will provide the necessary centripetal acceleration. By using Newton second law

Need Banking of tracks:

$$\boldsymbol{N\cos\theta = mg}$$

$$\boldsymbol{N\sin\theta = \frac{mv^2}{r}}$$

By dividing the equations we get $\boldsymbol{\tan\theta = \frac{v^2}{rg}}$

$$\boldsymbol{v = \sqrt{rg\tan\theta}}$$

Need Banking of tracks:

The banking angle θ and radius of curvature of the road or track determines the safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding. At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding. However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

11. Calculate the centripetal acceleration of Moon towards the Earth?

The centripetal acceleration is given by $a = \frac{V^2}{r}$ This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula

$$\omega^2 R_m = a_m$$

am is centripetal acceleration of the Moon due to Earth's gravity. ω is angular velocity. R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$R_m = 60R = 60 \times 6.4 \times 10^6 = 384 \times 10^6 m$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and $T = 27.3$ days = $27.3 \times 24 \times 60 \times 60$ second

$= 2.358 \times 10^6 sec$

By substituting these values in the formula for acceleration

$$a_m = \frac{(4\pi^2)(384 \times 10^6)}{(2.358 \times 10^6)^2} = 0.00272 ms^{-2}$$

The centripetal acceleration of Moon towards the Earth is 0.00272 m s^{-2}.

UNIT – 4 WORK, ENERGY AND POWER

TWO MARKS AND THREE MARKS:

01. Explain how the definition of work in physics is different from general perception.

1. Generally any activity can be called as work
2. But in physics, work is said to be done by the force when the force applied on a body displaces it.

02. Write the various types of potential energy. Explain the formulae.

1. The energy possessed by the body due to gravitational force gives rise to gravitational potential energy $U = mgh$.
2. The energy due to spring force and other similar forces give rise to elastic potential energy.
   $U = \frac{1}{2} Kx^2$.
3. The energy due to electrostatic force on charges gives rise to electrostatic potential energy.
   $U = - E.dr$

03. Write the differences between conservative and Non-conservative forces. Give two examples each.

Conservative forces	Non-conservative forces
Work done is independent of the path	Work done depends upon the path
Work done in a round trip is zero	Work done in a round trip is not zero
Total energy remains constant	Energy is dissipated as heat energy
Work done is completely recoverable	Work done is not completely recoverable
Force is the negative gradient of potential energy	No such relation exists.

04. Explain the characteristics of elastic and inelastic collision.

Elastic Collision	Inelastic Collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Total kinetic energy is not conserved
Forces involved are conservative forces	Forces involved are non-conservative forces
Mechanical energy is not dissipated	Mechanical energy is dissipated into heat, light, sound etc.

05. Define the following
a) Coefficient of restitution b) Power c) Law of conservation of energy
d) Loss of kinetic energy in inelastic collision.

a) Coefficient of restitution

b) t is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision, i.e.,

$$e = \frac{velocity\ of\ separation\ (after\ collision)}{velocity\ of\ approach\ (before\ collision)}$$

$= \frac{v_2 - v_1}{u_1 - u_2}$

c) Power

The rate of work done or energy delivered.

$$Power(P) = \frac{Workdone(W)}{Time\ taken\ (t)}$$

d) Law of conservation of energy

Energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

e) Loss of kinetic energy in inelastic collision

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision.

Total kinetic energy before collision,

$KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ ---------(1)

Total kinetic energy after collision,

$KE_f = \frac{1}{2}(m_1 + m_2)v^2$ -----------(2)

Then the loss of kinetic energy is Loss of KE, $\Delta Q = KE_i - KE_f$

$= \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_2u_2^2$ -------------(3)

Substituting equation (1) in equation (3), and on simplifying (expand v by using the algebra $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$Loss\ of\ KE, \Delta Q = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1 - u_2)^2$............(4)

FIVE MARKS QUESTIONS

01. Explain with graphs the difference between work done by a constant force and by a variable force.

Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation,

$dW = (Fcos\theta)dr$

The total work done in producing a displacement from initial position $\boldsymbol{r_i}$ to final position $\boldsymbol{r_f}$ is,

$$W = \int_{r_i}^{r_f} dW$$

$$W = \int_{r_i}^{r_f} (Fcos\theta)dr = (Fcos\theta)\int_{r_i}^{r_f} dr = (Fcos\theta)(r_i - r_f)$$

The graphical representation of the work done by a constant force is shown in Figure. The area under the graph shows the work done by the constant force.

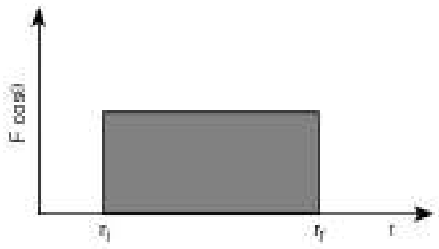


Figure 4.5 Work done by the constant force

The component $mg\ cos\theta$ and the normal force N are perpendicular to the direction of motion of the object, so they do not perform any work.

Work done by a variable force

When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation

$$dW = (Fcos\theta)dr$$

$$[Fcos\theta\ is\ the\ component\ of\ the\ variable\ force\ F]$$

where, F and θ are variables. The total work done for a displacement from initial position r_i to final position rf is given by the relation,

$$W = \int_{r_i}^{r_f} dW = \int_{r_i}^{r_f} Fcos\theta dr$$

A graphical representation of the work done by a variable force is shown in Figure. The area under the graph is the work done by the variable force.

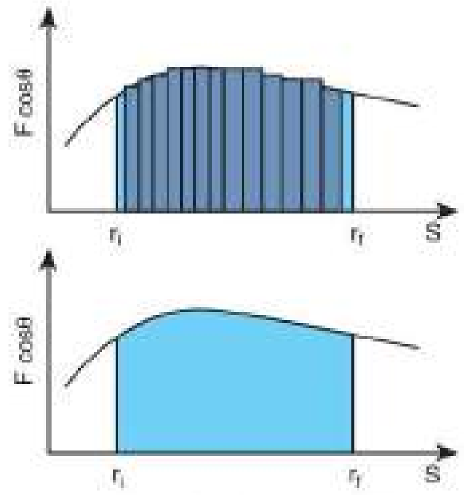


Figure 4.6 Work done by a variable force

02. State and explain work energy principle. Mention any three examples for it.

Work-Kinetic Energy Theorem

Work and energy are equivalents. This is true in the case of kinetic energy also. To prove this, let us consider a body of mass m at rest on a frictionless horizontal surface.

The work (W) done by the constant force (F) for a displacement (s) in the same direction is,

$W = Fs$ ……………(1)

The constant force is given by the equation,

$F = ma$ …………(2)

The third equation of motion can be written as,

$v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting for a in equation (2),

$F = m\left(\frac{v^2-u^2}{2s}\right)$…….(3)

Substituting equation (3) and (1),

$$w = m\left(\frac{v^2}{2s} - s\right) - m\left(\frac{u^2}{2s}s\right)$$

$w = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ………(4)

The expression for kinetic energy**:**

The term ½ mv^2 in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v).

$K.E = \frac{1}{2}mv^2$ ……….(5)

Kinetic energy of the body is always positive. From equations (4) and (5)

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Thus, $W = \Delta KE$ --------(6)

The expression on the right hand side (RHS) of equation (6) is the change in kinetic energy (ΔKE) of the body.

This implies that the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.

The work-kinetic energy theorem implies the following.

i. If the work done by the force on the body is positive then its kinetic energy increases.

ii. If the work done by the force on the body is negative then its kinetic energy decreases.

iii. If there is no work done by the force on the body then there is no change in its kinetic energy, which means that the body has moved at constant speed provided its mass remains constant.

03. Arrive at an expression for power and velocity. Give some examples for the same.
Relation between power and velocity

The work done by a force $\vec{F}$ for a displacement $d\vec{r}$ is

$W = \int \vec{F}.d\vec{r}$ …………..(1)

Left hand side of the equation can be written as

$W = \int dW = \frac{\int dW}{dt}dt$ ………….(2)

(multiplied and divided by dt)

Since, velocity is $\vec{v} = \frac{d\vec{r}}{dt}; d\vec{r} = \vec{v}dt$. Right hand side of the equation can be written as

$\int \vec{F}.d\vec{r} = \int \left(\vec{F}.\frac{d\vec{r}}{dt}\right)dt = \int \left(\vec{F}.\vec{v}\right)dt \left[\vec{v} - \frac{d\vec{r}}{dt}\right]$ ………..(3)

Substituting equation (2) and equation (3) in equation (1), we get

$\frac{\int dW}{dt}dt = \int (\vec{F}.\vec{v})dt$ ……………..(4)

$\int \left(\frac{dW}{dt} - \vec{F}.\vec{v}\right)dt = 0$

This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$$\frac{dW}{dt} - \vec{F}.\vec{v} = 0$$

Or

$$\frac{dW}{dt} = \vec{F}.\vec{v} \text{ ………………..(5)}$$

04. Arrive at an expression for elastic collision in one dimension and discuss various cases.

Elastic collisions in one dimension

Consider two elastic bodies of masses m1 and m2 moving in a straight line (along positive x direction) on a frictionless horizontal surface as shown in figure 4.16.

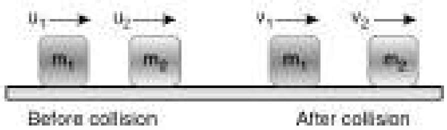


Figure 4.16 Elastic collision in one dimension

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

In order to have collision, we assume that the mass m1 moves faster than mass m_2 i.e., $u_1 > u_2$. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

	Momentum of mass m_1	Momentum of mass m_2	Total linear momentum
Before collision	$P_{t1} = m_1u_1$	$P_{t2} = m_2u_2$	$P_t = P_{t1} + P_{t2}$ $P_t = m_1u_1 + m_2u_2$
After collision	$P_{f1} = m_1v_1$	$P_{f2} = m_2v_2$	$P_f = P_{f1} + P_{f2}$ $P_f = m_1v_1 + m_2v_2$

From the law of conservation of linear momentum,

Total momentum before collision (pi) = Total momentum after collision (pf)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ …………..(1)

Or $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ………..(2)

Further,

	Kinetic energy of mass m_1	Kinetic energy of mass m_2	Total kinetic energy
Before collision	$KE_{t1} = \frac{1}{2}m_1u_1^2$	$KE_{t2} = \frac{1}{2}m_2u_2^2$	$KE_t = KE_{t1} + KE_{t2}$ $KE_t = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
After collision	$KE_{f1} = \frac{1}{2}m_1v_1^2$	$KE_{f2} = \frac{1}{2}m_2v_2^2$	$KE_t = KE_{t1} + KE_{t2}$ $KE_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ …………(3)

After simplifying and rearranging the terms,

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ ……………(4)

Using the formula $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as

$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$ …………..(5)

Dividing equation (5) by (2) gives,

$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)} = \frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$ ………(6)

$u_1 + v_1 = v_2 + u_2$ Rearranging,

$u_1 - u_2 = v_2 - v_1$

Equation can be written as

$u_1 - u_2 = -(v_1 - v_2)$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2,

$v_1 = v_2 + u_2 - u_1$………(7)

Or

$v_2 = u_1 + v_1 - u_2$ ---------(8)

To find the final velocities v_1 *and* v_2:

Substituting equation (8) in equation (2) gives the velocity of m_1 as

$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$

$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

Or $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$ -------------(9)

Similarly, by substituting (7) in equation (2) or substituting equation (9) in equation (8), we get the final velocity of m_2 as

$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$ …………(10)

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

equation(9) ⇒ $v_1 = (0)u_1 + \left(\frac{2m_2}{2m_2}\right)u_2$ …………..(11)

$v_1 = u_2$

equation(10) ⇒ $v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)u_2$…………(12)

$v_2 = u_1$

The equations (11) and (12) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$ and second body (usually called target) is at rest ($u_2 = 0$), By substituting $m_1 = m_2$ and $u_2 = 0$ in equations (9) and equations (10) we get,

From equation (9) ⇒ $v_1 = 0$ …………..(13)

From equation (10) ⇒ $v_2 = u_1$ …………..(14)

Equations (13) and (14) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3:

The first body is very much lighter than the second body $\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ then the ratio $m_1/m_2 \approx$

0 and also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{2}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_1 = \left(\frac{0 - 1}{0 + 1}\right) u_1$$

$$v_1 = -u_1 \ldots\ldots\ldots\ldots\ldots(15)$$

Similarly,

Dividing numerator and denominator of equation by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = (0)u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = 0 \ldots\ldots\ldots\ldots\ldots(16)$$

The equation implies that the first body which is lighter returns back (rebounds) in the opposite direction with the same initial velocity as it has a negative sign. The equation implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4:

The second body is very much lighter than the first body ($m_2 \ll m_1, \frac{m_2}{m_1} \ll 1$)

then the ratio $m_2/m_1 \approx 0$ and also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right) u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0) \ldots\ldots\ldots\ldots(17)$$

$$v_1 = \left(\frac{1 - 0}{1 + 0}\right) u_1 + \left(\frac{0}{1 + 0}\right)(0)$$

$$v_1 = u_1$$

Similarly, Dividing numerator and denominator of equation (4.58) by m_1, we get

$$v_2 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}\right)(0)$$

$$v_2 = \left(\frac{2}{1+0}\right)u_1$$

$$v_2 = 2u_1 \text{ ……………(18)}$$

The equation (17) implies that the first body which is heavier continues to move with the same initial velocity. The equation (18) suggests that the second body which is lighter will move with twice the initial velocity of the first body. It means that the lighter body is thrown away from the point of collision.

05. What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

In a perfectly inelastic or completely inelastic collision, the objects stick together permanently after collision such that they move with common velocity. Let the two bodies with masses m1 and m_2 move with initial velocities u_1 and u_2 respectively before collision. After perfect inelastic collision both the objects move together with a common velocity v as shown in Figure.

Since, the linear momentum is conserved during collisions,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

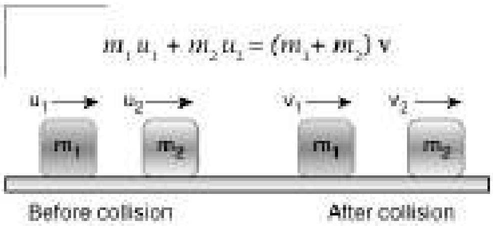


Figure 4.17 Perfect inelastic collision in one dimension

	Velocity		Linear momentum	
	Initial	Final	Initial	Final
Mass m_1	u_1	v	m_1u_1	m_1v
Mass m_2	u_2	v	m_2u_2	m_2v
Total			$m_1u_1 + m_2u_2$	$(m_1 + m_2)$ v

The common velocity can be computed by

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

1) In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e.,

2) Momentum is conserved. Kinetic energy is not conserved in elastic collision. Mechanical energy is dissipated into heat, light, sound etc. When a light body collides against any massive body at rest it sticks to it.
3) Total kinetic energy before collision ≠ Total kinetic energy after collision
4) Even though kinetic energy is not conserved but the total energy is conserved.
5) loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc.
6) if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision.
7) For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

UNIT – 5 MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES

TWO MARKS AND THREE MARKS:

01. Define center of mass.

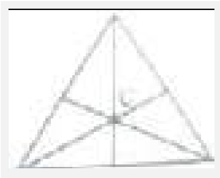
A point where the entire mass of the body appears to be concentrated.

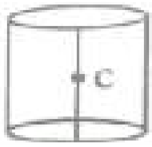
02. Find out the center of mass for the given geometrical structures.

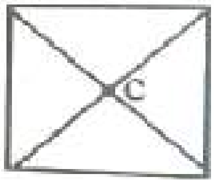
a) Equilateral triangle Lies in center

b) Cylinder
Lies on its central axis

c) Square
Lies at their diagonals meet







03. Define torque and mention its unit.

Torque is defined as the moment of the external applied force about a point or axis of rotation. The expression for torque is, $\vec{\tau} = \vec{r} \times \vec{F}$. Its unit is Nm.

04. What are the conditions in which force cannot produce torque?

The torque is zero when $\vec{r}$ *and* $\vec{F}$ are parallel or anti-parallel. If parallel, then θ=0 and sin 0 = 0. If anti-parallel, then θ = 180 and sin 180 = 0. Hence, τ = 0. The torque is zero if the force acts at the reference point. i.e. as r= 0, τ = 0.

05. What is the relation between torque and angular momentum?

An external torque on a rigid body fixed to an axis produces rate of change of angular momentum in the body about that axis. $T = \frac{dL}{dt}$.

06. What is equilibrium?

i) A rigid body is said to be in mechanical equilibrium when both its linear momentum and angular momentum remain constant.

ii) When all the forces act upon the object are balanced, then the object is said to be an equilibrium.

07. Give any two examples of torque in day-to-day life.

i) Opening and closing of a door about the hinges ii) Turning of a nut using a wrench
iii) Opening a bottle cap (or) water top

08. How do you distinguish between stable and unstable equilibrium?

Stable equilibrium	Unstable equilibrium
Linear momentum and angular momentum are zero.	Linear momentum and angular momentum are zero.
The body tries to come back to equilibrium if slightly disturbed and released.	The body cannot come back to equilibrium if slightly disturbed and released.
The center of mass of the body shifts slightly higher if disturbed from equilibrium.	The center of mass of the body shifts slightly lower if disturbed from equilibrium.
Potential energy of the body is minimum and it increases if disturbed.	Potential energy of the body is not minimum and it decreases if disturbed

09. Define couple.

Pair of forces which are equal in magnitude but **opposite in direction** and separated by a **perpendicular distance** so that **their lines of action do not coincide** that causes a turning effect is called a couple

10. State principle of moments.

When an object is in equilibrium the sum of the anticlockwise moments about a turning point must be equal to the sum of the clockwise moments.

11. Define center of gravity.

The point at which the entire weight of the body acts irrespective of the position and orientation of the body.

12. Mention any two physical significance of moment of inertia.

i) For rotational motion, moment of inertia is a measure of rotational inertia.

ii)The moment of inertia of a body is not an invariable quantity. It depends not only on the mass of the body, but also on the way the mass is distributed around the axis of rotation.

13. What is radius of gyration?

The radius of gyration of an object is the perpendicular distance from the axis of rotation to an equivalent point mass, which would have the same mass as well as the same moment of inertia of the object.

14. State conservation of angular momentum.

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

15. What are the rotational equivalents for the physical quantities, (i) mass and (ii) force?

i) For mass : Moment of inertia , $I = mr^2$ ii) For Force : Torque v = I α

16. What is the condition for pure rolling?

(i) The combination of translational motion and rotational motion about the center of mass.

(or) (ii) The momentary rotational motion about the point of contact.

17. What is the difference between sliding and slipping?

Sliding is the case when $vCM > R\omega$ (or $v_{TRANS} > v_{ROT}$). The translation is more than the rotation.

Slipping is the case when $v_{CM} < R\omega$ (or $v_{TRANS} < v_{ROT}$). The rotation is more than the translation.

<u>FIVE MARKS QUESTIONS</u>

01. Explain the types of equilibrium with suitable examples

Translational equilibrium

1) Linear momentum is constant 2) Net force is zero

Rotational equilibrium

1) Angular momentum is constant 2) Net torque is zero

Static equilibrium

1) Linear momentum and angular momentum are zero
2) Net force and net torque are zero

Dynamic equilibrium

1) Linear momentum and angular momentum are constant
2) Net force and net torque are zero

Stable equilibrium

1) Linear momentum and angular momentum are zero
2) The body tries to come back to equilibrium if slightly disturbed and released
3) The center of mass of the body shifts slightly higher if disturbed from equilibrium
4) Potential energy of the body is minimum and it increases if disturbed

Unstable equilibrium

1) Linear momentum and angular momentum are zero
2) The body cannot come back to equilibrium if slightly disturbed and released
3) The center of mass of the body shifts slightly lower if disturbed from equilibrium
4) Potential energy of the body is not minimum and it decreases if disturbed

Neutral equilibrium

1) Linear momentum and angular momentum are zero
2) The body remains at the same equilibrium if slightly disturbed and released
3) The center of mass of the body does not shift higher or lower if disturbed from equilibrium
4) Potential energy remains same even if disturbed

02. Explain the method to find the center of gravity of a irregularly shaped lamina.

1) The center of gravity of a uniform lamina of even an irregular shape by pivoting it at various points by trial and error.
2) The lamina remains horizontal when pivoted at the point where the net gravitational force acts, which is the center of gravity

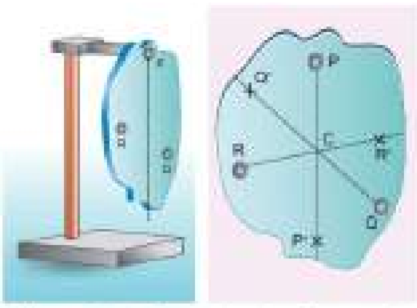


Figure 5.18. Determination of center of gravity of plane lamina by suspending

3) When a body is supported at the center of gravity, the sum of the torques acting on all the point masses of the rigid body becomes zero. Moreover the weight is compensated by the normal reaction force exerted by the pivot.

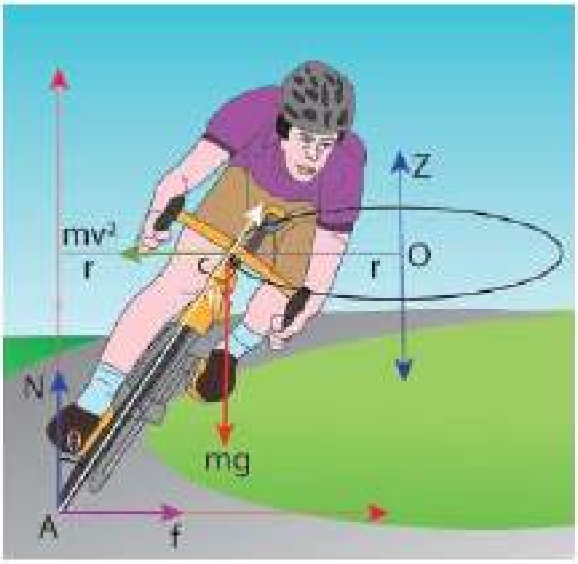
4) The body is in static horizontal.

5) There is also another way to determine the center of gravity of an irregular lamina. If we suspend the lamina from different points like P, Q, R , the vertical lines PP', QQ', RR' all pass through the center of gravity.
6) Here, reaction force acting at the point of suspension and the gravitational force acting at the center of gravity cancel each other and the torques caused by them also cancel each other.

03. Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.

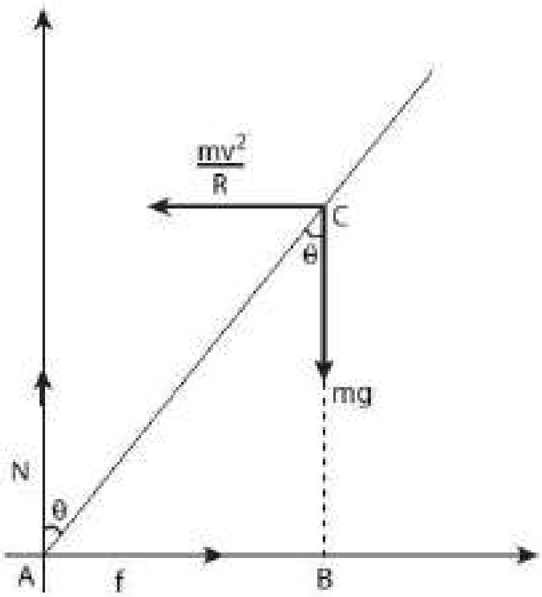
Bending of Cyclist in Curves

Let us consider a cyclist negotiating a circular level road (not banked) of radius r with a speed v. The cycle and the cyclist are considered as one system with mass m. The center gravity of the system is C and it goes in a circle of radius r with center at O. Let us choose the line OC as X-axis and the vertical line through O as Z-axis as shown in Figure.



The system as a frame is rotating about Z-axis. The system is at rest in this rotating frame. To solve problems in rotating frame of reference, we have to apply a centrifugal force (pseudo force) on the system which will be mv^2/r

This force will act through the center of gravity. The forces acting on the system are, (i) gravitational force (mg), (ii) normal force (N), (iii) frictional force (f) and (iv) centrifugal force (mv^2/r) . As the system is in equilibrium in the rotational frame of reference, the net external force and net external torque must be zero. Let us consider all torques about the point A in Figure.



For rotational equilibrium,

$$\vec{\tau}_{net} = 0$$

The torque due to the gravitational force about point A is $(mgAB)$ which causes a clockwise turn that is taken as negative. The torque due to the centripetal force is (mv^2/rBC) which causes an anticlockwise turn that is taken as positive.

$$-mgAB + \frac{mv^2}{r}BC = 0$$

$$mgAB = \frac{mv^2}{r}BC$$

From ΔABC,

$AB = ACsin\theta$ and $BC = ACcos\theta$

$$mgACsin\theta - \frac{mv^2}{r}ACcos\theta$$

$$tan\theta = \frac{v^2}{rg}$$

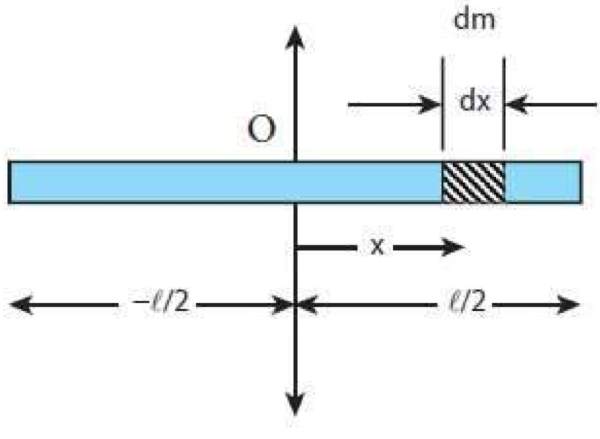
$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

While negotiating a circular level road of radius r at velocity v, a cyclist has to bend by an angle θ from vertical given by the above expression to stay in equilibrium (i.e. to avoid a fall).

04. Derive the expression for moment of inertia of a rod about its center and perpendicular to the rod.

Moment of Inertia of a Uniform Rod

Let us consider a uniform rod of mass (M) and length (l) as shown in Figure 5.21. Let us find an expression for moment of inertia of this rod about an axis that passes through the center of mass and perpendicular to the rod.



First an origin is to be fixed for the coordinate system so that it coincides with the center of mass, which is also the geometric center of the rod. The rod is now along the x axis. We take an infinitesimally small mass (dm) at a distance (x) from the origin. The moment of inertia (dI) of this mass (dm) about the axis is,

$$dI = (dm)x^2$$

As the mass is uniformly distributed, the mass per unit length (λ) of the rod is, $\lambda = M/l$

The (dm) mass of the infinitesimally small length as, $dm = \lambda dx = M/l\, dx$

The moment of inertia (I) of the entire rod can be found by integrating dI,

$$I = \int dl = \int (dm)x^2 = \int \left(\frac{M}{l}dx\right)x^2$$

$$I = \frac{M}{l}\int x^2 dx$$

As the mass is distributed on either side of the origin, the limits for integration are taken from $-l/2$ to $l/2$.

$$I = \frac{M}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$I = \frac{M}{l}\left[\frac{l^3}{24} - \left(-\frac{l^3}{24}\right)\right] = \frac{M}{l}\left[\frac{l^3}{24} + \frac{l^3}{24}\right]$$

$$I = \frac{M}{l}\left[2\left(\frac{l^3}{24}\right)\right]$$

$$I = \frac{1}{12}ml^2$$

05. Derive the expression for moment of inertia of a uniform ring about an axis passing through the center and perpendicular to the plane.
Moment of Inertia of a Uniform Ring

Let us consider a uniform ring of mass M and radius R. To find the moment of inertia of the ring about an axis passing through its center and perpendicular to the plane, let us take an infinitesimally small mass (dm) of length (dx) of the ring. This (dm) is located at a distance R, which is the radius of the ring from the axis as shown in Figure.

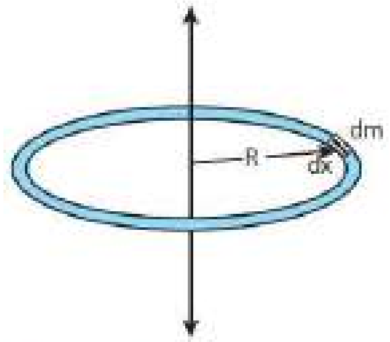


Figure 5.22 Moment of inertia of a uniform ring

The moment of inertia (dI) of this small mass (dm) is,

$$dI = (dm)R^2$$

The length of the ring is its circumference ($2\pi R$). As the mass is uniformly distributed, the mass per unit length (λ) is,

$$\lambda = \frac{mass}{length} = \frac{M}{2\pi R}$$

The mass (dm) of the infinitesimally small length is, $dm = \lambda dx = \frac{M}{2\pi R} dx$

Now, the moment of inertia (I) of the entire ring is,

$$I = \int dl = \int (dm)R^2 = \int \left(\frac{M}{2\pi R} dx\right) R^2$$

$$I = \frac{Mr}{2\pi} \int dx$$

To cover the entire length of the ring, the limits of integration are taken from 0 to 2πR.

$$I = \frac{MR}{2\pi} \int_0^{2\pi R} dx$$

$$I = \frac{MR}{2\pi} [x]_0^{2\pi R} = \frac{MR}{2\pi} [2\pi R - 0]$$

$$I = MR^2$$

$\frac{MR}{2n}[2nR - 0]$

$I = MR^2$

06. Derive the expression for moment of inertia of a uniform disc about an axis passing through the center and perpendicular to the plane.

Moment of Inertia of a Uniform Disc

Consider a disc of mass M and radius R. This disc is made up of many infinitesimally small rings as shown in Figure 5.23. Consider one such ring of mass (dm) and thickness (dr) and radius (r). The moment of inertia (dI) of the small ring is,

$$dI = (dm)r^2$$

As the mass is uniformly distributed, the mass per unit area (σ) is, $\sigma = \frac{mass}{area} = \frac{M}{\pi R^2}$

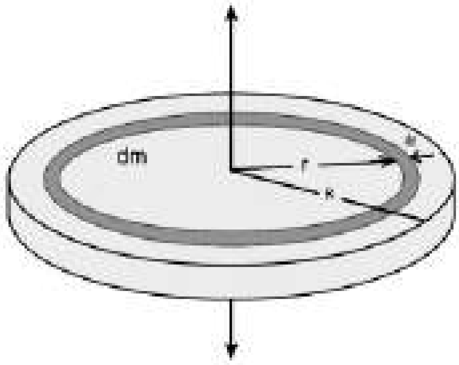


Figure 5.23 Moment of inertia of a uniform disc

The mass of the infinitesimally small ring is,

$$dm = \sigma 2\pi r dr = \frac{M}{\pi R^2} 2\pi r dr$$

where, the term ($2\pi r\, dr$) is the area of this elemental ring ($2\pi r$ is the length and dr is the thickness). dm = $2M/R^2\, rdr$

$$dI = \frac{2M}{R^2} r^3 dr$$

The moment of inertia (I) of the entire disc is,

$$I = \int dl$$

$$I = \int_0^R \frac{2M}{R^2} r^3 dr = \frac{2M}{R^2} \int_0^R r^3 dr$$

$$I = \frac{2M}{R^2} \left[\frac{r^4}{4}\right]_0^R = \frac{2M}{R^2} \left[\frac{R^4}{4} - 0\right]$$

$$I = \frac{1}{2} M R^2$$

07. Discuss conservation of angular momentum with example.

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

$$\tau = \frac{dL}{dt}$$

If $\tau = 0$ then, L=constant

As the angular momentum is L = I ω, the conservation of angular momentum could further be written for initial and final situations as,

$$I_i\omega_i = I_f\omega_f \ (or) \ I_\omega = constant$$

The above equations say that if I increases ω will decrease and vice-versa to keep the angular momentum constant.

There are several situations where the principle of conservation of angular momentum is applicable.

1) One striking Example : The ice dancer spins slowly when the hands are stretched out and spins faster when the hands are brought close to the body. Stretching of hands away from body increases moment of inertia, thus the angular velocity decreases resulting in slower spin.
2) When the hands are brought close to the body, the moment of inertia decreases, and thus the angular velocity increases resulting
in faster spin.

08. State and prove parallel axis theorem.

As the moment of inertia depends on the axis of rotation and also the orientation of the body about that axis, it is different for the same body with different axes of rotation. We have two important theorems to handle the case of shifting the axis of rotation.

(i) Parallel axis theorem:

Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

If I_C is the moment of inertia of the body of mass M about an axis passing through the center of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation,

$$I = I_c + Md^2$$

Let us consider a rigid body as shown in Figure 5.25. Its moment of inertia about an axis AB passing through the center of mass is IC. DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of IC. For this, let us consider a point mass m on the body at position x from its center of mass.

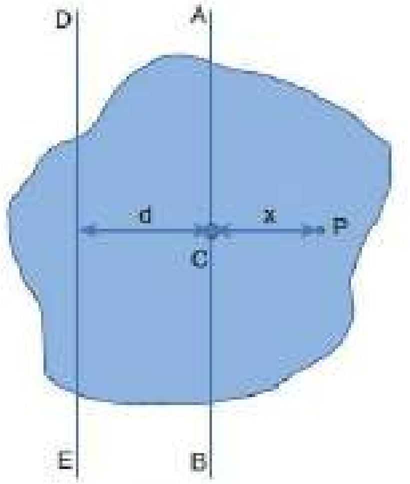


Figure 5.25 Parallel axis theorem

The moment of inertia of the point mass about the axis DE is, $m(x + d)^2$.
The moment of inertia I of the whole body about DE is the summation of the above expression.

$$I = \sum m(x + d)^2$$

This equation could further be written as,

$I = \sum m(x^2 + d^2 + 2xd)$

$I = \sum(mx^2 + md^2 + 2dmx)$

$I = \sum mx^2 + \sum md^2 + 2d\sum mx$

Here, $\sum mx^2$ is the moment of inertia of the body about the center of mass. Hence, $I_C = \sum mx^2$

The term, ∑mx = 0 because, x can take positive and negative values with respect to the axis AB. The summation ($\sum mx$) will be zero.

Thus, $I = I_C + \sum md^2 = I_C + (\sum m)d^2$

Here, Σm is the entire mass M of the object (∑m = M)

$I = I_C + Md^2$

09. State and prove perpendicular axis theorem.

(ii) Perpendicular axis theorem:
This perpendicular axis theorem holds good only for plane laminar objects.
The theorem states that the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.
Let the X and Y-axes lie in the plane and Z-axis perpendicular to the plane of the laminar object. If the moments of inertia of the body about X and Y-axes are I_X and I_Y respectively and I_Z is the moment of inertia about Z-axis, then the perpendicular axis theorem could be expressed as,

$$I_Z = I_X + I_Y$$

To prove this theorem, let us consider a plane laminar object of negligible thickness on which lies the origin (O). The X and Y-axes lie on the plane and Z-axis is perpendicular to it as shown in Figure 5.26. The lamina is considered to be made up of a large number of particles of mass m. Let us choose one such particle at a point P which has coordinates (x, y) at a distance r from O.

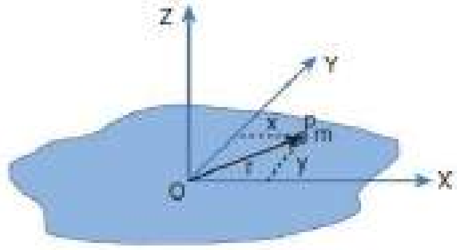


Figure 5.26 Perpendicular axis theorem

The moment of inertia of the particle about Z-axis is, mr2

The summation of the above expression gives the moment of inertia of the entire lamina about Z-axis as, $I_z = \sum mr^2$

Here, $r^2 = x^2 + y^2$

Then, $I_z = \sum m(x^2 + y^2)$

$I_z = \sum mx^2 + \sum my^2$

In the above expression, the term $\sum mx^2$ is the moment of inertia of the body about the Y-axis and similarly the term $\sum \text{my}^2$ is the moment of inertia about X-axis. Thus,

$I_X = \sum \text{my}^2$ and $I_Y = \sum mx^2$

Substituting in the equation for I_z gives,

$I_Z = I_X + I_Y$

Thus, the perpendicular axis theorem is proved.

10. Discuss rolling on inclined plane and arrive at the expression for the acceleration.
Rolling on Inclined Plane

Let us assume a round object of mass m and radius R is rolling down an inclined plane without slipping as shown in Figure 5.37. There are two forces acting on the object along the inclined plane. One is the component of gravitational force (mg sinθ) and the other is the static frictional force (f). The other component of gravitation force (mg cosθ) is cancelled by the normal force (N) exerted by the plane. As the motion is happening along the incline, we shall write the equation for motion from the free body diagram (FBP) of the object.

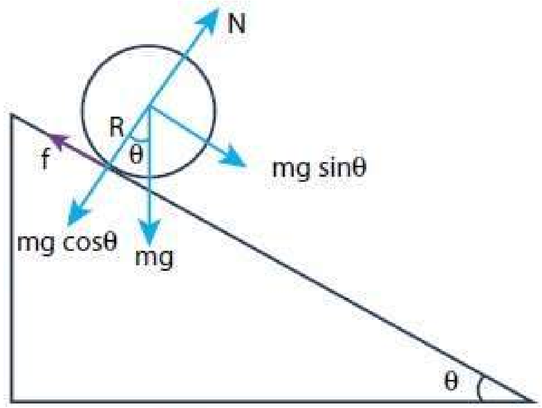


Figure 5.37 Rolling on inclined plane

For translational motion, $mg\ sin\theta$ is the supporting force and f is the opposing force,

$$mg\ sin\theta\ -\ f\ =\ ma$$

For rotational motion, let us take the torque with respect to the center of the object. Then mgsinθ cannot cause torque as it passes through it but the frictional force f can set torque of Rf.

$$Rf = I\alpha$$

By using the relation, a = r α, and moment of inertia I = mK2, we get,

$$Rf = mK^2\frac{a}{R};\ \ f = ma\left(\frac{K^2}{R^2}\right)$$

Now equation (5.59) becomes,

$$mg\ sin\theta - ma\left(\frac{K^2}{R^2}\right) = ma$$

$$mg\ sin\theta = ma + ma\left(\frac{K^2}{R^2}\right)$$

$$a\left(1 + \frac{K^2}{R^2}\right) = gsin\theta$$

After rewriting it for acceleration, we get,

$$a = \frac{gsin\theta}{\left(1 + \frac{K^2}{R^2}\right)}$$

We can also find the expression for final velocity of the rolling object by using third equation of motion for the inclined plane. $v^2\ =\ u^2\ +\ 2as$. If the body starts rolling from rest, u = 0. When h is the vertical height of the incline, the length of the incline s is, $s\ =\ h/sin\theta$

$$v^2 = 2\frac{gsin\theta}{\left(1+\frac{K^2}{R^2}\right)}\left(\frac{h}{sin\theta}\right) = \frac{2gh}{1+\frac{K^2}{R^2}}$$

By taking square root,

$$v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}$$

The time taken for rolling down the incline could also be written from first equation of motion as, $v = u + at$. For the object which starts rolling from rest, $u = 0$. Then,

$$t = \frac{v}{a}$$

$$t = \left(\sqrt{\frac{2gh}{1+\left(\frac{K^2}{R^2}\right)}}\right)\left(\frac{\left(1+\frac{K^2}{R^2}\right)}{gsin\theta}\right)$$

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{gsin^2\theta}}$$

The equation suggests that for a given incline, the object with the least value of radius of gyration K will reach the bottom of the incline first.